The Effect of DC Component on CMOS Injection-Coupled LC Quadrature Oscillator (IC-QO)

Marzieh Chaharboor
MS Degree Electrical and Electronics Engineering, Hadaf University, Sari, Iran
Marzieh.Chaharboor@gmail.com

Saman Mokhtabad
MS Degree Electrical and Electronics Engineering, Hadaf University, Sari, Iran
Saman.Mokhtabad@gmail.com

Hojat GHonoodi
PhD Degree Electrical and Electronics Engineering, NIT University, Babol, Iran
h.ghonoodi@gmail.com

Abstract
This paper creates a different insight to improve phase noise of Injection-Coupled quadrature oscillators (QOs). In fact, there are several phase noise functions and the important parameter is carrier power that considered here. The QO is analyzed and the mismatches between LC tanks that are the main proofs of phase error in this oscillator are shown. The main aim of this paper is focused on the reduction of phase noise by considering DC term. It is shown that the DC level which ignored in the most previous works is also important to improve phase noise by the carrier power. With due attention in the previous equations the phase noise can be reduced and the phase error can be cancelled or controlled by adjusting bias current. On the other word as a result, is obtained that increasing of the drain current and the voltage of LC tank decrease the phase noise and the phase error simultaneously. To confirm the proposed idea and analysis, a $5.5\,\text{GHz}$ QO is designed and simulated using $8.10\,\mu\text{m}$ TSMC CMOS technology. The simulation results show confirmation of the proposed idea.

Keywords: Quadrature, Phase Noise, Carrier Power, DC Component, Phase Error
Introduction

Quadrature outputs of LC oscillators are the most important sections of many communication systems (Razavi, B., 1991). A more common method to produce quadrature outputs is IC-QO mechanism, because of its lower phase noise than the other types (Chamas, I. R. and Raman, S., 1991). The Barkhausen’s phase criterion implies the $^\circ$ degree phase difference between IC-QO outputs. In an LC oscillator, the noise is originated in three blocks: the lossy LC tank, the transistors of differential pair and the tail current source (Lee, T. H., 1992). The quadrature LC oscillator without any mismatch have compensation of losses, and the argument of impedances $R, L, C, gm$ for phase difference is $\pm \pi/2$ with no phase error, but in two coupled oscillators with mismatches, we don’t have full compensation of losses and this is a quadrature error, $\Delta \phi$. But the important point is considering the drain current that contains DC current. The phase noise of each side reduced in this work by consideration of DC term that this term in the most cases of studies is ignored. It can be seen how it will improve phase noise, phase error and even effects on some mismatches here. This paper is organized as follows: The LC Quadrature Outputs and Phase Error, introduces the basic of LC quadrature outputs and the relation between phase noise and phase error for them. In Phase Noise and Carrier Power, the phase noise of LC oscillator is analyzed and the carrier power is taken into consideration as a crucial parameter. In Drain Current and DC Component, IC-QO mechanism is considered for the idea of this paper and the drain current is adjusting the tank voltage under effects of DC current. The Proposed Idea describes how based on the previous analysis, phase noise will be reduced and as result phase error will be improved and mismatches will be lower or cancelled. To make sure, a GHz source injection QO is simulated and the results will be shown.

LC Quadrature Outputs and Phase Error

Passing two differential outputs at frequency of $\omega_0$ through a frequency divider is a conventional method of generating quadrature outputs at frequency of $\omega_1$. The basic of this method is very simple. The two differential outputs have a phase of $\lambda \circ\circ\circ$ and therefore dividing them by $\gamma$ generates two signals with phase difference of $\lambda \circ\circ\circ$ (Mazzanti A. et al, 1993). The mismatch like device mismatches and parasitic effects cause to find out new approaches in order to reduce the phase error. Figure $\gamma$ shows the quadrature oscillator phasor diagram with mismatches. The phase difference deviates from $\lambda \circ\circ\circ$ because of the mismatches (Oliveira, L. B et al, 1993).
With considering a phase error between outputs of two oscillators following equations can be written:

\[ \Delta \theta_1 = \theta_1 - \theta \]  
\[ \Delta \theta_2 = \theta_2 - \theta \]  
\[ \theta_2 - \theta_1 = \left( \frac{\pi}{2} + \Delta \phi \right) \]

An RLC circuit with high parallel resistance has a high quality factor, and a small derivation from the resonance gives a significant phase variation, as shown in fig 9. In coupled LC oscillators with high Q resonators, small mismatches produced a high quadrature error. In first quadrature oscillators with low Q integrated inductors can reach to a good quadrature relationship (Rofougaran, A. et al, 7991). But with considering equations (1) and (3) and (3), a relationship between phase noise and phase error is obtained and implies that the phase error and the mismatch also can be reduced by the good phase difference. So a good quadrature relationship can be reached by a low phase noise, too.

