Optimal Ordering Policy Under Acceptance Sampling Plan and Trade Credit Financing

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Abstract. Acceptance of sampling plans and trade credit has become increasingly common in today’s business. These two issues should be considered simultaneously when determining an ordering decision. This paper uses EOQ to model the decision under the acceptance sampling plan and trade credit; meaning, how often it would be necessary to order to minimize the total related cost. We develop theorems based on optimal lemmas to solve the problem. Computational analyses are given to illustrate the solution procedures and we discuss the influence of credit period, acceptance sampling plan, holding cost and ordering cost on the total cost, and the ordering decision. We conclude with a computational analysis that leads to a variety of managerial insights.

Keywords: Ordering; Inventory; Acceptance sampling plan; Trade credit; Optimization.

INTRODUCTION

Most logistic and retailing companies suffer from returned items considered as defective by customers. Practically, in the United States, companies spend roughly $950 billion annually on logistics, and 4.5% of this, or $43 billion, are for returns [1]. Generally, the most fundamental reason for defects comes from the manufacturing site due to imperfect production. Therefore, knowing how to prevent the selling of defective products to the customer is an important problem.

Acceptance sampling, one topic in statistical quality control, involves inspecting a sample of a product and comparing it with a quality standard. If the sample fails to test, then that implies poor quality, and the entire group of items from which the sample was taken is rejected [2]. An acceptance sampling plan is an application of statistical techniques to determine whether a quality of material should be accepted or rejected. In the serial or complex non-serial manufacturing systems, research suggests several ways to use acceptance sampling to prevent defects [3]. In particular, acceptance sampling is a key ingredient of the total quality control system and useful when the inspection process is expensive, destructive or takes a long time [4].

Reyniers and Tapiero [5] and Starbird [4] suggested economic contract models to find the supplier’s optimal quality and identify the effect of the contract design on equilibrium behavior. Other research focuses on the optimal inspection policy of each or both participants; supplier and retailer. In their research, the optimal inspection policy and how it can be found through a game theoretical model in the manufacturing system or supply chain network [3] was explained. Most studies considered how to determine the optimal inspection policy. In our paper, we incorporate the acceptance sampling plan into the inventory model and determine the optimal ordering policy.

Also the traditional Economic Order Quantity (EOQ) model tacitly assumes that a buyer must pay the vendor for products immediately after the buyer receives them. In practice, a vendor may often provide forward financing to the buyer. This means that the vendor allows the buyer a certain fixed period (credit period) to settle the amount owed, and does not charge any interest on the amount owed during this period. In this case, the buyer can sell the goods...
and deposit the accumulated revenue in the bank to earn interest.

Since the publication of Goyal’s [6] paper almost 25 years ago, a lot of papers have appeared in the literature dealing with a variety of trade credit situations. Teng [7] amended the model of Goyal [6] by considering the difference between unit price and unit cost. Huang [8] considered not only that the supplier offers a credit period to the retailer, but also that the retailer offers a credit period to customers. Then Huang [9] incorporated the models of Teng [7] and Huang [8] under a limited storage space. Ouyang et al. [10] considered the ordering policy with cash discount and payment delay. Chung and Huang [11] considered an inventory model to allow items with imperfect quality with a permissible delay in payment.

Recently, Tsao and Sheen [12] considered pricing and replenishment decisions for deteriorating items with lot-size and time dependent purchasing cost under a credit period. Sheen and Tsao [13] discussed channel coordination under trade credit with freight cost and quantity discounts. Ouyang et al. [14] demonstrated that a significant profit increase for the entire supply chain can be achieved by linking both trade credit and freight rate policies. Tsao and Sheen [15] determined the dynamic pricing, promotion and replenishment policies for a deteriorating item under trade credits. Ouyang et al. [16] considered deteriorating items with partially permissible delay in payments linked to order quantity. Teng [17] considered that the retailer offers either a partial or a full trade credit to his customers. Chen and Kang [18] developed integrated inventory models considering two-level trade credit and a price-negotiation scheme. Huang [19] considered an integrated inventory model under an order processing cost reduction and delay in payment. Tsao [20] considered multi-echelon, multi-item channels subject to a supplier’s credit period and the retailer’s promotional effort. Therefore, field researchers find the issue of trade credit very popular. It is essential to consider trade credit when developing an inventory model.

