Passive balancing of a rotating mechanical system using genetic algorithm

H. Taplak a, S. Erkaya b, İ. Uzmay c

a Vocational School of Kayseri, Erciyes University, 38039 Kayseri, Turkey
b Faculty of Engineering, Department of Mechatronics Engineering, Erciyes University, 38039, Kayseri, Turkey
c Faculty of Engineering, Department of Mechanical Engineering, Erciyes University, 38039, Kayseri, Turkey

Received 12 December 2011; revised 6 September 2012; accepted 1 October 2012

KEYWORDS
Unbalanced disc; Optimization; Bearing vibration; Genetic algorithm; Rotating machinery.

Abstract
The main objective of this study is to perform the passive balancing of a rotating mechanical system having multi-discs. Reducing the bearing vibrations is considered an optimization problem, and the Genetic Algorithm (GA) approach is used for solving it under the appropriate constraints. Therefore, bearing amplitudes are formulated as an objective function, and the eccentricity directions of 2nd, 3rd, ..., nth discs are defined as design variables relative to the first one. The conditions making the forces at the bearings minimum are taken as the nonlinear constraints in the optimization process. On the other hand, a rotating mechanical system having three unbalanced discs is evaluated as a numerical example to show the effectiveness of the proposed approach. Commercial software, ADAMS, is also used for simulating the system and testing the accuracy of the obtained values for the design variables. Theoretical and simulation results show that the proposed approach gives a certain decrease in bearing amplitudes and forces.

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1. Introduction

Rotating machines comprise such machine elements as shafts, discs, gears or blades. Some well-known examples of these machines are machine tools, turbo-machinery and gas-turbines. One major problem encountered in these mechanical systems is vibration. Mainly, these vibrations arise from unbalance in the rotor or in the shaft, misalignment in the coupling, and lack of straightness of the shaft, etc.

Owing to the growing demands for high-power, high-speed and light-weight rotor-bearing systems, computations of critical speeds and steady-state responses at synchronous and subcritical resonances become essential for system design, identification, diagnosis, and control [1]. Vibration control is especially important in improving machine surface finishes; achieving longer bearings, spindles, and tool life in high-speed machining; and reducing the number of unscheduled shut-downs [2]. Optimum design of a flexible rotor supported on three-lobe bearings was presented by using the GA approach for optimal performance, considering system stability [3]. In order to develop an efficient optimum design method for tilting-pad bearings, the GA method was used by considering some constraints and a global objective function [4]. Balancing of a flexible rotor was formulated as a minimax optimization problem [5]. The sequential quadratic programming (SQP) method was also employed to solve that problem. A balancing example was used to demonstrate the effectiveness of that formulation. Liu and Qu [6] presented a new balancing method for rotor systems. GA optimization and computer simulation were used to simplify the balancing process. The proposed method decreased test numbers and increased the precision and efficiency of field balancing. Liu [7] developed a novel balancing method based on the holospectrum technique to balance a flexible rotor without test runs at high speeds. Chung and Jang [8] studied dynamic stability and time responses for an automatic ball balancer of a rotor with a flexible shaft. The stability of the ball balancer around the balanced equilibrium position was analyzed. Polymeric materials as bearing supports were investigated to improve the dynamic performance of rotor-shaft systems [9].
Abbreviations