**Figure 1 - Quadrature oscillator phasor diagram with mismatches**

**Phase Noise and Carrier Power**
To consider the phase noise of coupled oscillator, it is better to investigate a single LC oscillator phase noise, first. Each differential pair switches the tail current $I_T$ into the branch of LC resonators. This current is proportioned to the drain current of differential transistors. The transistors are ideal switchers that connect in parallel RLC tanks. The oscillator output voltage is approximately sinusoidal but the LC tank acts as a filter thus it represents as a square wave form (Rael, J. J. and Abidi, A. A., 1990). The voltage amplitude of LC tank can be written as (Hajimiri, A. and Lee, T. H., 1999):

$$V_{\text{Tank}} = \frac{4}{\pi} I_T R_p$$  \hspace{1cm} (4)

The voltage harmonics are attenuated by LC tank in each oscillator so the impedances of inductors and the capacitors are canceled and only paralleled resistances are leave (Rael, J. J. and Abidi, A. A., 1990) as shown in figure $\gamma$.

![Figure $\gamma$ – Behavior of RLC tank in resonant frequency](image)

To calculate the phase noise contribution of tank, assuming that the noise source is the thermal noise (Lee, T. H., 1992) and the equation (6) represents a current source across the tank with spectral density:

$$S(i_n) = \frac{4KT}{R_p}$$  \hspace{1cm} (6)

And the voltage noise source is as:

$$S(v_n) = S(i_n) \cdot |Z|^2$$  \hspace{1cm} (7)

Where $Z$ is the tank impedance. The small offset frequencies $\omega_m$ will be as:

$$\omega_m \ll \frac{\omega_0}{2Q}$$  \hspace{1cm} (7)

And the impedance of LC tank is (Razavi, B., 1991) (Lee, T. H., 1994):

$$|Z(\omega_0 + \omega_m)|^2 \approx R_p^2 \frac{1}{4Q^2} \frac{\omega_0^2}{\omega_m^2}$$  \hspace{1cm} (8)

The $Q$ is defined as equation (9) (Razavi, B., 1991):

$$Q = \frac{\omega_0}{2} \sqrt{\left(\frac{d\Delta}{d\omega}\right)^2 + \left(\frac{d\theta}{d\omega}\right)^2}$$  \hspace{1cm} (9)
Where \( A = |Z(j\omega)| \), \( \theta = \arg[Z(j\omega)] \), \( \omega_0 = \sqrt{L/C} \) is the resonance frequency. In an LC oscillator \( \frac{dA}{d\omega} = 0 \) and for \( Q = Q(\omega_0) \) (Razavi, B., 1989):

\[
Q_0 = \frac{\omega_0}{2} \left| \frac{d\theta}{d\omega} \right|_{\omega=\omega_0} = R_p \sqrt{\frac{C}{L}} = \frac{R_p}{\omega_0 L} \tag{71}
\]

\[
Q_0 = R_p \sqrt{\frac{C}{L}} = \frac{R_p}{\omega_0 L} \tag{71}
\]

Typically, the losses in the capacitors are so low and they will be ignored and because the losses in the inductors are considered, the resonator quality factor is determined mainly by inductor and the parallel resistance is obtained from the inductor quality factor (Lee, T. H., 1992). Using equations (71) and (72):

\[
S(v_n) = \frac{4KT}{P_{\text{Carrier}}} \left| \frac{R_p}{2Q} \frac{\omega_0}{\omega_m} \right|^2 = 4KTP_{\text{Carrier}} \left( \frac{\omega_0}{2Q\omega_m} \right)^2 \tag{72}
\]

From above equation can conclude that increasing \( Q \) leads to noise spectral density reduction when all other parameters keep without any changes. It shows that the output noise is corresponding to frequency that is due to LC tank filtering and the spectral density is inversely proportional to the square of the offset frequency. This behavior is due to voltage frequency response of an RLC tank rolls off as \( \frac{1}{f^2} \) to each side of the center frequency (Lee, T. H., and A. Hajimiri, 2001).

