Simulation Run Time Reduction using Model Order Reduction Techniques

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ABSTRACT

Optimization and real-time control of production scenarios in oil/gas reservoirs is time-consuming and requires powerful computers with high amount of storage capacity. This is because these systems are of large scale. The time-consuming problem of running such models is handled in two ways: Parallel Computing and Model Simplification. Model simplification is implemented in two ways of Black box modeling and white box modeling [1].

Recently Model Order Reduction (MOR) techniques are proposed which benefit from strong points of so-called white-box and black box traditional modeling approaches. Since governing equations of multi-phase fluid flow in the reservoir are mainly nonlinear; therefore nonlinear model order reduction methods like Proper Orthogonal Decomposition (POD) are employed to reduce the system order and accelerate the simulation process.

As it is reported in the literature MOR by POD has resulted in speedup factor of 3-4 when it is applied to system of equation solved by Newton method. By applying Newton method greater speedup factors could not be achieved since full order Jacobian matrix should be computed in every iteration in order to obtain reduced Jacobian matrix (J_r) [2]. In this work POD is applied to a system of equations which is solved by IMPES method and encouraging speedup factor of 6 or more is obtained.

Nomenclature:

B_α: Formation Volume Factor Of Phase Alpha
D: Depth
K_r: Relative Permeability
N_c: Number Of Blocks
R: Radius
S: Saturation
T_α: Transmissibility Factor Of Phase Alpha
<Greek Letters>
Δ: Difference
Λ: Mobility Ratio
<Subscripts>
R: Reduced
A: Phase A
O: Oil
W: Water
<Superscripts>
T: Transpose

1. Introduction

Reservoir engineering like any other area of science which is concerned with solving large scale system of equations (10^5 - 10^6 order of system) encounter the problem of long simulation run-time.

Governing equations of fluid flow in reservoir simulation are nonlinear. This is because relative permeability is a non-linear function of saturation and also rock and fluid compressibility are functions of pressure [3]. Therefore, among different methods which are used for Model Order Reduction, proper orthogonal decomposition (POD) is applied

which is used for nonlinear systems [4]. This entails running a full simulation model, saving a number of snapshots (state vectors) at different time steps and then constructing a set of basis vectors from the snapshots. The basis vectors are created through singular value decomposition (SVD) of snapshots matrix [5]. All these basis vectors are orthogonal to each other, time independent and show the spatial distribution of transfer function of the system. By selecting the most important basis vectors, order of obtained equation set is reduced to much less than the full model while the basic features of the system are preserved. So, the computational time is reduced while the accuracy is retained. For this purpose, initially the equations for simulating two phase flow of oil and water are presented and the procedure of solving the nonlinear system of equations by IMPES method is explained. Then A two dimensional heterogeneous model is simulated by IMPES method. After that, POD is applied to the full model data and basis vectors of the system are obtained. At the next step, MOR is used to reduce system of equations obtained by IMPES. Later on, ROM is run with the same scenario used for training and also a different scenario and finally the results are compared to those of the full model.

2. IMPES and MOR combination for oil-water flow

In this part, flow equations of the two phase fluid (oil-water) in porous media are explained briefly. Then, by discretization, the system of equation which should be solved by IMPES method is obtained. Afterwards by applying POD, the two-phase reduced order model is obtained.

2.1Oil-water formulation and discretization: By combining mass conservation equations with modified form of Darcy equation for multiphase flow, the required equation to model two phase flow of immiscible oil and water can be derived. Then, for each component /phase α (with α=O for oil and α=w for water) one can write [6]:

$$-\nabla.(\rho_\alpha u_\alpha)+\rho_\alpha q_\alpha = \frac{\partial}{\partial t}(\emptyset \rho_\alpha s_\alpha) \qquad (1)$$

By using modified form of Darcy equation for multi-phase flow the volumetric velocity (u_α) is given:

$$u_\alpha = \frac{k_{r\alpha}k}{\mu_\alpha}\nabla(p_\alpha - \rho_\alpha g D) \qquad (2)$$

By combining equation (1), (2), saturation constraint ($s_w + s_o$=1), capillary pressure relationship ($p_c(s_w) = p_o - p_w$), and doing some manipulation the following equations are obtained:

$$\begin{cases} \nabla.[\lambda_o k(\nabla p_o - \rho_o g D] + q_o = \frac{\partial}{\partial t}(\emptyset\, \rho_o s_o) \\ \nabla.[\lambda_w k(\nabla p_w - \rho_w g D] + q_w = \frac{\partial}{\partial t}(\emptyset\, \rho_w s_w) \end{cases} \qquad (3)$$

