Estimation of strain hardening factor and ductility of a reduced beam section (RBS) connection

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Abstract
This paper presents a reduced beam section (RBS) approach via the introduction of two pairs of longitudinal voids in the beam web to enhance the ductility of post-Northridge connections. In order to achieve the highest connection ductility, using finite element method a parametric study was done on the geometry of such voids by considering three different sizes of SAC specimens, SAC3, SAC5 and SAC7. To generalize the design procedure and to make it to be applicable for other beam sections, the suitable equations were proposed to estimate the strain hardening factor, limiting value and ductility of RBS connections with longitudinal voids.

Keywords: Strength, Ductility, Strain hardening factor

1. Introduction

Steel moment resisting frames had extensive brittle fractures in their welded connections during the earthquakes in Northridge (1994) and Kobe (1995) [1]. Since then modifications to design procedure of pre-Northridge connections and its welding has been introduced. E70T-4 type welding changed to E70-TGK2 with smooth welding access holes and backing bar removed from the bottom beam flange [2, 3].

This type of connection is now known as post-Northridge connection. Reducing the beam section is one known and easy method to improve the strength and ductility of the post-Northridge connections by transferring the stress concentrations to a region away from the column face. Reduced beam web (RBW) connections include the wedge design beam connections [4] and reduced beam web with circular voids [5], rectangular long voids [6], drilled voids [7] and RBW with arch-shape cuts at the beam web [8].

In 2009, Hedayat and Celikag [6] proposed the use of single pair of rectangular long voids at the beam web to enhance the connection ductility of post-Northridge connections (Figure 1). This method was effective for beams with the maximum depth equal to 600 mm.

This study was aimed to increase the strength and ductility of post-Northridge connections with deep beams by creating two pairs of longitudinal voids at the beam web (Figure 2). To find out the best connection configuration, a parametric study was done with respect to the size and the location of the voids. This parametric study was carried out through the 141 models and it was based on the finite element method. At the end to generalize the design procedure and to make it applicable for other beam sections (e.g. European sections) the maximum moment that can be developed at column face was evaluated by estimating the strain hardening factor. Finally, using analytical results, suitable equations were proposed to estimate the strain hardening factor and ductility of reduced beam web connections with multi longitudinal voids.
Figure (1): Single longitudinal voids with stiffeners and tube at the center of voids proposed in [6]

Figure (2): Modified post-Northridge connections with multi longitudinal voids

2. Finite Element Method

Three non-modified post-Northridge connections, comprised of three pre-tested specimens, SAC3 (beam: W24 × 68; column: W14 × 120), SAC5 (beam: W30 × 99; column: W14 × 176) and SAC7 (beam: W36 × 150; column: W14 × 257) from [3] were modeled using the general purpose finite element program ANSYS [9]. These specimen sizes were chosen since they are considered as good representatives of the conventional pre/post Northridge specimen sizes, small, medium and large size connections [3]. The length of the beam (L/2) and the column for all these specimens were 3429 mm and 3658 mm respectively. The proposed beam end configuration with different values of design parameters was then applied to all these non-modified post-Northridge connections to create modified specimens.

After Northridge earthquake, Miller [2] inspected more than 100 damaged buildings and also experimental tests were conducted by the SAC group (e.g. [3]) on the pre and the post-
Northridge connections all there showed that, the failure of this type of connection is not often due to the failure of bolts. Therefore, in the finite element model, the bolts were not exactly modeled but shear tab, bolt holes and interaction between the shear tab and the beam web were modeled to achieve a realistic model. In finite element models, both welds and base metals were modeled using shell elements and the associated material property was defined for each one. SHELL43 was used to model weld, shear tab, continuity plates and column plates whereas SHELL181 was used to model the beam plates. SHELL43 and SHELL181 are one-layer four-node and multi-layer eight-node shell elements respectively. These elements have six degrees of freedom at each node and all of them have plasticity, large deflection, and large strain capabilities. In the case of using SHELL181, each element was separated into five layers across the thickness. The number of layers was selected based on the finite element study carried out by Gilton and Uang [10].