But another method in order to decrease the phase noise can be achieved by one of the most used and well-known phase noise model, Leeson-Cutler semi empirical equation (Leeson, D. B., 1992) (Baghdady, E. J. et al., 1995) (Cutler, L. S. and Searle, C. L., 1992). It is based on the assumption that the oscillator is a linear time invariant system. The following equation for \( \ell(\omega_m) \) is obtained (Hajimiri, A. and Lee, T. H., 2001):

\[
\ell(\omega_m) = 10 \log \left\{ \frac{2FKT}{P_{\text{Carrier}}} \left[1 + \left( \frac{\omega_0}{2Q\omega_m} \right)^2 \right] \left[1 + \left( \frac{\omega_0}{\omega_m} \right)^3 \right] \right\} \tag{73}
\]

Where \( k \) is Boltzmann constant, \( T \) is absolute temperature, \( P_{\text{Carrier}} \) is carrier power dissipated in the resistive part of the tank, \( \omega_0 \) is oscillation frequency, \( Q \) is quality factor, \( \omega_m \) is offset from the carrier, \( \omega_0/\gamma \) is corner frequency between \( \gamma/\gamma' \) and \( \gamma/\gamma' \) zones of the noise spectrum and \( F \) is empirical parameter called excess noise factor which includes nonlinear effects for LC oscillators (Leenaerts, D. et al., 2001). The phase noise is usually obtained from division by the carrier power as shown in equation (73). The carrier power equation is as:

\[
P_{\text{Carrier}} = \frac{V_{\text{tank}}^2}{R_p} \tag{74}
\]

As is shown in equation (74) the power and the square of voltage of tank are in direct proportional, so from equation (73) and (74) can concluded that with increasing the voltage of tank, phase noise could be decreased and this significant point will be used for the idea of this paper. The main point
that is used in this paper is the voltage of LC tank and the power that is in direct proportional to the square of this voltage.

**Drain Current and DC Component**

Typically, quadrature waveform is generated from the outputs of two coupled LC oscillators and the LC QOs are attractive due to their low phase noise (Rofougaran A. et al, \( \ddagger \ddagger \ddagger \ddagger \)) (Mazzanti A. et al, \( \ddagger \ddagger \ddagger \ddagger \)) (Chamas, I. R. and Raman, S., \( \ddagger \ddagger \ddagger \ddagger \)) (Djurhuus T. et al, \( \ddagger \ddagger \ddagger \ddagger \)) (Romano L. et al, \( \ddagger \ddagger \ddagger \ddagger \)). Different methods have been proposed to couple two CMOS LC tank oscillators and the coupling is performed in several approaches. The super harmonic coupling using tail resonator technique is proposed in previous works (Gierkink, S. L. et al, \( \ddagger \ddagger \ddagger \ddagger \)). The cross-coupled differential voltage-controlled oscillators operate as a frequency divider for the signals injected at the common source node (Ghonoodi, H. and Miar-Naimi, H., \( \ddagger \ddagger \ddagger \ddagger \)).

In this work, the IC-QO mechanism is considered because it can acts as a combination of two frequency dividers and also two frequency doublers so that the coupling transistor pairs operate as frequency doublers (Chamas, I. R. and Raman, S., \( \ddagger \ddagger \ddagger \ddagger \)). Figure 4 represents a frequency divider diagram and operation.