Where λ_α is the α-phase mobility and is defined as:

$$\lambda_\alpha(s_\alpha) = \frac{k_{r\alpha}}{\mu_\alpha B_\alpha} \qquad (4)$$

The ρ_α can be written as $\rho_\alpha = \frac{\rho_{\alpha,sc}}{B_\alpha}$, where B_α is formation volume factor of phase α and $\rho_{\alpha,sc}$ is standard density of phase α. By stipulating the boundary and initial conditions the model is finalized. For the case of two phase flow of oil

and water, four equations and four unknowns (p_o, s_o, p_w, s_w) are involved. p_o and s_o are selected as the primary unknowns and when these two are computed, p_w, and s_w can be calculated from capillary pressure relationship and saturation constraint. These equations can be solved numerically. Therefore, using finite difference method these equations are discretized. Figure 1 shows a discretized portion of one-dimensional grid.

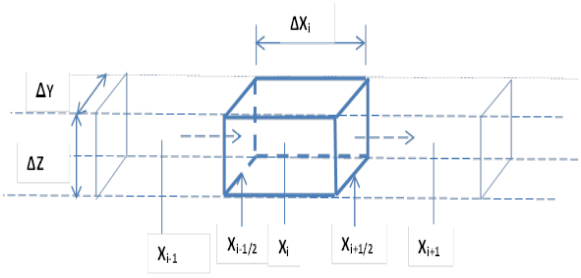

Figure 1

Portion of a one-dimensional grid

By discretizing the equation set (3), the following system of equation is obtained:

$$\begin{cases} [T_o][p_o] + [D_o][s_o] = [Q_o] \\ [T_w][p_w] + [D_w][s_w] = [Q_w] \end{cases} \quad (5)$$

Where T represents flow term and is function of pressure and saturation, D represents accumulation term which is function of pressure only and Q represents source/sink term witch is a function of pressure and saturation.

This is a non-linear set of equations and it is solved by IMPES solution method. P_o and S_o are taken as main variables. By omitting S_w and P_w from above system by using saturation constraint and capillary pressure relationship and after omitting the S_o, the following nonlinear relationship for P_o is obtained:

$$[D_w]^{n+1,k}[T_t]^n[P_o]^{n+1,k+1} - [D_o]^{n+1,k}[T_w]^n[P_c]^n + [D_w]^{n+1,k}[Q_o]^n + [D_o]^{n+1,k}[Q_w]^n = 0 \quad (6)$$

Where n is current time step and n+1 is the next time step and k is iteration level. By considering the constraint for size of time step for stability of solution, P_o at the next time step (n+1) can be solved from the above system and subsequently S_o at the next time step (n+1) will be calculated explicitly.

Therefore, the only system of equations that is solved at each iteration of every time step is:

$$[F]^k[p]^{k+1} = [b]^k \quad (7)$$

The goal is to reduce the number of N_c equations (N_c is the number of blocks) in this system to n_r equation ($r << N_c$) by applying Model Order Reduction (MOR) method.

2.2 Model order reduction of reservoir model:

Now a technique based on the orthogonal decomposition of a collection of measurements of physical quantities (such as pressure) in position and time (signals) is used to reduce the complexity of models. Following ideas from Fourier series expansions [7, 8], signals are represented as series of orthonormal functions [9]. These so-called basis functions approximate the spatial distribution of the signal while the coefficients of the basis functions represent the time-varying dynamics. The basis functions are derived from measured or simulated data and are

physically relevant. Therefore, the transfer function of the system can be stated as a linear combination of its basis functions. Here because discrete form of the pressure function is involved, basis vectors are used instead of basis functions (basis vectors):

$$P(t) = a_1(t)\psi_1 + \cdots + a_k(t)\psi_k \qquad (8)$$

Where ψ_i are the basis vectors and $a_i(t)$ are the Fourier coefficients. The basis vectors ψ_i are the eigenvectors of correlation matrix which is equal to $C = X_p^T \times X_p$ where X_p is pressure snapshot matrix and X_p^T is its transpose. The basis vectors can also be obtained by singular value decomposition of snapshot matrix X_p.