In order to determine the appropriate mesh density, a mesh sensitivity study was done for both modified and non-modified specimens based on the recommendation given by the ANSYS program and by comparing the analytical results with experimental results of References [3]. Figure 3 shows the finite element mesh for a typical modified specimen with multi longitudinal voids. A very fine mesh size was used for the beam flange and web at the voids area to accurately capture the local buckling of the beam flange and web at this region. The number of elements for specimens (SAC3, SAC5 and SAC7) in average was 27000 that around of 30% to 50% of this amount were due to the size of the voids located at the beam web.

To perform material nonlinearity analyses, plasticity behavior was based on the Von-Mises yielding criteria and the associated flow rule. Isotropic hardening was assumed for the monotonic analysis, whereas kinematic hardening was assumed for the cyclic analysis as used by Mao et al. [11] and Ricles et al. [12]. A bilinear material response with a post yielding stiffness equal to 4% of the modulus of elasticity of steel was used for the base metals in accordance with the material properties given by Lee et al. [3]. For weld metals, a multi-linear material response based on the material property given by Mao et al. [11] and Ricles et al. [12] was used.

The analysis with monotonic loading were conducted by applying a monotonic vertical displacement load to the beam tip until achieving more than 4% total rotation at the column web center, whereas the load history recommended by FEMA [13] was utilized for analyses with cyclic loading. When applied loads are in the vertical direction only, then the out-of-plane deformations (normal to the web) may not occur. Therefore, in order to ensure that buckling occurs when the model becomes unstable, the imperfect model was analyzed under cyclic or monotonic loadings. In this study, in order to determine the imperfect model, first the buckling mode shapes were computed in a separate buckling analysis and then were implemented to perturb the original perfect geometry of the model as it was done by SAC group [14].

In order to verify the validity of the numerical research, Hedayat et al. ([6], [7]) prepared finite element models for the specimens SAC3, SAC5 and SAC7 of the experimental study conducted by [3]. The numerical results agreed suitably with the experimental ones.
3. Details of the proposed beam end configuration (BEC)

Details of the proposed BEC are shown in Figure 2. The equation for minimum required shear depth (equation 1) [15] can be used to determine the minimum clear vertical distance, parameters $a_1$ and $a_2$, between the two voids.

$$\phi R_n = 0.9 \times 0.6 \times f_y \times A_g$$

(1)

where, $\phi R_n$ is total shear force which is equal to the expected beam plastic moment capacity at column face divided by half of the total beam length ($L_b/2$), $f_y$ is the beam nominal yield strength, and $A_g$ is gross shear area ($A_g = a \times w_t$, $t_w$ is the beam web thickness). If the over strength factor is taken as 1.2 then the equation (1) can be simplified to find the minimum required shear depth, $a_1$ and $a_2$, as follows:

$$a_1 = \frac{5.29 \times Z_b}{L_b \times t_w}$$

(2)

where, $Z_b$ is the plastic section modulus of the beam. Parameter $a_1$ is equal to 465 mm, 300 mm and 210 mm for specimens SAC7, SAC5 and SAC3 respectively. The horizontal length of each void is 1.25 times the beam overall depth ($L_{V1}=L_{V2}=1.25D$). The minimum value of parameter $b$ (see Figure 2) is 1.4 times the parameter $c$, where $c$ is the roots radius of the beam ($b=1.4c$). The factors 1.25 and 1.4 were selected based on the parametric study done by Hedayat et al. [6] for RBW connections with single longitudinal voids. The first pair of voids was located as such that their distance from center of voids to the face of column was equal to the overall depth of the beam (Figure 2). Void depth $D_{v1}$ was achieved by equation (3), where $t_f$ is the beam flange thickness and $b_1$ is as shown as the inset figure in Figure 3. In this study, for all cases, same size the voids were used ($D_{v1}=D_{v2}$).