![Figure 4-Block diagram of a frequency divider](image)

The injection current of IC-QOs can be calculated by the equation (\( \ddagger \ddagger \ddagger \ddagger \)):

\[
I_{inj1,2} = \frac{K_{p,n}}{2} (V_{m1,2} - V_{th})^2
\]  

(75)

Where \( K_{p,n} = \mu_{p,n} C_{ox} \) and \( V_{m1,2} \) is the maximum value of output voltage of each sides of \( \ddagger \ddagger \ddagger \ddagger \) and \( \ddagger \ddagger \ddagger \ddagger \). So the injection current in an oscillatory condition can be obtained as (Ghonoodi, H. and Miar-Naimi, H., \( \ddagger \ddagger \ddagger \ddagger \)):

\[
I_{inj1,2}(t) = I_m \cos(2\omega t + \phi_{1,2}) + I_m
\]

(71)

\[
I_m = \frac{I_{inj1,2}}{2}
\]

(72)

Where \( \phi_1 \) and \( \phi_2 \) are the phases of the second-order frequency component of injection current in oscillators. In equation (71), the injection current is considered as sum of a DC level and a sinusoidal waveform. Sinusoidal approximation of the injection current has been used in previous works but the DC term is ignored (Chamas, I. R. and Raman, S., \( \ddagger \ddagger \ddagger \ddagger \)). But this section shows that the DC component directly contributes in calculations and effects on outputs (Ghonoodi, H. and Miar-Naimi, H., \( \ddagger \ddagger \ddagger \ddagger \)).
The differential structure of cross-coupled causes a frequency of twice oscillator frequency for the injection current at the source. The IC-QO topology generates four quadrature outputs but because of the mismatches, the relative phase difference deviates from $90^\circ$.

Assuming the phase error of $\Delta \phi$ from quadrature condition between outputs of two oscillators, so following equations could be written as:

\[ V_1(t) = V_{m1} \cos(\omega t + \theta_1) \]
\[ V_2(t) = V_{m2} \cos(\omega t + \theta_2) \]
\[ \theta_2 - \theta_1 = \left( \frac{\pi}{2} + \Delta \phi \right) \]

Where $\omega$ is angular frequency and $\Delta \phi$ is phase deviation amount from $90^\circ$. If assumed that the currents inject to the source node, they will be switched by the cross coupled core transistors, as if they are multiplied by a square wave of frequency of $\omega$. When the switching transistors are turned on, they conduct the whole source current. The drain current of switching pairs with considering the DC term can be written as:

\[ I_{d(1)}(t) = I_{max(1)}(t) + I_{mix2(1)}(t) + I_{dc(1)}(t) \]
\[ I_{d(2)}(t) = I_{max(2)}(t) + I_{mix2(2)}(t) + I_{dc(2)}(t) \]

According to equations (78) and (79) drain currents consist of $I_{max(1)}(t)$, $I_{max(2)}(t)$ and $I_{dc}$ current for both oscillators and they are source injected current, capacitor current and DC current respectively. Referring to (Ghonoodi, H. and Naimi, H. M., 2015) with considering Fourier’s theory $I_{dc}$ can be written as:

\[ I_{dc(1,2)}(t) = \frac{2}{\pi} (I_{R(1,2)} - I_{m(1,2)}) \cos(\omega t + \theta_{1,2}) \]

Note that the DC current at the source node consists of the tail bias current and injection current, while $I_{max(1)}(t)$ in equations (78) and (79) is the drain current raised by injection current and indeed, $I_{max(1)}(t)$ is derived from the multiplying the source current and the square wave due to gate voltage (Ghonoodi, H. and Miar-Naimi, H., 2015). As resultant, if $Z$ is considered as the impedance of tank, the voltage of each LC tanks will be as:

\[ V_{1\pm} = -I_{d(1\pm)} Z_1(j\omega) \]
\[ V_{2\pm} = -I_{d(2\pm)} Z_2(j\omega) \]

Where $V_{1\pm}$ is the phasor voltage of LC tank, $V_{1\pm}$, $t$ (t) and $I_{d\pm}$ is the phasor of drain current, $I_{d\pm}$, $t$ (t) , Also the phasor relationship between drain current and voltage of tank can be as equation (80) (Ghonoodi, H. and Miar-Naimi, H., 2015):

\[ \angle V_{1(2)}(t) = \angle I_{d(1,2)}(t) + \angle Z_{L(1,2)}(j\omega) \]
So the DC current is valuable in calculating of the drain current. Considering DC current causes the drain current increases and this leads to voltage of tank to be increased according to represented equations. In this section, it is shown that the amplitude of drain current raise by the injection current, capacitor current and DC current that causes the better performance of tank voltage. The important role of the DC level for drain current and $V_{tank}$ observed here as another significant point for the purposed idea of this paper.