In this study, we formulate a model that includes trade credit, together with the acceptance sampling plan. To our knowledge, this is the first study to consider the inventory replenishment problem under an acceptance sampling plan and trade credit financing. The objective is to determine the optimal replenishment cycle time, while minimizing the total cost. We provide theorems based on optimization to solve the problem. From computational analysis, we illustrate the solution procedures and discuss the effects of system parameters on total cost and the replenishment decision. We conclude with analyses that lead to a variety of managerial insights. The retailer’s supply chain is shown in Figure 1.

The contributions of this paper to the literature are as follows. First, the acceptance sampling plan is incorporated into the inventory model in this paper. This is the first study to consider the inventory replenishment problem under an acceptance sampling policy. Secondly, for studies about trade credit, we are also the first to consider the acceptance sampling. Thirdly, we develop easy-to-use theorems for solving the problems described. An overview of the rest of this paper is also as follows. First, the assumptions and the notations used in this study are described. Then, the inventory model under an acceptance sampling policy and trade credit financing are formulated, and theorems are provided to solve the problem. Subsequently, the computational analysis to illustrate the solution procedures is presented and parameter sensitivities are discussed. Finally, some conclusions are drawn.

**NOTATIONS AND ASSUMPTIONS**

We use the following variable and parameter definitions to construct our model:

- $p$: unit retail price,
- $w$: unit wholesale price,
- $m$: unit inspection cost,
- $A$: ordering cost per order,
- $H$: unit inventory holding cost per unit time,
- $g$: goodwill cost for selling a defective product to a

![Figure 1. The retailer in the supply chain system.](www.SID.ir)
MODEL FORMULATION

In this research, we consider that the retailer applies the acceptance sampling plan to an incoming lot at the inspection stage. Then, the retailer can make a decision as to whether or not to accept the lot, which is based on the result of the inspection. If the number of defective product from the sample size is larger than critical level, the lot will be rejected; If the number of defective products from the sample size is less than, or equal to, critical level, the lot will be accepted. Though the lot is accepted, the supplier has to return or repair the defective products, which are detected by the inspection. In this case, the retailer will receive a discount of defective products from the supplier. If the incoming lot is rejected, a cost for rejecting the lot will be incurred for the retailer, since the retailer should pay the supplier a penalty for rejecting good products.

When the quality of incoming product is $q$, the probability of acceptance can be expressed as follows:

$$P(k \leq c) = \sum_{X=0}^{c} \binom{n}{X} q^X (1-q)^{n-X}, \quad (1)$$

and the probability of rejection is:

$$P(k > c) = 1 - \sum_{X=0}^{c} \binom{n}{X} q^X (1-q)^{n-X}. \quad (2)$$

The total related cost consists of the following elements:

1. Annual ordering cost $= A/T$.
2. Annual inventory holding cost $= HDT/2$.
3. Annual inspection cost $= mc/T$.
4. Annual goodwill cost for selling defective products to customers $= \frac{p(k \leq c)}{2} \frac{T^{\frac{M-n}{T}}}{T}$.
5. Annual discount of defective products $= P(k \leq c)$.
6. Cost for rejecting the product per time per year $= P(k > c) \frac{1}{T}$.
7. There are two cases occurred in interest earned per year:
   - Case 1: when $T \geq M$
     $$\text{Annual interest earned} = \frac{p.Ie.DM^2}{2T}.$$
   - Case 2: when $T \leq M$
     $$\text{Annual interest earned} = \frac{p.Ie.DT}{2} + \frac{p.Ie.D(M - T)}{2}.$$
8. There are two cases occurred in interest paid per year:

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Case 1: when $T \geq M$
Since the average unit cost purchasing from the supplier is $\frac{[wD - P(k \leq c)r.w.n.q]}{D}$, then annual interest paid = $\frac{[wD - P(k \leq c)r.w.n.q]Ip(T-M)^2}{2T}$. 

