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MEMS Magnetometer Calibration Using Modified Ellipsoid Fitting Method

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Abstract
Low-cost sensors based on Micro-Electro-Mechanical-Systems (MEMS) are typically used for attitude determination in navigation systems. Measured value of magnetometer is subjected to different kind of errors such as random noise, constant bias, nonorthogonality, scale factor deviation and more importantly hard iron and soft iron effects. Therefore, it is needed to calibrate these sensors for accurate measurement. One of the most common methods in calibrating magnetic sensors is least squares ellipsoid fitting. But, the common least squares ellipsoid fitting can be inefficient for real time applications. In this paper, a modified ellipsoid fitting method is proposed. In this method, nonlinear optimization is developed to minimize a novel cost function. The efficacy of the new algorithms is demonstrated through experimental results.

Keywords: magnetometer- calibration- ellipsoid fitting- nonlinear optimization

1. Introduction
One of the most important problems in control and navigation is determining the attitude. This problem is often solved by an Attitude Heading Reference System (AHRS) using gyros that are corrected by gravity sensors (pitch and roll) and magnetic field sensor (yaw). Rate gyros suffer from bias and random walk errors which degrade the accuracy of attitude determination. Increasing the accuracy requires very expensive sensors which have long-term bias stability such as mechanical, fiber-optic or ring laser gyroscopes [1, 2]. The cost of this kind of sensors limits the AHRS to expensive applications.

The rapid development of Micro-Electro-Mechanical Systems (MEMS) along with compensation algorithms makes it possible to use low-cost and lightweight sensors in a wide range of applications specially AHRS. Although MEMS sensors are less accurate than expensive inertial sensors for use in navigation, compensation algorithms and extra sensors can improve the accuracy of MEMS. MEMS-based AHRS are consisted of micro-electro-mechanical magnetometers, accelerometers and gyroscopes which sense earth’s magnetic field, tri-axial acceleration and angular rate respectively [3, 4]. These measurements are combined to achieve the best accuracy.

MEMS- based AHRS are used widely in different application such as Unmanned Aerial Vehicle (UAV) [2, 5-7], mobile applications, autonomous boats [8-10] and etc. A critical point in using a magnetometer is calibration task. Because of offset, alignment errors, soft iron and hard iron effects, the measured data are inaccurate. Therefore before using the sensor data, it should be calibrated to compensate these errors. Ellipsoid fitting is common method for calibration the magnetometer [11-15]. In [11] an algorithm for calibrating strapdown magnetometers in the magnetic field domain is presented. The calibration algorithm uses an iterated, batch least-squares estimator that is initialized using a two-step nonlinear estimator. Given the fact that the error model of magnetic compass is an ellipsoid, [14] presented a constraint least square method to estimate the magnetic calibration parameters. In [15] extended a hyper least square estimator for ellipsoid problem of TAM calibration.

However, the ellipsoid fitting can be poor and ellipsoids maybe fit in non-ellipsoidal curves as well for real time applications in common least squares method. In this paper, a modified ellipsoid fitting method is proposed. In this method, nonlinear optimization is developed to minimize a novel cost function.

The outline of the paper is as follows: In Section II, the ellipse fitting problem is formulated and several important algorithms are reviewed. In Section III, a new modified least squares ellipsoid fitting is proposed, which is followed by the experimental results in Section IV.

2. Ellipsoid fitting formulation and background
Rotating an ideal 3-axis magnetometer in the calibration area and drawing the locus of outputs will results in a sphere, because the geomagnetic field magnitude is constant in that area. On the other hand, the output locus of a non-ideal magnetometer is an ellipsoid because of the error sources such as hard iron and soft iron effects. General form of the quadratic surface equation is:

\[ F(\theta, H_m) = aH_{mx}^2 + bH_{mx}H_{my} + cH_{my}^2 + dH_{mx}H_{mx} + eH_{my}H_{mx} + fH_{mx}^2 + pH_{mx} + qH_{my} + rH_{mx} + s = 0 \]
where $\theta = (a, b, c, d, e, f, p, q, r, s)^T$. Equation (1) can be written as:
\[
X^TAX - 2X^TA\xi + \xi^TAX_0 = 1
\]
where, $A = \begin{bmatrix} a & b/2 & c/2 \\ b/2 & d/2 & e/2 \\ c/2 & e/2 & f/2 \end{bmatrix}$ is the coefficient matrix and $X_0 = -A^{-1}\xi$ are the center of ellipsoid.