$$D_{v1} = [(D-2t_f-a_1)/[2((b_1/D_{v1})+1))]$$

(3)
4. Design parameters

All specimens SAC3, SAC5 and SAC7 were used for the parametric study which was carried out on the geometry of the voids by defining three design parameters, α, β and γ. These parameters are defined as follows:

Parameter α: This parameter is ratio of $b_1$ to $D_{v1}$ ($\alpha = \frac{b_1}{D_{v1}}$ where $b_1$ and $D_{v1}$ are shown in Figure 2). The values used for parameter α were 2, 3 and 4. Hence, by assuming the value of this parameter the first pair of voids depth (parameter $D_{v1}$) can be obtained by using equation 3.

Parameter β: This parameter is the ratio of $b_2$ to $b_1$ which are shown in Figure 3 ($\beta = \frac{b_2}{b_1}$). Four different values were used for parameter β; 1, 0.75, 0.5 and 0.25. Hence, by assuming the value of this parameter and knowing $D_{v2}$ ($D_{v2} = D_{v1}$), the perpendicular distance of the second pair of voids (parameter $a_2$) can be obtained. However, this value cannot be less than the value obtained from equation 2. It should be noted that, in this study, the value of parameter $a_1$ was directly obtained from equation 2.

Parameter γ: This parameter is the ratio of the horizontal clear distance between the first and the second voids to the void length ($\gamma = \frac{CD}{L_v}$). The values used for parameter γ were 0.1, 0.15, 0.2 and 0.25.

5. Effect of design parameters on the strength and ductility of the modified post-Northridge connections

By considering all design parameters mentioned in the previous paragraph, totally 141 models were analyzed. Table 1 summarizes the results of some of them. Connection ductility was evaluated by using parameter $\theta_{CWC}$, which is the total rotation of the connection at the column web center. It is calculated by dividing the beam tip deflection by a distance measured from the beam tip to the column web center. Connection strength was evaluated by using the $M/M_P$ ratio which is the ratio of the applied moment measured at the column face level at the failure time to the full beam plastic moment capacity at the column face level. This comparison was done for monotonic loading. Based on the ANSI/AISC 341-10 [16], beam-to-column connections used in the seismic force resisting system (SFRS) shall satisfy the following requirements:

1) The connection shall be capable of accommodating a story drift angle of at least 0.04 rad
2) The measured flexural resistance of the connection, determined at the column face, shall equal at least 0.80$M_P$ of the connected beam at a story drift angle of 0.04 rad.

Based on the analytical results presented in Table 1 and with respect to the requirements mentioned in the above paragraph, from the strength and ductility point of views, for all modified SAC specimens, the highest connection performance might be achieved for $\alpha=2$, $\beta=0.75$ and $\gamma=0.1$. These specimens easily achieved the minimum required strength. The maximum ductility achieved for these specimens were 5.0, 4.02 and 3.91 percent radian for beam depths 912 mm, 750 mm and 600 mm respectively.
### Table (1): Strength and ductility of some modified specimens

<table>
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<th>SPECIMEN</th>
<th>SAC7</th>
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<td>β</td>
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### 6. Generalization of the Design Procedure

The modification procedure presented in the previous section is easy for application. However, a few key parameters, such as, gravity effect, length of the beam, and moment gradient of the beam were neglected. This section consideration is to use these parameters to generalize the design procedure that will capable the proposed modifications to be applicable to the other sections. This design procedure is based on the one presented in FEMA350 [13] and is similar to the one presented by Engelhardt and Uang [17] for RBS and welded haunch connections.
The design method is based on the limiting moment, \( M_{pd} \) (equation 4), and the associated shear force, \( V_{pd} \) (equation 5), at critical plastic section which is the start point of the first pair of voids. The critical plastic section is denoted in Figure 2 by parameter \( S_C \) and can be obtained by using equation 6.