Case 2: when $T \leq M$
Annual interest charge = 0. In this case, no interest charge is paid for the items. Therefore, the total related cost, $TC_i$, has two different expressions as follows:

Case 1: when $T \geq M$:

$$TC_1(T) = \frac{A}{T} + \frac{DTH}{2} + \frac{mn}{T} + \frac{g(DT - n)q}{T} - P(k \leq c) \cdot \frac{r.w.n.q}{T}$$
$$+ P(k > c) \cdot \frac{s}{T} - pIe \cdot \frac{DM^2}{2T}$$
$$+ \frac{[wD - P(k \leq c)r.w.n.q]Ip(T-M)^2}{2T}.$$  

(3)

Case 2: when $T \leq M$:

$$TC_2(T) = \frac{A}{T} + \frac{DTH}{2} + \frac{mn}{T} + \frac{g(DT - n)q}{T} - P(k \leq c) \cdot \frac{r.w.n.q}{T}$$
$$+ P(k > c) \cdot \frac{s}{T} - pIe \cdot \frac{DM^2}{2T}$$
$$- pIe \cdot D(M - T).$$  

(4)

OPTIMAL ORDERING POLICY

The problem is to determine the optimal ordering cycle time, $T^*$, while minimizing $TC_i$, where $TC_i(T) = TC_1(T)$ when $T \geq M$ and $TC_i(T) = TC_2(T)$ when $T \leq M$. The first-order and second-order derivative of $TC_1(T)$ and $TC_2(T)$, with respect to $T$, are as follows:

$$dTC_1 \over dt = \frac{-2[A + mn + sP(k > c)] - ng(g + P(k \leq c)rw) - DM^2(wIp - pIe) - P(k \leq c)nquwIp(T - M^2) + DT^2(H + wIp)}{2T^2},$$  

(5)

$$dTC_2 \over dt = \frac{-2[A + mn + sP(k > c)] - ng(g + P(k \leq c)rw) + DT^2(H + pIe)}{2T^2}.$$  

(6)

$$d^2TC_1 \over dt^2 = \frac{2[A + mn + sP(k > c)] - ng(g + P(k \leq c)rw) + DM^2(wIp - pIe) - P(k \leq c)nquwIpM^2}{T^3},$$  

(7)

$$d^2TC_2 \over dt^2 = \frac{2[A + mn + sP(k > c)] - ng(g + P(k \leq c)rw)}{T^3}. $$  

(8)

Because $A$ and $mn$ are relatively large, and $P(k \leq c)$, $P(k \leq c)$, and $q$ are all less than 1, we know $A + mn + sP(k > c) - ng(g + P(k \leq c)rw) > 0$ in normal cases. This means $d^2TC_i \over dt^2 > 0$ is satisfied. Therefore, Equation 8 implies that $TC_2(T)$ is convex on $T > 0$. Let:

$$\Delta_1 = 2[A + mn + sP(k > c)] - ng(g + P(k \leq c)rw).$$

Equation 7 implies that $TC_1(T)$ is convex on $T > 0$ if

$$\Delta_1 + DM^2(wIp - pIe) - P(k \leq c)nquwIpM^2 > 0.$$  

From $TC_1(M) = TC_2(M), TC_i(T)$ is continuous and defined on $T > 0$. We have the following lemma.

**Lemma 1**

(a) If $\Delta_1 + DM^2(wIp - pIe) - P(k \leq c)nquwIpM^2 > 0$, $TC_i(T)$ is a convex function of $T$.

(b) If $\Delta_1 + DM^2(wIp - pIe) - P(k \leq c)nquwIpM^2 \leq 0$, $TC_i(T)$ is a convex function of $T$ on $(0, M)$ and a concave function of $T$ on $[M, \infty)$.