The purpose of magnetometer calibration is finding the best ellipsoid which fits on the set of measurement data and derive the coefficient matrix $A$ and the center of ellipsoid $X_0$.

The methodology of ellipsoid fitting is looking for an ideal ellipsoid in which the following cost function is minimized:
\[
\min_{\theta \in \mathbb{R}^{10}} \sum_{i=1}^{N} \|F(\theta, H_{mi})\|^2
\]

or
\[
\min_{\theta \in \mathbb{R}^{10}} \theta^T D \theta
\]
where $D$ is
\[
\begin{bmatrix}
\|e_{h1}\| & \ldots & \|e_{hN}\| \\
\|e_{r1}\| & \ldots & \|e_{rN}\| \\
\|e_{a1}\| & \ldots & \|e_{aN}\| \\
\|e_{s1}\| & \ldots & \|e_{sN}\|
\end{bmatrix}
\]
To get sure that the fitted surface is an ellipsoid one can impose the equality constraint $4ab - b^2 = 1$ and without loss of generality in matrix form:
\[
\theta^T C \theta = 1
\]

where $C = \begin{bmatrix} C_1 & 0_{3 \times 7} \\ 0_{7 \times 3} & \theta_{7 \times 7} \end{bmatrix}$ and $C_1 = \begin{bmatrix} 0 & 0 & 2 \\ 0 & -1 & 0 \\ 2 & 0 & 0 \end{bmatrix}$.

Using Lagrange multiplier $\lambda$, the simultaneous equation can be obtained from (4) and (5) as follows:
\[
G(\theta) = \theta^T D \theta + \lambda (1 - \theta^T C \theta).
\]

In order to find extremum, the first order derivative of (6) should be obtained and set it equal to zero. Hence,
\[
D^T D \theta = \lambda C \theta.
\]

Therefore, the extremum of constrained problem could be obtained:
\[
\begin{align*}
\{D^T D \theta = \lambda C \theta \\
\theta^T C \theta = 1
\end{align*}
\]

Further calculation will results in the ellipsoid coefficient matrix $A$ and ellipsoidal center coordinates $X_0$.

3. New modified ellipsoid fitting
Ellipsoid fitting based on least square (LS) method is a common method in ellipsoid fitting. A disadvantage of this method is when the data is corrupted with noise (as in practical data) or has some out of range points and also if there is a concentration of data. In these cases, fitted ellipsoid will intend toward them. As a result, it may lead us to a wrong fitted ellipsoid. Hence it is more efficient that the fitted ellipsoid passes through more points with less distance error from experimental data. Hence a criterion is needed to weight the data.

The proposed method in this paper is based on introducing a novel cost function and minimizing it using nonlinear optimization. The weighting criterion can be defined based on standard deviation of error. Error is calculated using
\[
E = \sqrt{\sum (x_n - x)^2 + (y_n - y)^2 + (z_n - z)^2}
\]
and the final cost function is:
\[
f = 1 - \frac{1}{n_p} \sum_{i=0}^{100} N_i \left(1 - \frac{\sigma_i}{\sigma}\right)
\]

$\sigma$ is the standard deviation of $E$, $\sigma_i = 0.01 \cdot \sigma, (i = 0: 100)$ and $N_i$ is number of data which $E < \sigma_i$.

As an illustration example, suppose that $\sigma = 1$ and $\sigma_i = 0.01$. Number of data which their errors are less than $\sigma_i$ is $N_i$. They are appropriate data and it is desirable to pass through these set. Hence it can be weighted as $N_i \left(1 - \frac{\sigma_i}{\sigma}\right) = 0.99 N_i$ to show their importance. The goal is to fit an ellipsoid which has more data in first interval and then in second and so on.