\( G_i \): Geometric center of \( i \)th disc
\( D_i \): Hole diameter of \( i \)th disc
\( d \): Shaft diameter
\( e_i \): Eccentricity value of \( i \)th disc
\( F \): External force
\( F_{\text{elastic}} \): Total elastic force
\( F_{\text{ext damping}} \): Total external damping force
\( F_{\text{inertia}} \): Total inertial force
\( F_{\text{int damping}} \): Total internal damping force
\( f(\chi) \): Objective function
\( GA \): Genetic Algorithm
\( G_i \): Gravity center of \( i \)th disc
\( H_i(\chi) \): Nonlinear constraints
\( h_i \): Number of nonlinear constraints
\( K \): Stiffness matrix
\( k_{\text{LB}} \): Stiffness coefficient of left bearing
\( k_{\text{RB}} \): Stiffness coefficient of right bearing
\( m \): Mass
\( m_{\text{LB}} \): Left bearing mass
\( m_{\text{RB}} \): Right bearing mass
\( m_i \): Mass of \( i \)th disc
\( l_i \): Distance between \( i \)th disc and left bearing
\( L_i \): Distance between hole and disc centers for \( i \)th disc
\( LB \): Left bearing
\( RB \): Right bearing
\( s \): Number of considered points in objective function
\( t_i \): Thickness of \( i \)th disc
\( W \): Weighting factor in objective function
\( x_i \): Design variable vector
\( x_{i,\text{max}} \): Upper bound of design variable
\( x_{i,\text{min}} \): Lower bound of design variable
\( \eta_i \): Angular directions of \( i \)th eccentricity
\( \theta_i \): Angular position of the shaft in \( z \) axis
\( \Omega \): Forcing frequency

Theoretical study shows that the use of such sectors reduces the rotor unbalanced response, increases the limit stability speed for simple rotor-shaft systems, which all improve the dynamic characteristics.

Vibration of a cracked rotor sliding bearing system with rotor-stator rubbing was investigated by Wan et al. [10]. The dynamic response of the rotor was calculated using the Newmark method by considering some nonlinear factors. Yang et al. [11] investigated the vibration of a low-pressure steam-turbine rotor in a nuclear power plant. The hybrid GA method was used by considering the resonance response as an objective function. It is possible to reduce excessive response at critical speed and obtain more stable conditions using this approach. Li and Yu [12] developed a nonlinear coupled lateral torsional vibration model of a rotor-bearing-gear coupling system based on the engagement conditions of gear couplings. Al-Bedoor [13,14] presented the coupled torsional and lateral vibrations of unbalanced rotors, including the effect of the rotor-to-stator rubbing, and investigated a model to evaluate this type of transient vibration using the simple Jeffcott rotor. The existence of inertial coupling and nonlinear interaction between the torsional and lateral vibrations was demonstrated. Shape optimization to reduce the vibration level in rotors was presented by Straub et al. [15]. Mass was constituted as an objective function and a nonlinear constraint was used to reduce the rotor weight, vibration and noise. Kang et al. [16] simulated the balancing of flexible rotor-bearing systems under various arrangements of sensors and planes using finite element analysis. The accuracy and efficiency of rotor balancing were improved by selecting sensor locations and balancing. In the study by Hredzak and Guo [17], the feasibility of an active balancing method on a hard disc drive was investigated. Active and passive balancing (adding counterweights) methods were compared with each other, and the advantages and disadvantage of the proposed method were discussed. Ishida [18] introduced new passive vibration suppression methods using combinations of well-known passive balancing devices, such as a ball balancer, a centrifugal pendulum absorber and a roller-type centrifugal absorber.

When the literature is investigated, it can be seen that there have been many studies about rotating mechanical systems over the last 20 years. These studies are especially focused on reducing vibrations, either designing suitable bearings or controlling these vibrations by using different control techniques (active balancing of systems). Passive balancing was studied by some researchers [17,18]. But, passive solution studies, without adding any counterweight or devices for vibrations, i.e. adjusting the values of only suitable design parameters using optimization techniques, on rotating mechanical systems, have not been exactly considered. In this study, in order to determine suitable balancing conditions for unbalanced discs, theoretical analysis of a rotating mechanical system, having many unbalanced discs, was performed. After the vibration analysis of the system, two numerical examples with three unbalanced discs were implemented to clarify the application of the proposed approach. Three discs, causing three centrifugal forces, were located at random angular positions relative to each other. Related vibrations in \( x \) and \( y \) directions were determined and an appropriate objective function was constituted. The GA approach was used to solve the optimization problem for finding the optimum values of design variables. This work proposes a practical method for reducing the bearing vibration by using an optimization technique.