Firstly, we discuss how to determine the optimal ordering cycle time under the situation when:

$$\Delta_1 + DM^2(wIp - pIe) - P(k \leq c)nquwIpM^2 > 0.$$  

If:

$$\Delta_1 + DM^2(wIp - pIe) - P(k \leq c)nquwIpM^2 \leq 0,$$

we can obtain the optimal ordering cycle time for each case (by solving $\frac{dTC_i(T)}{dt} = 0$):

$$T_1^* = \sqrt{\frac{2[A + mn + sP(k > c)] - ng(g + P(k \leq c)rw) + DM^2(wIp - pIe) - P(k \leq c)nquwIpM^2}{D(H + wIp) - P(k \leq c)nquwIp}},$$  

(9)

$$T_2^* = \sqrt{\frac{2[A + mn + sP(k > c)] - ng(g + P(k \leq c)rw)}{D(H + pIe)}.}$$  

(10)
From Lemma 1, when:

$$\Delta_1 + DM^2(w.Ip - p.Ie) - P(k \leq c)mqrwIpm^2 > 0,$$

$TC_1(T)$ is convex on $T$. Therefore:

$$\frac{dT C_1(T)}{d^T} = TC_1'(M) \begin{cases} 
    > 0, & \text{if } T > T^* \\
    = 0, & \text{if } T = T^* \\
    < 0, & \text{if } T < T^* 
\end{cases}$$

Similarly,

$$\frac{dT C_1(T)}{d^T} = TC_2'(M) \begin{cases} 
    > 0, & \text{if } T > T^* \\
    = 0, & \text{if } T = T^* \\
    < 0, & \text{if } T < T^* 
\end{cases}$$

Also, we know that:

$$TC_1'(M) = TC_2'(M)$$

$$= \frac{-2[A + mn + s.P(k > c) - nq(g + P(k \leq c)r_w)] + DM^2(H + pIe)}{M^2}.$$ 

Let:

$$\Delta_2 = -2[A + mn + s.P(k > c) - nq(g + P(k \leq c)r_w)] + DM^2(H + pIe),$$

consequently, $\Delta_2 > 0$ if, and only if, $TC_1'(M) = TC_2'(M) > 0$. Then we have the following lemma.

**Lemma 2**

(a) If $\Delta_2 > 0$, then $T_1^* < M$ and $T_2^* < M$.

(b) If $\Delta_2 < 0$, then $T_1^* > M$ and $T_2^* > M$.

(c) If $\Delta_2 = 0$, then $T_1^* = T_2^* = M$.

We develop the following theorem for determining the optimal ordering cycle time, $T^*$, based on the above analysis.

**Theorem 1**

When $\Delta_1 + DM^2(w.Ip - p.Ie) - P(k \leq c)mqrwIpm^2 > 0$,

(a) If $\Delta_2 > 0$, then $T^* = T_2^*$.

(b) If $\Delta_2 < 0$, then $T^* = T_1^*$.

(c) If $\Delta_2 = 0$, then $T^* = T_1^* = T_2^* = M$.

**Proof**

(a) If $\Delta_2 > 0$, Lemma 2 (a) implies that $T_1^* < M$ and $T_2^* < M$. According to the convexities and the definitions of $TC_1(T)$ for Case 1, and $TC_2(T)$ for Case 2, we find that $TC_1(T)$ is increasing on $[M, \infty)$ and $TC_2(T)$ is increasing on $[T_2^*, M]$. This means $TC_1(T)$ has the minimum value at $M$, and $TC_2(T)$ has the minimum value at $T_2^*$. Therefore, from $TC_2(T_2^*) \leq TC_2(M) = TC_1(M)$, we know that $TC(T)$ has the minimum value at $T_2^*$, i.e.: $T^* = T_2^*$.