To obtain the snapshot matrix, the full model should be ran with a specific scenario (specific internal boundary condition which is well bottom hole pressure). At each time step variable states which are pressures in this case are saved. Next, pressure snapshots are selected and pressure snapshot matrix (X_p) is built. Now suppose that K pressure snapshots are selected, i.e. X_p matrix is of dimension $N_c \times K$, where N_c is the number of grid blocks and K is the number of snapshots.

$$X_p = \left[x_p^1, x_p^2, x_p^3, \ldots, x_p^k\right]_{Nc\times k} \quad or$$

$$X_p = \begin{bmatrix} p_1^1 \cdots & p_1^2 \cdots & p_1^k \\ p_2^1 & p_2^2 & p_2^k \\ \vdots \cdots & \vdots \cdots & \vdots \\ p_{N_c}^1 \cdots & p_{N_c}^2 \cdots & p_{N_c}^k \end{bmatrix}$$

A large number of eigenvectors (Columns of ψ matrix) has very low levels of energy, and can be excluded without affecting the overall system energy significantly. In other words, the most important basis vectors of the system are selected such that at least 99% of the total system energy is preserved. Mathematically, this is stated as [4]:

$$\frac{\sum_{i=1}^{i}\lambda_i}{\sum_{i=1}^{k}\lambda_i} > 0.99 \qquad (9)$$

Where λ_i is eigenvalue corresponding to that eigenvector (basis vector) and r is the number of main basis vectors.

The Matrix form of equation (8) is presented as:

$$[p]_{N_c\times 1} = [\psi]_{N_c\times k}[a]_{k\times 1} \text{ or } P = \psi a \qquad (10)$$

By selecting the main (r) basis vectors, the relation (10) is written as:

$$[p]_{N_c\times 1} \cong [\psi]_{N_c\times r}[a]_{r\times 1} \qquad (11)$$

Now to get the reduced set of equation which is solved in ROM, the relation (11) is inserted to equation (10) (full model system equation) and pre-multiplied by ψ_l^T . The result is the system of equation in ROM which is written as:

$$[A_r]_{r\times r}[a]_{r\times 1} = [B_r]_{r\times 1} \qquad (12)$$

Where

$$[A_r]_{r\times r} = [\psi]^T{}_{r\times N_c}[A]_{N_c\times N_c}[\psi]_{N_c\times r}$$

$$[B_r]_{r\times 1} = [\psi]^T{}_{r\times N_c}[B]_{N_c\times 1}$$

Now at this reduced form, in order to obtain the pressure distribution at each time step, it is only required to calculate $\underline{r}$ number of $\underline{a_i}$ coefficients. In other words the unknown parameters are reduced to $\underline{r}$ compared to $\underline{N_c}$ number of pressure (P) terms in full model. Since $r << N_c$ computation time is reduced significantly and the model is simulated in a much shorter period of time.

3. Model Order Reduction Implementation

Since source codes of a professional simulator like Eclipse were not available, simulation codes are generated in MATLAB [10] environment originally. Afterwards the validity of MATLAB codes is verified with Eclipse [11, 12], these codes are employed to construct the reduced order model (ROM). Then the ROM is run two times for two different purposes. In the first run, the same scenario which is used to run the full model is employed in order to verify the ROM validity and in the second run a completely different scenario is applied to evaluate the generalization capability of ROM.

3.1. Model Specification: The model is rectangular and similar to well-known Odeh model (SPE series 1) [13]. The schematic diagram of two-dimensional model is illustrated in Figure 2. The model consists of 20×20 blocks with no-flow boundary. Each block is 500 ft in length, 500 ft in width and 15 ft in height ($500\times500\times15$ ft^3). The permeability and porosity of the blocks are 400 md and 0.3 respectively. The model has four production wells (each one in a corner) and a water injection well in block (i=3, j=3). The reservoir initial pressure and saturation are 5000 psi and 0.25 respectively.

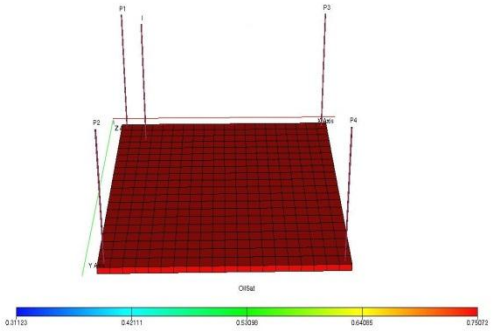

Figure 2

Schematic diagram of reservoir model

3.2 MOR construction and validation: The model described in previous section is run for 1000 days while the producer well is producing with constant pressure of 4000 psi and injection well injects water with constant bottom hole pressure of 5500 psi. The pressure of all blocks for every 30 days is specified as the pressure snapshot matrix. 34 snapshots are obtained. By decomposing the snapshot matrix to its singular values, the basis vectors of the system are obtained. There are 34 basis vectors.