2. Materials and methods

2.1. System description and modeling

A rotating mechanical system having multi-discs is described as in Figure 1. Each disc is located at a distance relative to the left bearing side, and the bearings are located at the end of the shaft. The schematic representation of the unbalanced disc is given in Figure 2.

The governing equation of the system can be defined as [19]:

\[
F_{\text{inertia}} = F_{\text{elastic}} + F_{\text{int damping}} + F_{\text{ext damping}}. \tag{1}
\]

where \( F_{\text{inertia}} \) is the total inertial force and \( F_{\text{elastic}} \) is the total elastic force. \( F_{\text{int damping}} \) and \( F_{\text{ext damping}} \) are the total internal and external damping forces, respectively. If the internal and external damping forces are neglected, and also inertial and elastic forces are expanded, the mathematical model is given as [20]:

\[
[M] \ddot{\mathbf{q}}(\theta) + [K] \mathbf{q}(\theta) = \mathbf{F}(\theta), \tag{2}
\]

where \( M \) and \( K \) are the mass and stiffness matrices, respectively. The dimensions of these matrices are determined by the
number of nodes in the model. \( M \) is a diagonal matrix and consists of masses of bearings and each unbalanced disc. Stiffness matrix \( K \) also consists of the stiffness coefficients of the corresponding shaft sections. This matrix can be easily found from the influenced coefficients matrix. Column matrices \( F \) and \( q \) denote the external forces acting on related nodes and generalized coordinates, respectively. \( \theta \) is the angular position of the shaft in the \( z \) axis. Column matrix \( F \) is given for \( x \) and \( y \)-directions, as follows:

\[
F_x = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
m_1 e_1 \Omega^2 \cos(\theta + \eta_1) \\
m_2 e_2 \Omega^2 \cos(\theta + \eta_2) \\
\vdots \\
m_n e_n \Omega^2 \cos(\theta + \eta_n) \\
0 \\
m_1 e_1 \Omega^2 \sin(\theta + \eta_1) \\
m_2 e_2 \Omega^2 \sin(\theta + \eta_2) \\
\vdots \\
m_n e_n \Omega^2 \sin(\theta + \eta_n)
\end{bmatrix},
\]

\[
F_y = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
m_1 e_1 \Omega^2 \cos(\theta + \eta_1) \\
m_2 e_2 \Omega^2 \cos(\theta + \eta_2) \\
\vdots \\
m_n e_n \Omega^2 \cos(\theta + \eta_n) \\
0 \\
m_1 e_1 \Omega^2 \sin(\theta + \eta_1) \\
m_2 e_2 \Omega^2 \sin(\theta + \eta_2) \\
\vdots \\
m_n e_n \Omega^2 \sin(\theta + \eta_n)
\end{bmatrix}
\]

where \( F_x \) and \( F_y \) are external forces in \( x \) and \( y \) directions, respectively. \( e_i \) denotes the eccentricity values of \( i \)th discs. \( \Omega \) is the forcing frequency on the system. \( n \) denotes the number of unbalanced discs. \( \eta_i \) are the angular directions of eccentricities for the \( i \)th unbalanced disc. The generalized coordinates can be described as physical displacements in \( X \) and \( Y \) directions.

\[
X = \begin{bmatrix} x_{LB} & x_1 & x_2 & x_3 & \cdots & x_n & x_{RB} \end{bmatrix}^T
\]

\[
Y = \begin{bmatrix} y_{LB} & y_1 & y_2 & y_3 & \cdots & y_n & y_{RB} \end{bmatrix}^T.
\]

Assuming that the motion is periodic and is simply composed of harmonic components of various amplitudes:

\[
x_j = A_j \cos(\theta) \quad (j = LB, RB, 1, 2, 3, \ldots, n).
\]

By substituting Eqs. (3)–(5) into the mathematical model (Eq. (2)) to define the characteristics of \( A_j \) and \( B_j \) [19,20], the steady-state responses for \( x \) and \( y \) directions can be obtained as symbolic characteristics using the MATLAB/Symbolic Toolbox [21].