\[
M_{pd} = C_{pr} \times Z_{RBWS} \times F_{ye} \tag{4}
\]
\[
V_{pd} = 2M_{pd} / L + wL/2 \tag{5}
\]
\[
S_C = D - L_{V1}/2 + r_V \tag{6}
\]
\[
Z_{RBWS} = Z_b - D_{V1} \times t_w \times (a_1 + D_{V1}) \tag{7}
\]

In these equations \( F_{ye} \) is the expected yield stress of material, \( Z_{RBWS} \) is plastic section modulus at RBW area (equation 7), \( w \) is the gravity factored loads on the beam and parameter \( L' \) is shown in Fig.2. The moment at column face, \( M_f \), is

\[
M_f = M_{pd} + V_{pd} \times S_C \tag{8}
\]

By substituting equations (4) and (5) into equation (8) and normalizing both sides with respect to the full section plastic moment of beam \((Z_b, F_{ye})\), the maximum normalized moment at column face, \( \eta_t \), will be:

\[
\eta_t = \eta_g + \eta_e = \frac{w \times L \times S_C}{2Z_b} + C_{pr} \times \frac{Z_{RBWS}}{Z_b} \times \frac{1}{1 + \frac{2S_C}{L'}} \tag{9}
\]

The first term, \( \eta_g \), considers the effect of gravity loads and the second term, \( \eta_e \), considers the effect of seismic loads. For simplicity, the influence of the portion of gravity load within the length \( S_C \) was neglected. To enhance the ductility of a post-Northridge connection, the configuration of voids (or the values of parameters \( \alpha, \beta \) and \( \gamma \)), must be chosen as such to keep the value of \( \eta_t \) in an appropriate range to avoid beam flange fracture at both RBW area and WAH region before achieving adequate connection ductility, 4% total rotation at column web center.

For any modified connection \( \eta_t \) can be determined using experimental and analytical results. For example, \( \eta_t \) is 1.05 [17] and 1.15 [13] for RBS connections of bottom flange cut and both top and bottom beam flange cut respectively. However, in the study done by Hedayat et al. [6] for RBW connections with single rectangular voids, a range of appropriate value for \( \eta_t \) (between 1.05 and 1.14) depending on the beam overall depth and the connection type was proposed. The normalized moment developed at the column face, \( M/M_p \), for all modified specimens are presented in Table 1. This is the parameter \( \eta_t \). Appropriate values of parameter \( \eta_t \) can be determined by comparing the values of \( \eta_t \) with the connection ductility, \( \theta_{cw} \). Despite being difficult to select, based on the data presented in Table 1, a value between 0.97 and 1.01 might be the best value of parameter \( \eta_t \). However, according to the finite element results, a value closer to the lower bound might be more appropriate for deeper beams (beam depth\( \geq 750 \) mm) while a value closer to the upper bound might be more suitable for shallower beams.

\( C_{pr} \) in equation 4 is a factor to account for the peak connection strength, including strain hardening, local restraint and additional reinforcement. In FEMA350 [13], the \( C_{pr} \) factor is given by equation \((f_y+f_u)/2f_y\), where \( f_y \) and \( f_u \) are the specified minimum yield and tensile stress of material respectively. FEMA350 [13] proposes the use of value 1.2 for any case of
modified connections except where otherwise noted in the individual connection design procedure. This factor is the ratio of the measured moment at the start point of the first pair of voids (i.e. at the critical plastic section) at the connection failure time to the beam plastic moment capacity at this location. For the proposed BEC, this factor is a function of the configuration and the size of voids. Therefore, in this study the nonlinear model given in equation 10 was used to estimate the $C_{pr}$ factor, based on the design parameters $\alpha$, $\beta$, $\gamma$ and beam flange and web slenderness ratios. In this equation $b_f$, $t_f$ and $t_w$ are the beam flange width and thickness and the beam web thickness respectively. Constant $C_1$ and exponents $C_2$ to $C_6$ were determined using regression analyses and are summarized in Table 2. The last column of Table 2 gives the observed average error. This error is the mean square error which emphasizes the effect of large errors ($\sum_{i=1}^{n}(\text{real } C_{pr} - \text{estimated } C_{pr})^2$, $n$ is total number of data).