By trial and error it was found that when more than four basis vectors are selected and applied no considerable improvement is obtained and error values are almost the same as it is shown in Figure 3. Hence it is decided to use 5 basis vectors as a suitable choice. Table 1 presents Root Mean Square Error (RMSE) and Mean

Absolute Relative Error (MARE) for field average pressure, field oil cut and field water cut of the reduced order model when basis vectors ranging from 1 to 5.

Now the reduced order model is constructed with these five basis vectors. At this step, the scenario used to run the reduced order model is the same as it was used for full order model. Table 2 shows RMSE and MARE for this scenario. The obtained results show satisfactory agreement between ROM and full model. To see MOR time reduction effects, simulation time and solver time (Time to solve pressure equation set) of both MOR and Full model are compared in Table 3. The results show that MOR has resulted in a speedup factor of 6 in simulation time and a factor of 50 in solver time.

Table 1
Error analysis for different number of basis vectors

	Error, Pave		Error, Oil cut		Error, water cut	
	RMSE	MARE	RMSE	MARE	RMSE	MARE
1 basis	48.5	1 %	0.12	11.4 %	0.12	56.5 %
2 basis	9.3	0.2 %	0.08	7.4 %	0.08	37.4 %
3 basis	4.7	0.1 %	0.01	1.52 %	0.01	7.03 %
4 basis	1.45	0.03 %	0.0087	0.85 %	0.0087	3.63 %
5 basis	1.25	0.02 %	0.0058	0.54 %	0.0058	2.11 %

Table 2
ROM output errors for training scenario

Error, P_{ave}		Error, Oil cut		Error, water cut	
RMSE	MARE%	RMSE	MARE%	RMSE	MARE%
1.25	0.02 %	0.0058	0.54 %	0.0058	2.11 %

3.3 Model Order Reduction Testing: In this part of the study, the reduced order model estimation capability for new different scenarios is evaluated. The ROM obtained in the previous section is run with a different scenario. In the previous scenario the bottom hole pressure of oil production wells was 4000 psi but it is considered 4400 psi in the new one which is a complete different scenario .The results are demonstrated in Figures 6 to 7. These figures show that ROM results match those of full model very well. Again to investigate ROM ability in time reduction the simulation and solver time are recorded and summarized in Table 3. The results show significant time reduction factors for this scenario and it is about 5.5 for simulation time and 38 for solver time. The error analysis between MOR and full model are also shortened in Table 4. The results clearly illustrate that good agreement exits between MOR and full model.

Table 3
ROM and full model time comparison for training scenario

Times / Models	Full Order Model	Reduced Order Model
Simulation run time	262 sec	42.6 sec
Solver time	58.47 sec	1.52 sec

Table 4
ROM and full model time comparison for testing scenario

Table 5
Errors of ROM output for testing scenario

Times / Models	Full Order Model		Reduced Order Model	
Error, P_{ave}	Error, Oil cut		Error, water cut	
Simulation time	225 sec		40.9 sec	
RMSE	MARE%	RMSE	MARE%	RMSE	MARE%
Solver time	58.26 sec		1.52 sec	
1.19	0.02 %	0.0046	0.39 %	0.0046	2.19 %