2.2. Optimization process

The goal of the optimum design approach is to find the optimum value of the objective function in a given domain [11]. There are many numerical optimization methods for determining appropriate design parameters. The genetic algorithm is a guided random search technique. It uses objective function information instead of derivatives like those in gradient-based methods. On the other hand, Genetic algorithms are simple to implement and involve evaluations of only the objective function, and the use of certain genetic operators, such as selection, crossover, mutation and reproduction, to explore the design space [22]. A selection operator is used to choose the parents for the next generation. Also, reproduction controls how the genetic algorithm creates the next generation. Crossover is a very important operator in the success of genetic algorithms. This operator is a primary source of new candidate solutions and provides the search mechanism that efficiently guides the evolution through the solution space towards the optimum. The mutation operator must be used to prevent the nonoptimal solution of the genetic algorithm, that is, premature convergence. Mutation is a process of randomly altering a part of an individual for producing a new individual. This operator explores new probabilities for optimal solution, but should be selected carefully, not leading to loss of the good characteristics of strings [4,21]. The flowchart of the optimization process is outlined in Figure 3.

The bearing amplitudes are formulated as an objective function for minimizing the vibrations. The symbolic derivations of these amplitudes are implemented in the symbolic toolbox of MATLAB.
Minimize \( f(\chi) = \sum_{i=1}^{s} \left[ W_1|x_i| + W_2|y_i| \right] + W_3|x_i| + W_4|y_i| \)  

Subject to \( H_r(\chi) = 0 \)  

\( \chi_{\text{min}} \leq \chi \leq \chi_{\text{max}} \)  

where \( f(\chi) \) denotes the objective function, which comprises the amplitudes of the bearings in \( x \) and \( y \) directions. \( s \) is the number of considered points during the optimization. \( W_1, W_2, W_3 \) and \( W_4 \) denote the weighting factors, which must satisfy the condition:  

\( 0 \leq W_i \leq 1 \) and \( \sum_{i=1}^{4} W_i = 1 \).

Each weighting factor is considered to have an identical effect on the objective function. \( H_r(\chi) \) are the nonlinear constraints, which consist of force equilibrium conditions in \( x \) and \( y \) directions of each bearing. \( h \) are the number of these nonlinear constraints. Although each constraint, which has a deterministic continuous character, increases the complexity of the solution, it is necessary to have a functional system. \( \chi \) is a vector comprising the independent design variables, \( \chi_{\text{min}} \) and \( \chi_{\text{max}} \) are the lower and upper bounds of the selected design variables, respectively. The range of each design variable must comprise the intervals of \( 0 \sim 2\pi \) radians. The design variable vector is given as:

\[ \chi = [\eta_2 \eta_3 \ldots \eta_n]^T. \]

### 3. Case studies

A rotating mechanical system, having three unbalanced discs, was considered as a numerical example (Figure 4). The shaft and each disc were assumed as steel. The disc was located at an equal distance relative to each other. It was also assumed that the shaft was supported at both ends with rigid supports and that damping was negligible. The mass of shaft and its static deflection were neglected, and two case studies were performed. The eccentricity values and disc characteristics for two cases are given in Table 1.