**The Proposed Idea**

Variation of circuit parameters makes the prediction of phase noise difficult (Razavi, B., 1991). Two typical phase noise equations mentioned in equations (79) and (83). Most of the phase noise equations often characterized in terms of the single side band noise spectral density, $\ell (\omega)$ that is expressed in decibels below the carrier per hertz (dBc/Hz). So the total characterization can be valid for all types of oscillation could be considered as:

$$\ell (\omega_m) = \frac{P(\omega_m)}{P(\omega_0)}$$

Where $P (\omega_m)$ is the single sideband noise power at a distance of $\omega_m$ from the carrier (in Hz bandwidth, $P (\omega_c)$ is the carrier power (Oliveira, L. B et al., 1998). Regards to most of the phase-noise equations can conclude that whenever carrier power increased the phase noise will be decreased. According to equation (79) and they are in inversely proportional to each other. As mentioned in equation (81) the carrier power is in direct relation with the voltage of LC tank.

The effects of DC current on drain current and voltage of tank have been shown by equations (97), (99) (Note that $V_7$ is $V_{tank}$). The equations (97) and (99) also show that the drain currents are affected by this DC component.

From the results of analysis in previous sections, considering the DC term causes increasing of the carrier power and based on the phase noise equations, the phase noise can be reduced and also the outputs will be improved respectively. In previous works the relationship between phase difference and $\Delta \phi$ were obtained ($\Delta \phi \approx \Delta \theta$) in some cases, (Oliveira, L. B et al., 1998) and also refers to equations (79) and (99), $\theta = \theta_9 - \theta_7 = (\pi / 3 + \Delta \phi)$ phase error is in direct relation with phase differences of two sides 7 and 9, so the reduction of phase noise of each oscillator causes the better phase errors, thus the mismatches will be lower in some cases and small mismatches will be removed.

**Simulation Results**

As described in the Drain Current and DC Component, Injection-Coupled Quadrature Oscillators can be considered as a combination of two frequency dividers and two frequency doublers which the coupling transistor pairs works as frequency doublers. In IC-QO structure, the NMOS and PMOS have similar mechanism, but PMOS coupling has several advantages over NMOS coupling and the main advantage is that, at the source node there is no current sharing.

In order to simulate the result of study, the IC-QO in figure 5 was implemented. The transistors Mcp7 and Mcp9 are the coupling transistors which generate the frequency of $\omega_0$ (twice the oscillation frequency) and inject the current to the source of Msw7 and Msw9 as oscillation core component. The oscillation frequency $\omega_0$, $\phi$ GHz @ $\lambda$ MHz offset is considered as fundamental frequency and the coupled oscillator have TSMC $\lambda$ standard parameters. To prove the analytical results, the above IC-QO has been simulated in different experiments. Constant quality factor ($Q = \lambda$) is considered. The coupling and switching transistors have same aspect ratio (W/L) of
\[700\, \mu \text{m}/0.78\, \mu \text{m}\]. The LC tank comprise of a \(70\)-nH inductor and \(290\)-fF capacitor. These parameters are considered constants for all simulation and just the value of the DC level will be changed to confirm the proposed idea.

As a result, it is apparent in figure 1 that with increasing the DC level, the value of Phase Noise will be decreases. As well as, with decreasing the phase noise, the phase error will decrease respectively and also small mismatches are cancelled as is observable in figure 6.

Figure 5 – Proposed CMOS Injection-Coupled LC Quadrature Oscillator

Figure 6 – Plot of Phase Noise with DC level changes
Conclusion
In this paper, an IC-QO is investigated with a deeper insight and more precise observation. Analysis show that the phase noise is a function of drain current that consists of DC component and also the phase error and mismatches in LC tanks are dependent to this current. The effect of decreasing the amount of phase noise would be favorable for the quadrature oscillator outputs and phase error, too. By adjusting the tail bias current in QOs, this can be observed as it has been shown in analytical and simulation results that the phase noise and phase error decreased by increasing the DC component, while the quality factor (Q) is keep unchanged. Due to phase noise reduction, the improvement of outputs performance and excluding the effect of mismatches would be achievable. To verify the proposed idea, an Injection-Coupled quadrature oscillator with TSMC CMOS 0.78 µm standard in 5.5 GHz frequency with considering different value of DC level is employed. It is obvious that the simulation results confirm the proposed attitude.

References


