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Nonwandering flows of some spaces

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Abstract

In this paper, Our effort is studying nonwandering flows, Planar flows, and their properties. We have shown that the set of periodic (noncritical) points is open. When S is connected, such a flow has a simple characterization; namely, it is nonwandering. We have given conditions on some spaces by their flows that proves when a space is disconnected.

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1 Introduction

The theory of prolongation, introduced by T. Ura [1], has proven to be a rather useful apparatus in studying the structure of dynamical systems. In [2], the first author studied planar flows in which the positive prolongation of each point coincided with the closure of the positive semitrajectory through the point. Such flows were referred to as flows of characteristic 0^+. Such flows were subsequently studied over more general phase spaces in [3], [4], [5], [6], and [7]. Knight [8] carried on a similar study for planar flows of characteristic 0; these are flows where the prolongation of each point coincides with the closure of the trajectory through the point. The structure of these flows turned out to be surprisingly simple. In this paper, we study nonwandering flows, planar flows and their properties. An interesting characterization, which is somewhat surprising, is that if the phase space is Hausdorff then the flow is nonwandering if and only if the positive prolongation of each point coincides with its negative prolongation.We have shown that the set of periodic (noncritical) points is open. When S is connected, such a flow has a simple characterization; namely, it is nonwandering if and only if every point of $R^2 - S$ lies on a cycle surrounding S ; then we prove that if S is unbounded and the flow is nonwandering, S is disconnected.

Definition 1.1. Dynamic System(Continues Flow) Let X be a topological space and let R denote the additive group of real num- bers with the usual topology. The pair (X, π) is called a dynamical system or a continuous flow if $\pi : X \times R \to X$ is a continuous mapping such that for each $x \in X$ and $s, t \in R$, $\pi(x, 0) = x$ and $\pi(\pi(x, s), t) = \pi(x, s+t)$.

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For convenience we shall denote $\pi(x,t)$ by x.t. For each $x \in X$, we let $C^+(x)(C^-(x))(C(x))$ denote the positive trajectory (negative trajectory) (trajectory) through x. We let $K^+(x) = \overline{C^+(x)}$, $K^-(x) = \overline{C^-(x)}$, and $K(x) = \overline{C(x)}$. The positive limit set (negative limit set) (limit set) of x is denoted by $L^+(x)(L^-(x))(L(x))$. We denote the positive prolongation (negative prolongation) (prolongation) of $mathrmx$ by $D^+(x)(D^-(x))(D(x))$. Similarly, the positive prolongational limit set (negative prolongational limit set) (prolongational limit set) of x shall be denoted by $J^+(x)(J^-(x))(J(x))$.

Definition 1.2. positively stable subset For a point x of X (a subset M of X) we let $\eta(x)(\eta(M))$ denote the neighborhood filter of x (of M). A subset M of X is said to be positively stable if to each $U \in \eta(M)$ corresponds $V \in \eta(M)$ such that $C^+(V) \subset U$. The negative and bilateral versions are defined similarly.

Definition 1.3. Poisson stable point A point x of X is said to be Poisson stable (positively Poisson stable) (negatively Poisson stable) if $x \in L^+(x) \cap L^-(x)(x \in L^+(x))(x \in L^-(x))$. A point x of X is said to be nonwandering if $x \in J^+(x)$. A flow (X,π) is said to be nonwandering if each of its points is nonwandering.

We let int(M) denote the interior of a subset M of $X, \overline{M}$ the closure of M, and ∂M the boundary of M. In particular, if x is a cyclic (i.e. periodic but not critical) point of a planar flow, then int $(C(x))(\text{ext}(C(x)))$ shall denote the bounded (unbounded) component of the complement of the Jordan curve $C(x)$. The trajectory of a cyclic point is called a cycle. Throughout this paper the phase space X of any dynamical system will be assumed to be Hausdorff.

2 Nonwandering and Plana flows

Lemma 2.1. *For any flow (X,π) , the following two conditions are equivalent:*
(a) $D^+(x) \subset D^-(x)$ for all $x \in X$;
(b) $D^+(x) = D^-(x)$ for all $x \in X$.

Theorem 2.2. *For any flow (X,π) , the following statements are mutually equivalent:*
(i) for each $x \in X$, $D^+(x) = D^-(x)$;
(ii) the flow is nonwandering;
(iii) the set of nonwandering points is dense in X.

Theorem 2.3. *If the phase space ofa nonwandering dynamical system is metric and either locally compact or complete, then the set of Poisson stable points is dense in X.*

we assume a given planar flow (R^2,π) . The set of critical points of this flow shall be denoted by S, and that of cyclic points by P . For a point $x \in P$, we shall let $M_x = C(x) \cup$ int $(C(x))$, so that $M_x = \overline{\text{int}(C(x))}$. We say that a cyclic trajectory $C(x)$ surrounds a set F if $F \subset$ int $(C(x))$. We further assume that $R^2 \neq S$. We note that periodic points are necessarily nonwandering. In planar flows a point is Poisson stable if and only if it belongs to $P \cup S$.