The bearing amplitudes in \( x \) and \( y \) directions are considered as an objective function, and the eccentricity directions...
of the second and third discs relative to the first one’s are defined as the design variables. The conditions making the forces at the bearings minimum are taken as nonlinear constraints. A stochastic uniform was applied as a selection function for choosing the next generation. Crossover and mutation probabilities were adjusted as 80 and 20% for the next generation, respectively [23]. The crossover function was selected as scattered. This type of crossover creates a random binary vector. So, the genes are selected from the first parent, where the vector is a 1, and from the second one, where the vector is 0, and combines the genes to form the first child. The opposite rule is used to form the second one (see Figure 5). The Gaussian function was adjusted as the mutation operator. Population size was adjusted as 20. Two elite individuals in the current generation were considered to guarantee the survival of the next generation [23].

For solving the optimization problem, 50 generations were implemented. 360 points during the periodic motion of the shaft were considered in the optimization process. This approach was achieved by the use of the genetic algorithm and direct search toolbox in MATLAB, and a PIV processor with a CPU speed of 3.2 MHz and 1024 Mb Ram is used.

4. Results

A rotating mechanical system comprising the centrifugal forces arising from unbalanced discs was considered for investigation and for reducing system vibrations. In order to reduce the corresponding vibrations, no balancing mass was used at the balancing plane of the left and right bearings, or any control technique. Therefore, as the main goal, it was intended to design a self-balancing system for reducing these vibrations by using an optimization technique. The Genetic algorithm approach was implemented to minimize the related vibrations under the nonlinear constraints. As a numerical example, three unbalanced discs were located on a flexible shaft and the running speed of the system was taken as 4000 rpm. As a result of optimization, the obtained angular configurations for disc eccentricities were adapted to the model system, and the verification of the proposed optimization process was also evaluated by using ADAMS software [24]. After the optimization process, the obtained values of design variables for two case studies are given in Table 2.

For Case I, the convergence history during the optimization is outlined in Figure 6. The algorithm shows good convergence. After 50 generations, the best individual fitness stays as $1.364 \times 10^{-4}$ and the average fitness occurs as $1.366 \times 10^{-4}$. The eccentricity directions of discs in the optimized system (with optimum adjusting of eccentricity directions for discs) are also outlined in Figure 7 for Case I.
Table 2: Eccentricity directions (before and after optimization).

<table>
<thead>
<tr>
<th>Case</th>
<th>Original</th>
<th>Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Case II</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>Case I</td>
<td>30</td>
<td>185.87</td>
</tr>
<tr>
<td>Case II</td>
<td>130</td>
<td>216.53</td>
</tr>
<tr>
<td>Case I</td>
<td>60</td>
<td>13.02</td>
</tr>
<tr>
<td>Case II</td>
<td>240</td>
<td>38.64</td>
</tr>
</tbody>
</table>

After the optimization, the obtained values of design variables for Case I are re-evaluated in theoretical analysis. Original and optimized system outputs, as amplitudes, at the bearings for x and y directions, are given in Figure 8. This figure shows that there is a certain decrease in bearing amplitude when using the optimum value of design variables. The decreases in amplitudes for horizontal directions are read as 82.87% and 58.63% for left and right bearings, respectively. Also, the decrease in amplitudes for vertical directions appears, approximately, by 83.54% and 59.10% for left and right bearings, respectively.

Bearing forces in x and y directions, obtained for original and optimized systems for Case I, are outlined in Figure 9. As seen from this figure, the application of optimization yields a certain decrease in bearing force for each direction. The drops in force for x direction are by 78.93% and 66.76% for the left and right bearings, respectively. Also, the decreasing ratio of this force for the y direction occurs as 79.74% and 67.18% for the left and right bearings, respectively.

