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Word
Relationship of soft hypermodules and soft fuzzy hypermodules

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Abstract

The concepts of soft hypermodules and soft fuzzy hypermodules are introduced by applying soft set theory to hypermodules and fuzzy hypermodules. Also, the connection between soft hypermodules and soft fuzzy hypermodules is verified by associated (fuzzy) hyperoperations and homomorphisms.

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1 Introduction and preliminaries

Molodtsov in [3] proposed soft set theory for dealing with uncertainties in many areas such as economics, engineering, environmental sciences, medical sciences and social sciences which can not be deal with by classical methods, because classical methods have inherent difficulties. Then Maji et al. introduced several operations on soft sets. Aktaş and Çağman defined soft groups and obtained the main properties of these groups. Feng et al. defined soft semirings and soft ideals on soft semirings. Moreover, the concepts of soft rings and soft modules defined by Acar et al. and Qiu-Mei Sun et al., respectively. Moreover, this theory was discussed on some hyperstructures such as hypergroupoids, polygroups, semihyperrings and Γ-hypermodules.

Fuzzy hyperstructures is one of the applications of fuzzy set theory ([6]) to algebra which was initiated by Rosenfeld, who defined fuzzy groups. To associate a fuzzy set with each pair of elements of a set is one of the directions of research in the study of fuzzy hyperstructures. This idea was introduced by Corsini and Tofan and then, Sen, Ameri and Chowdhury introduced and analyzed fuzzy semi hypergroups. This idea was extended to fuzzy hyperrings and fuzzy hypermodules by Davvaz and Leoreanu-Fotea.

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Now, we focus on this direction, and apply soft set theory to hypermodules and fuzzy hypermodules, and investigate the connection between the defined notions by this application. Hence we give some definitions of (fuzzy) hyperstructures in the following.

Let $H$ be a nonempty set and let $\mathcal{P}^*(H)$ be the set of all nonempty subsets (nonzero fuzzy subsets) of $H$. A (fuzzy) hyperoperation on $H$ is a map $\circ : H \times H \to \mathcal{P}^*(H)$, and the couple $(H, \circ)$ is called a (fuzzy) hypergroupoid. A hypergroupoid $(H, \circ)$ is called a (fuzzy) semihypergroup if for all $x, y, z \in H$, we have $(x \circ y) \circ z = x \circ (y \circ z)$, which means that $\bigcup_{u \in x} u \circ z = \bigcup_{v \in y} v \circ x$, (where for all $\mu \in \mathcal{F}^*(H)$ and $A \subseteq H$, we have $(x \circ \mu)(r) = \bigvee_{t \in H} ((x \circ t)(r) \land \mu(t))$ and $(x \circ A)(r) = \bigvee_{a \in A} (x \circ a)(r)$

for all $r \in H$. Also, for two nonzero fuzzy subsets $\mu$ and $\lambda$ of fuzzy semihypergroup $(H, \circ)$, we have $(\mu \circ \lambda)(t) = \bigvee_{p, q \in H} (\mu(p) \land (p \circ q)(t) \land \lambda(q))$, for all $t \in H$. We say that a (fuzzy) semihypergroup $(H, \circ)$ is a (fuzzy) hypergroup if $x \circ H = H = H \circ x$ $(x \circ H = \chi_H = H \circ x)$ for all $x \in H$.

**Definition 1.1.** The triple $(R, \ast, \circ)$ is a (fuzzy) hyperring, if

1. $(R, \ast)$ is a commutative (fuzzy) hypergroup;
2. $(R, \circ)$ is a (fuzzy) semihypergroup;
3. "$\circ$" is distributive over "$\ast$".

**Definition 1.2.** Let $(R, \ast, \circ)$ be a (fuzzy) hyperring. A nonempty set $M$, endowed with a (fuzzy) hyperoperation "$+$", and external (fuzzy) hyperoperation "$\cdot$" is called a left (fuzzy) hypermodule over $(R, \ast, \circ)$ if the following conditions hold:

1. $(M, +)$ is a commutative (fuzzy) hypergroup;
2. $\cdot : R \times M \to \mathcal{P}^*(M)$ ($\mathcal{F}^*(M)$) is such that for all $a, b$ of $M$ and $r, s$ of $R$ we have
   
   (i) $r \cdot (a + b) = (r \cdot a) + (r \cdot b)$;
   (ii) $(r \ast s) \cdot a = (r \ast a) + (s \ast a)$;
   (iii) $(r \circ s) \cdot a = r \ast (s \ast a)$.