(b) If $\Delta_2 < 0$, Lemma 2 (b) implies that $T_1^* > M$ and $T_2^* > M$. According to the convexities and the definitions of $TC_1(T)$ for Case 1, and $TC_2(T)$ for Case 2, we find that $TC_1(T)$ is decreasing on $[M, T_1^*]$ and $TC_2(T)$ is decreasing on $[0, M]$. This means $TC_1(T)$ has the minimum value at $T_1^*$ and $TC_2(T)$ has the minimum value at $M$. Therefore, from $TC_2(M) = TC_1(M) \geq TC_1(T_1^*)$, we know that $TC(T)$ has the minimum value at $T_1^*$, i.e.: $T^* = T_1^*$.

(c) If $\Delta_2 = 0$, Lemma 2 (c) implies that $T_1^* = T_2^* = M$. According to the convexities and the definitions of $TC_1(T)$ for Case 1, and $TC_2(T)$ for Case 2, we find that $TC_1(T)$ is increasing on $[M, \infty)$ and $TC_2(T)$ is decreasing on $[0, M]$. This means both $TC_1(T)$ and $TC_2(T)$ have minimum values at $M$. Therefore, from $TC_2(M) = TC_1(M)$, we know that $TC(T)$ has the minimum value at $M$, i.e.: $T^* = T_1^* = T_2^* = M$.

Then, we discuss how to determine the optimal ordering cycle time under a situation when:

$$\Delta_1 + DM^2(w.Ip - p.Ie) - P(k \leq c)mqrwIpm^2 \leq 0.$$ 

If:

$$\Delta_1 + DM^2(w.Ip - p.Ie) - P(k \leq c)mqrwIpm^2 \leq 0,$$

we consider the following lemma.

**Lemma 3**

If:

$$\Delta_1 + DM^2(w.Ip - p.Ie) - P(k \leq c)mqrwIpm^2 \leq 0,$$

then, $T_2^* < M$.

**Proof**

If $T_2^* \geq M$, then:

$$\frac{2[A + mn + s.P(k > c) - nq(g + P(k \leq c)r_w)]}{D(1 + pIe)} \geq M^2.$$
Arranging the inequality, we obtain:

$$\Delta_1 \geq D(1 + pIe)M^2.$$  

Then, adding:

$$DM^2(wIp - pIe) - P(k \leq c)mqrwIpM^2,$$

into both sides, we have:

$$\Delta_1 + DM^2(wIp - pIe) - P(k \leq c)mqrwIpM^2 \geq DM^2(1 + wIp) - P(k \leq c)mqrwIpM^2.$$  

Because $D$ is relatively large, and both $P(k \leq c)$ and $q$ are less than 1, we know:

$$DM^2(1 + wIp) - P(k \leq c)mqrwIpM^2 > 0,$$

in normal cases. This is a contradiction, therefore, $T_2^* < M.$ □

Then, we develop Theorem 2 for determining the optimal cycle time, $T^*$, based on the above analysis:

**Theorem 2**

When:

$$\Delta_1 + DM^2(wIp - pIe) - P(k \leq c)mqrwIpM^2 \leq 0,$$

then $TC(T)$ has the minimum value at $T^* = T_2^*$.

**Proof**

From Lemma 3 and the feasible cycle time interval ($T > M$ in Case 1 and $T \leq M$ in Case 2), it can be proved. □

Therefore, we can use Theorem 1 to determine the optimal replenishment cycle time, when:

$$\Delta_1 + DM^2(wIp - pIe) - P(k \leq c)mqrwIpM^2 > 0,$$

and use Theorem 2 to determine the optimal replenishment cycle time, when:

$$\Delta_1 + DM^2(wIp - pIe) - P(k \leq c)mqrwIpM^2 \leq 0.$$

**COMPUTATIONAL ANALYSES**

The purposes of the computational analysis are as follows:

1. To illustrate the solution procedure;
2. To use sensitivity analysis to highlight the influence of the length of the credit period, acceptance sampling plan, holding cost and ordering cost.