Theorem 2.4. *The planar flow (R^2,π) is nonwandering if and only if the set $P \cup S$ is dense in R^2 .*

46th Annual Iranian Mathematics Conference
25-28 August 2015
Yazd University
Poster Nonwandering flows of some spaces pp.: 3–4

Lemma 2.5. *For each $x \in R^2$, $L^{\pm}(x)$ is a cycle if and only if $x \in P$.*

Lemma 2.6. *For each $x_0 \in P$, there exists an invariant neighborhood U of M_{x_0} such that for each $x \in U - M_{x_0}$, $C(x)$ is a cycle surrounding M_{x_0} with int $(C(x)) \cap S =$ int $(C(x_0)) \cap S$.*

Lemma 2.7. *For any $x_0 \in P$, there exists an invariant neighborhood W of $C(x_0)$ such that for each $x \in W\cap$ int $(C(x_0))$, $C(x)$ is a cycle with int $(C(x)) \cap S =$ int $(C(x_0)) \cap S$*

Theorem 2.8. *For any $x_0 \in P$, there exists an invariant neighborhood V of $C(X_0)$ such that for each $x \in V$, $C(x)$ is a cycle with int $(C(x)) \cap S =$ int $(C(x_0)) \cap S$.*

We note that The set P is open.

Lemma 2.9. *Let S_0 be the set of critical points surrounded by the cycle $C(x_0)$. Then all other points within $C(x_0)$ are cyclic provided every cycle within $C(x_0)$ surrounds S_0 .*

3 Main results

Definition 3.1. A nonempty subset S_0 of S_0 is said to be a central set if it has a neighborhood N such that for each $x \in N - S_0$, $C(x)$ is a cycle surrounding S_0 . Further, S_0 is called global if we can have $N = R^2$; otherwise, it is called local.

Theorem 3.2. *Let $x \in P$. If* $\mathrm{S_x} = \mathrm{int}\ (C(x)) \cap S$ *has only a finite number of components, then at least one of them is a central set.*

Theorem 3.3. *If S has only a finite number of components, then at least one of them is a central set.*

Notation: If S is finite, then some point of S is a Poincare center.
Our effort is proving the following theorem.

Theorem 3.4. *Let (R^2, π) be a planar flow in which the set S, $S \neq R^2$, of critical points is connected. Then (R^2, π) is nonwandering if and only if S is a global central set which is a simply connected continuum, and $P \cup S = R^2$.*

Proof. Suppose that (R^2, π) is nonwandering. Then it follows from notation that S is a central set. In order to show that S is a global center it suffices to show that $P \cup S = R^2$. We note that $S \subset$ int $(C(x))$ for every $x \in P$, as S is connected and int $(C(x)) \cap S \neq \emptyset$. It follows from Lemma 2.9 that for each $x \in P$, int $(C(x)) - S \subset P$. Set $\mathrm{A} = P \cup S$. Then, A is open since $\mathrm{A} = \cup$ int $(C(x))|x \in P$ and P is open by previous notations. Next, we wish to show that $\partial \mathrm{A} = \emptyset$. Assume $\partial \mathrm{A} \neq \emptyset$, and let $x_0 \in \partial \mathrm{A}$. Since A is open, $\partial \mathrm{A}$ does not contain any cyclic or critical points. Furthermore, $L(x_0) \subset \partial \mathrm{A}$, as $\partial \mathrm{A}$ is invariant and closed. Therefore, we must have $L(x_0) = \emptyset$ (see [6, p.184]). But this implies that $C(x_0)$ separates the plane into two open invariant regions V_1 and V_2 (see e.g. [8, 1.7]). Assume, without loss of generality, that $S \subset V_1$. By Theorem 2.4, the nonempty open set V_2 contains a point $\tilde{z} \in P$. But then $\overline{\mathrm{int}(C(\tilde{z}))} \cap S \neq \emptyset$ implies that $\overline{\mathrm{int}(C(\tilde{z}))} \cap V_1 \neq \emptyset \neq \overline{\mathrm{int}(C(\tilde{z}))} \cap V_2$. Further, $\overline{\mathrm{int}(C(\tilde{z}))} \cap \partial \mathrm{A} = \emptyset$, as $\overline{\mathrm{int}(C(\tilde{z}))} \subset P \cup S$. Therefore, $\overline{\mathrm{int}(C(\tilde{z}))} \subset V_1 \cup V_2$, contradicting the fact that $\overline{\mathrm{int}(C(\tilde{z}))}$ is connected. The fact that S is a simply connected continuum is clear, since $S = \cap\{\overline{\mathrm{int}(C(x))}|x \in P$. The converse implication is obvious from Theorem 2.4. □

Notation: Let (R^2, π) be a planar flow with one critical point s_0 . Then (R^2, π) is nonwandering if and only if s_0 is a global Poincare center.

Result: If S is unbounded and the flow is nonwandering, S is disconnected.

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