$$C_{pr} = C_1 \times \alpha^{C_2} \times \beta^{C_3} \times \gamma^{C_4} \times [(D - 2t_f)/t_w]^{C_5} \times (b_f/t_f)^{C_6}$$

(10)

After finalizing the geometry of proposed BEC, the connection ductility can also be estimated using equation 11. Constant $C_1$ and exponents $C_2$ to $C_7$ were determined using regression analyses and are summarized in Table 2. The last column of Table 2 gives the average of mean square error observed for all SAC specimens.

$$\theta_{CWC} = C_1 \times \alpha^{C_2} \times \beta^{C_3} \times \gamma^{C_4} \times [(D - 2t_f)/t_w]^{C_5} \times (b_f/t_f)^{C_6} \times (L_b/D)^{C_7}$$

(11)

Note that in order to use equations 11 and 12, all design principles presented in section 4 should be considered (i.e. $L_{v1}=L_{v2}=1.25D$; $D_{v1}=D_{v2}$; $a_i = 5.29 \times Z_b/(L_n\cdot t_w)$; $a_2 \geq a_1$ and the distance from the center of the first pair of voids to the column face is equal to the beam overall depth, $D$).

| Table (2): Variables C1 to C7 to predict $C_{pr}$ and $\theta_{CWC}$ |
|--------------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| Equation                  | C1  | C2  | C3  | C4  | C5  | C6  | C7  | Err |
| Eq. 10 to predict $C_{pr}$| 1.831 | 0.169 | 0.121 | 0.1181 | -0.783 | 0.979 | - | 0.016 |
| Eq. 11 to predict $\theta_{CWC}$ | 2.459 | -0.160 | -0.107 | -0.191 | 1.189 | -1.480 | -0.300 | 0.006 |

7. Conclusion

The aim of this study was to find practical and effective way to enhance the ductility and strength of post-Northridge connections so that they are better applicable for new and existing buildings. For this purpose multi longitudinal voids horizontally opened in the beam web where the distance of the centerline of the first pair of voids from the face of the column was equal to the beam depth. All voids had same length (1.25 times the beam overall depth) and same depth. Design parameters $\alpha$, $\beta$ and $\gamma$ were defined to change the geometrical location of the voids. A parametric study was carried out with respect to these parameters to find the optimum location of voids to achieve the highest connection strength and ductility.
This finally led to modeling of the 141 post-Northridge specimens of different beam overall depths.

Analytical results showed that the presence of the second pair of voids were efficient in uniformly distributing the plastic equivalent strains along the beam length and, therefore, significantly reducing the plastic equivalent strain concentration at the column face level, weld access hole region and at the beam flanges at the void areas. It finally led to the achievement of the adequate strength and ductility for the specimens of the proposed BEC. Results also showed that the location and size of voids can influence the performance of the modified connections. The effect of the configuration of voids was investigated using design parameters \( \alpha, \beta \) and \( \gamma \). Results indicated that the highest connection strength and ductility can be achieved for \( \alpha, \beta \) and \( \gamma \) equal to 2, 0.75 and 0.1 respectively. These specimens easily achieved the minimum required strength. From the ductility point of view, however, the proposed method caused a remarkable increase in the ductility of all connections, its efficiency was much more for deeper beams (beams of overall depth greater than 750 mm) such that the deep beam specimens SAC7 (with beam overall depth equal to 912 mm) easily achieved five percent total rotation.

In order to generalize the design procedure to be applicable for any beam section and to take other design parameters such as beam length, beam moment gradient and beam gravity loads were taken into account, equations 10 and 11 were proposed to estimate the best configuration of voids. Finally, the best location of voids was controlled by using the parameter \( \eta_t \). It is expected that any modified specimen (even in the case of a shallow beam) with appropriate value of parameter \( \eta_t (0.98 \leq \eta_t \leq 1.01) \) achieves both adequate connection’s strength and ductility simultaneously.

8. References