For original and optimized configurations of disc eccentricities, the rotating mechanical system for Case I is simulated by using ADAMS software. Amplitude variations for original and optimized systems are shown in Figure 10. Also, an isometric view of the system for Case I, eigenfrequencies, and eigenmodes of the flexible shaft obtained from simulation are given in the Appendix. At the beginning of the motion, there is a certain deviation between theoretical and simulation results. For the case of steady-state response, the decreases in amplitudes for the x direction occur as 79.48% and 62.03% for the left and right bearings, respectively. For the y direction, the drops in amplitudes are read as 80.33% and 62.96% for the left and right bearings, respectively. The deviations between theoretical and simulation studies are mainly originated from the definition of the governing equation, that is, neglecting the gyroscopic effect, and internal and external damping in the theoretical analysis. In the simulation step, all these characteristics are considered.
The amplitude and force of Case II are similar to Case I, as figural characteristics. However, the magnitudes of these characteristics are not the same, due, primarily, to the different eccentricity values and disc masses. For Case II, the decreasing ratios in amplitudes for the $x$ direction are read as 78.48 and 64.58% for left and right bearings, respectively. Also, the decreases in amplitudes for the $y$ direction appear as 78.95 and 65.07% for left and right bearings, respectively. In cases of bearing force, these decreases in the left and right bearings for the $x$ direction are by 74.37 and 69.41%, respectively. For the $y$ direction, the decreasing ratio of this force occurs as 75.04 and 69.98% for the left and right bearings, respectively. Both the theoretical and the simulation case studies show that if the suitable values of the design variables are defined using optimization, there is a certain decrease in the amplitudes and bearing forces.

5. Conclusions

This study proposes a simple method based on the optimization theory to determine the appropriate relative positions of the unbalanced discs to each other for obtaining a self-balancing condition. As a result of optimization, the absolute values of investigated parameters, such as amplitudes and forces, are closer to zero than those of the original configuration. It is also concluded that the decreasing values of these investigated parameters are obtained using the appropriate weighting factors for the objective function and the constraints. For the case of steady-state response, there are little deviations between theoretical and simulation results, due, primarily, to some assumptions, such as neglecting the internal and external damping, and the gyroscopic effect, in theoretical investigation. Both theoretical and simulation case studies show that the suitable values of the design variables can be defined using the optimization approach. Since the theoretical study and its verification with mechanical system simulation are evaluated together, the proposed approach is a versatile method to carry out the reducing of system vibrations. In middle and nearly low speed applications, this result can be considered as a solution. In cases of very high-speed applications, this approach can be considered a pre-application before the classical balancing method, such as adding counterweights, etc. (This automatically decreases the value of counterweight which has to be added). This approach can be easily adapted to rotating mechanical systems having multi-discs with eccentricity.

Appendix

The isometric view of the system on simulation software is given in Figure A.1, and eigenmodes and eigenfrequencies of the flexible shaft are outlined in Figure A.2, respectively.
Figure 10: Amplitude variations of bearings for simulation of Case I.

Figure A.1: Isometric view of mechanical system in simulation of Case I: (a) Initial configuration, and (b) optimized configuration.

Figure A.2: Eigenmodes and eigenfrequencies of flexible shaft.

References


Hamdi Taplak graduated in 1993 from the Mechanical Engineering Department at Erciyes University, Turkey, from where he also received M.S. and Ph.D. degrees. He currently works at the Vocational School of Kayseri at Erciyes University, Turkey. His research areas include: mechanical vibrations, rotating mechanical systems, and condition monitoring of mechanical systems.

Selçuk Erkaya graduated in 2001 from the Mechanical Engineering Department at Erciyes University, Turkey, from where he also received M.S. and Ph.D. degrees, and where he currently works in the Mechatronics Engineering Department. His research areas include: mechanism and machine theory, noise and mechanical vibrations, neural networks and optimization, robust design of mechanical systems.

İbrahim Uzmay graduated from the Mechanical Engineering Department at Yıldız Technic University, Turkey, in 1978. He also received M.S. and Ph.D. degrees from Yıldız Technic University, Turkey, and Istanbul Technic University, Turkey, respectively. He currently works in the Mechanical Engineering Department at Erciyes University, Turkey. His research areas include: mechanism and machine theory, mechanical vibrations and noise control, system dynamics and control, robotics.