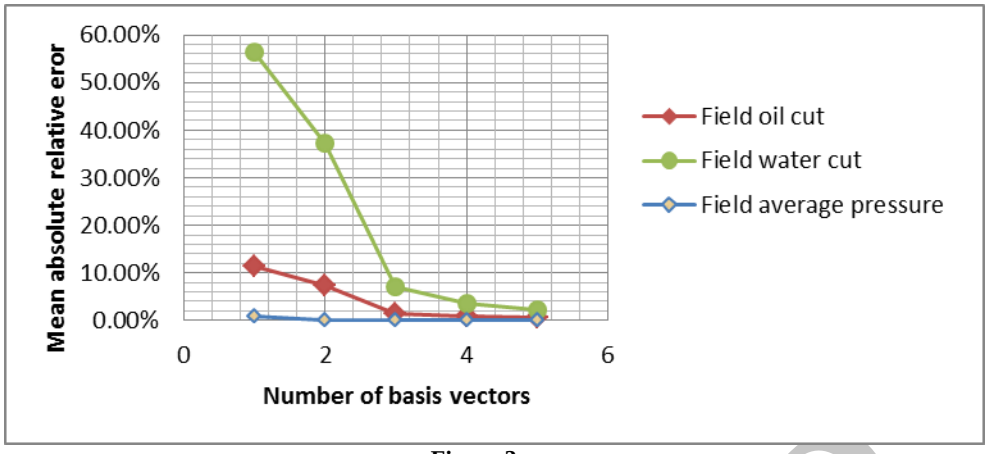

Figure 3
Relative error vs. number of basis vectors

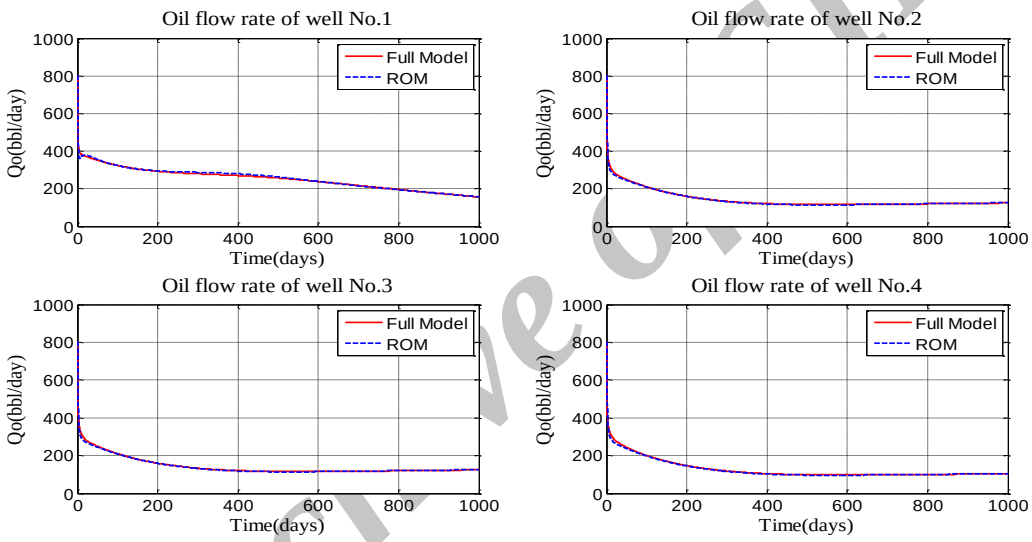

Figure 4
Oil flow rate comparison of ROM and Full model run with testing scenario

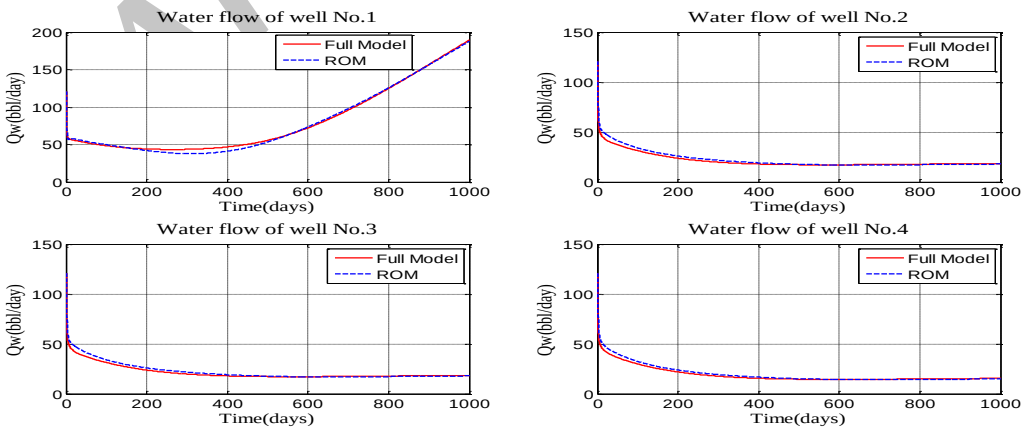

Figure 5
Water flow rate comparison of ROM and Full model run with testing scenario

4. Conclusion:

The constructed ROM from full model simulation data is ran with the same bottom hole pressure used for the training simulation and results for oil production rate and water production rate perfectly match those of full order model. The ROM is also ran with a complete different bottom hole pressure (different scenario) than the one used for training simulation. Again ROM results match those of full order model remarkably. For testing scenario, the mean absolute relative error percentages for field average pressure, field oil cut and field water cut are 0.02 %, 0.39 % and 2.19 % respectively.

Reduced order model run-time is 6 times faster than the full model, although number of equations has decreased from 400 to 5. This is because construction of reduced equation set necessitates the construction of full equation set in every iteration. Therefore in every iteration, the full equation set is constructed but the reduced set is solved. As a result, speedup factor obtained for solver time (Time to solve the equation set) is more significant than that obtained for simulation run-time.

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