As a result
\[
\sum_{i=0}^{100} N_i \left(1 - \frac{\sigma_i}{\sigma}\right)
\]
should be maximum. To maximize this term, $N_i$ for first intervals should be large. We are looking for a minimization problem to use MATLAB toolboxes. Therefor (11) is normalized and subtracted from 1 and then, (10) is obtained. To calculate $n_p$, suppose an ideal case where there is no error, so $N_i = n$. Hence
\[
\sum_{i=0}^{100} N_i \left(1 - \frac{\sigma_i}{\sigma}\right) = n + 0.99n + \ldots + 0.01n
\]

\[
= \frac{1}{100} n(100 + 900 + \ldots + 1) = \frac{1}{2} n
\]

and consequently, $n_p = \frac{101}{2} n$.

The starting point of this algorithm is obtained from LS method. The cost function allocates more weight to the points which their error standard deviation is less than a threshold. It tries to find a set of centers and radius of an ellipsoid by searching the space and minimizing the cost function (10).

4. Experimental results
To verify the proposed algorithm ADIS16488 is used. This sensor is composed of 3 accelerometers, 3 gyroscopes and 3 magnetometers. The device is shown in Fig. 1. In this paper, our focus is on magnetometer calibration. The data are collected by circulating the device in 3D space and in all possible directions. Data collection should be done carefully to collect a set of data which approximately covers all the space. Collected data are imported in the MATLAB and the proposed algorithm is applied.
In Fig. 2 the ellipsoid obtained from the experimental data (blue) and the one from LS algorithm (red) is shown. It can be seen that the LS ellipsoid is out of range in some sections. The

Fig. 3 shows experimental ellipsoid and the one from modified ellipsoid method. In comparison with Fig. 2 it is clear that the modified ellipsoid fits the experimental data better than LS ellipsoid. Difference between proposed ellipsoid and LS ellipsoid is depicted in Fig. 4. The cumulative sum error $E$ in LS method is about $1.2372e+03$ and in modified method is about $1.0717e+03$. As a result there is about 20 percent improvement in fitting accuracy.

To see the result better, LS ellipsoid and modified ellipsoid are shown in X-Y, X-Z and Y-Z plane separately. From Fig. 5, it’s clear that the LS ellipse in the left part of figure does not cover the data appropriately. On the other hand modified ellipse covers the experimental data better. The difference between LS ellipse and modified ellipse in Y-Z plane is shown in Fig. 6. The same results in X-Y plane is depicted in Fig. 7.

To better comparison of the two methods some criteria is check and the result is showed in table 1.

<table>
<thead>
<tr>
<th></th>
<th>LS ellipsoid fitting</th>
<th>Modified ellipsoid fitting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root sum of square of error</td>
<td>1.2372e+03</td>
<td>1.0717e+03</td>
</tr>
<tr>
<td>RMS of error</td>
<td>45.5393</td>
<td>28.5915</td>
</tr>
<tr>
<td>Standard deviation of error</td>
<td>38.3036</td>
<td>21.8150</td>
</tr>
</tbody>
</table>

5. Concluding Remarks
Ellipsoid fitting is common method for calibration the magnetometer. However, the ellipsoid fitting can be poor and ellipsoids maybe fit in non-ellipsoidal curves as well for real time applications in common least squares method. In this paper, a modified ellipsoid fitting method is proposed. In this method, nonlinear optimization is developed to minimize a novel cost function. Experimental data is collected using ADIS16488. The data are collected by circulating the device in 3D space and in all possible directions. Experimental results show that the proposed method increases the accuracy about 20 percent in the ellipsoid fitting.
Fig. 5: Experimental data vs. LS method (up) and modified ellipsoid (down) in X-Z plane

Fig. 6: Experimental data vs. LS method (up) and modified ellipsoid (down) in Y-Z plane

Fig. 7: Experimental data vs. LS method (up) and modified ellipsoid (down) in X-Y plane
6. References


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