A nonempty subset $N$ of a hypermodule $M$ is called a subhypermodule of the hypermodule $(M, +, \cdot)$, if $(N, +)$ is a hypergroup and $R \cdot N \subseteq \mathcal{P}^*(N)$. Also, a nonempty subset $N$ of a fuzzy hypermodule $M$ is called a subfuzzy hypermodule if for all $x, y \in N$ and $r \in R$, we have

1. $(x + y)(t) > 0$ implies that $t \in N$;
2. $x + N = \chi_N$;
3. $(r \cdot x)(t) > 0$ implies that $t \in N$.
Example 1.3. Let $R = [0,1]$ and define the hyperoperation $\oplus_{\max}$ for all $x, y \in R$ by
\[
x \oplus_{\max} y = \begin{cases} 
\max\{x, y\}, & x \neq y \\
[0, x] & x = y
\end{cases}
\]
Then, $(R, \oplus_{\max}, \cdot)$ is a hyperring (Krasner hyperring) where “$\cdot$” is the ordinary multiplication on real numbers. Also, $I = [0, 0.5]$ is a hyperideal of $R$. Now, define $a \circ b = (a \cdot b) \oplus_{\max} I$ for $a, b \in R$. Then, $(R, \oplus_{\max}, \circ)$ is a hyperring. Set $R/I = \{r + I \mid r \in R\}$, where “$+$” is the ordinary additive, and define the following hyperoperations on $R/I$:
\[
(a + I) \boxplus (b + I) = \{c + I \mid c \in a \oplus_{\max} b\}, \quad r \boxminus (z + I) = \{t + I \mid t \in r \odot z\}.
\]
Then, $(R/I, \boxplus, \boxminus)$ is a hypermodule over the hyperring $(R, \oplus_{\max}, \circ)$.

Example 1.4. ([1]) Let $(M, +, \cdot)$ be a module over a ring $(R, +, \cdot)$ without unity. Define the following fuzzy hyperoperations for all $a, b \in M$ and $r, s \in R$:
\[
r \boxplus s = X_{[r+s]}, \quad r \boxminus s = X_{[r,s]}, \quad a \oplus b = X_{[a+b]}, \quad (r \odot a)(t) = \begin{cases} 1/2, & \text{if } t = ra \\
0, & \text{otherwise}
\end{cases}
\]
Then $(M, \oplus, \odot)$ is a fuzzy hypermodule over the fuzzy hyperring $(R, \boxplus, \boxminus)$.

Let $(M_1, +_1, \cdot_1)$ and $(M_2, +_2, \cdot_2)$ be two (fuzzy) hypermodules over a hyperring $R$. The map $f : M_1 \rightarrow M_2$ is called a (strong) homomorphism of (fuzzy) hypermodules if for all $x, y \in M_1$ and $r \in R$, we have $f(x +_1 y) \subseteq (\leq) f(x) +_2 f(y)$ and $f(r \cdot_1 x) \subseteq (\leq) r \cdot_2 f(x)$ ($f(x +_1 y) = f(x) +_2 f(y)$ and $f(r \cdot_1 x) = r \cdot_2 f(x)$).

2 Soft (fuzzy) hypermodules

Let $X$ be an initial universe set and $E$ be a set of parameters. $\mathcal{P}(X)$ denotes the power set of $X$ and $A \subseteq E$. Then, $F_A$ is called a soft set ([3]) over $X$, where $F$ is a mapping given by $F : A \rightarrow \mathcal{P}(X)$. In fact, a soft set over $X$ is a parameterized family of subsets of the universe $X$. For $e \in A$, $F(e)$ may be considered as the set of $e$-approximate elements of the soft set $F_A$.