**Numerical Examples**

In this section, a real case of an automotive industry is used to verify the model. XYZ Technologies Co., is an automotive parts manufacturer in Taiwan. Its products include relevant materials, such as electronic parts and the electronic connector of the automotive. The parts we consider in this paper are supplied for the switch module of an electronic transmission of the Chrysler 300 C. Please note that the company name has been concealed and the data used here has been scaled to protect confidentiality. The parameters used in this study are:

- $A = 50$ dollars/order,
- $p = 6$ dollars/unit,
- $w = 3$ dollars/unit,
- $H = 0.1$ dollars/unit/year,
- $m = 0.1$ dollars/unit,
- $s = 1$ dollars/unit,
- $n = 20$ units,
- $c = 2$ units,
- $g = 1.5$ dollars/unit,
- $r = 0.2$,
- $Ip = 0.15$ per dollars,
- $Ic = 0.1$ per dollars,
- $M = 0.3$ years,
- $D = 1000$ units/year.

To solve this problem, we firstly calculate:

$$\Delta_1 + DM^2(wIp - pIe) - P(k \leq c)mqrwIpM^2,$$

and $\Delta_2$. Since:

$$\Delta_1 + DM^2(wIp - pIe) - P(k \leq c)mqrwIpM^2 = 83.51 > 0,$$

and:

$$\Delta_2 = -34.02 < 0,$$

then, we can utilize Theorem 1 to get the optimal cycle time, $T^* = T_1 = 0.39$ year; the total related cost $TC_1 = 229.33$ dollars. A graphic representation of $TC_1$ is shown in Figure 2.

**Effects of Acceptance Sampling Plan and Credit Period**

It is also important to discuss the influence of the credit period, the acceptance sampling plan, holding cost and ordering cost to the total cost, and the ordering decision. The computational results shown in Figure 3 indicate the following managerial phenomena: When the vendor provides a longer credit period, $M$, the buyer will replenish the goods more often. In other
words, the retailer will shorten the replenishment cycle time to take advantage of the longer credit period.

An examination of Figures 4 and 5 gives the following results:

1. When the sample size, m, increases, the retailer will shorten the replenishment cycle time.
2. When the critical level, c, increases, the retailer will shorten the replenishment cycle time to reflect this.

From Figures 6 and 7, we obtain:

1. When the holding cost, \( H \), increases, the buyer
will shorten the replenishment cycle time. It is reasonable that when the holding cost increases, the buyer will then shorten the cycle time in an effort to lower his inventory cost.

2. When the ordering cost, $A$, increases, the buyer will increase the replenishment cycle time to reflect this. If the ordering cost increases, it is reasonable that the buyer lengthens the cycle time to reduce the frequency of replenishment.

CONCLUSION

In this study, we explore the optimal replenishment cycle time decision, while still minimizing cost under an acceptance sampling plan and trade credit financing. Theorems based on optimization are provided to solve the problem. From computational analysis, we illustrate the solution procedures and discuss the impact of the related system parameters. The results show that the company will replenish the goods more often when credit period, sample size, critical level and holding cost increase or when ordering cost decreases. The total cost will increase as the credit period decreases or as holding cost and ordering cost increase.

A future research topic may be directed to consider the acceptance sampling plan as decisions. This means that decisions about ordering and an acceptance sampling plan should be determined simultaneously.

Proving an efficient algorithm to solve the problem with multiple decision variables is a challenge for this topic. Another future research topic is probability demand. In our research, we consider a deterministic demand function. It is worth extending our research to model the probability demand problem and develop a stochastic solution approach to solve it.

REFERENCES


**BIOGRAPHY**

Yu-Chung Tsao is Assistant Professor in the Department of Business Management at Tatung University in Taiwan. He received his Ph.D. in Industrial Management from the National Central University. He was a visiting scholar doing research in the School of Industrial and Systems Engineering at Georgia Institute of Technology. His research interests are in the areas of Decision Sciences, Production and Marketing Models, Supply Chain and Logistics Management.