Example 2.1. ([4]) Consider a soft set $F_E$, which describes the attractiveness of houses that one is considering for purchase. Suppose that there are six houses in the universe $X$, given by $X = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$ is a set of decision parameters, where $e_i (i = 1, 2, 3, 4, 5)$ stand for the parameters “expensive”, “beautiful”, “wooden”, “cheap”, and “in green surroundings”, respectively. Consider the mapping $F$ defined by $F(e_1) = \{h_2, h_4\}$, $F(e_2) = \{h_1, h_3\}$, $F(e_3) = \{h_3, h_4, h_5\}$, $F(e_4) = \{h_1, h_3, h_5\}$ and $F(e_5) = \{h_1\}$. The soft set $F_E$ is a parameterized family $(F(e_i) \mid 1 \leq i \leq 5)$ of subsets of the set $X$, and can be viewed as a collection of approximations:

$F_E = \{\text{expensive houses} = \{h_2, h_4\}, \text{beautiful houses} = \{h_1, h_3\}, \text{wooden houses} = \{h_3, h_4, h_5\}, \text{cheap houses} = \{h_1, h_3, h_5\}, \text{in green surroundings houses} = \{h_1\}\}.$

For a soft set $F_A$, the set $\text{Supp}(F_A) = \{x \in A \mid F(x) \neq \emptyset\}$ is called the support of the soft set $F_A$. If $\text{Supp}(F_A) \neq \emptyset$ then the soft set $F_A$ is called non-null.
Definition 2.2. Let $F_A$ be a non-null soft set over a (fuzzy) hypermodule $M$. Then $F_A$ is called a soft (fuzzy) hypermodule over $M$ if $F(x)$ is a sub(fuzzy) hypermodule of $M$, for all $x \in \text{Supp}(F_A)$.

Definition 2.3. Let $F_A$ and $G_B$ be soft hypermodules over two hypermodules $M$ and $M'$, respectively, and $f : X \rightarrow X'$ and $g : A \rightarrow B$ be two functions. A pair $(f, g)$ is called a homomorphism of soft (fuzzy) hypermodules, denoted by $(f, g) : F_A \rightarrow F_B$, if $f(F(x)) = G(g(x))$ for all $x \in A$, and $f$ is a homomorphism of (fuzzy) hypermodules.

Theorem 2.4. Let $F_A$ be a soft fuzzy hypermodule over a fuzzy hypermodule $(M, \oplus, \odot)$ on fuzzy hyperring $(R, \oplus, \odot)$. Then $F_A$ is a soft hypermodule over associated hypermodule $(M, +, \cdot)$ on associated hyperring $(R, \oplus, \odot)$.

Theorem 2.5. $F_A$ is a soft hypermodule over a hypermodule $(M, +, \cdot)$ on a hyperring $(R, \oplus, \odot)$ if and only if $F_A$ is a soft fuzzy hypermodule over the associated fuzzy hypermodule $(M, \oplus, \odot)$ on the associated fuzzy hyperring $(R, \oplus, \odot)$.

Theorem 2.6. Let $(F_A, \oplus_1, \odot_1)$ and $(G_B, \oplus_2, \odot_2)$ be two soft fuzzy hypermodules over fuzzy hypermodules $(M_1, \oplus_1, \odot_1)$ and $(M_2, \oplus_2, \odot_2)$, respectively, $(F_A, +_1, \cdot_1) = \Phi_S((F_A, \oplus_1, \odot_1))$, and $(G_B, +_2, \cdot_2) = \Psi_S((G_B, \oplus_2, \odot_2))$ be associated soft hypermodules. If $(f, g)$ is a homomorphism of soft fuzzy hypermodules, then $(f, g)$ is a homomorphism of soft hypermodule homomorphism, too.

Theorem 2.7. Let $(F_A, +_1, \cdot_1)$ and $(G_B, +_2, \cdot_2)$ be two soft hypermodules over the hypermodules $M_1$ and $M_2$ and $(F_A, \oplus_1, \odot_1) = \Phi_S((F_A, +_1, \cdot_1))$, and $(G_B, \oplus_2, \odot_2) = \Phi_S((G_B, +_2, \cdot_2))$ be the associated soft fuzzy hypermodules. $(f, g)$ is a homomorphism of soft fuzzy hypermodules if and only if $(f, g)$ is a homomorphism of soft hypermodules.

References


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Proc-272
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