کارگاه‌های آموزشی مرکز اطلاعات علمی

آموزش مهارت های کاربردی در تدوین و چاپ مقاله

اصول تنظیم قراردادها

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An Efficiency Improvement Approach in Data Envelopment Analysis (DEA)

Mohammad.Reza. Alirezaee, Mohsen. Afsharian* and Sara Kamali

Iran University of Science and Technology,Mathematics Faculty, Narmak, Resalat, Tehran, Iran

** afsharian@iust.ac.ir*

1. Introduction

There are some ways to achieve the targets for inefficient decision making units (DMUs) in basic Data Envelopment Analysis (DEA) models which are used in efficiency improvement approaches. But, there are some difficulties for using efficiency improvement approaches which obtain by DEA models. In other word, there isn't any choice for efficiency improvement level; hence, these efficiency improvement approaches aren't selective and gradual. Different methods have been introduced under this circumstance (see 2, 3), which Inverse DEA models are the important group of them [3], but, these methods make some assumptions which, are sorely restrictive in real cases. Hence, a general and efficient method under this circumstance is necessary.
In this paper, we propose an efficiency improvement approach which decision make can choose growth in targets for next period and it doesn't have any forces for becoming efficient.

2. The Proposed Method

Consider observed output $Y_j = (y_{1j},...,y_{sj}) \geq 0$ and input $X_j = (x_{1j},...,x_{mj}) \geq 0$, $X_j \neq 0$, $Y_j \neq 0$ for *DMUj*, $j=1,...,n$. and, the variables are u_r and v_i as u_r: weight assigned to output r ($r=1,...,s$), v_i: weight assigned to input i ($i=1,...,m$).

$$CCR\ \ Model:$$

$$EFF_p = Max \quad \sum_{r=1}^{s} u_r y_{rp} \Big/ \sum_{i=1}^{m} v_i x_{ip}$$

$$s.t. \quad \sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{i=1}^{m} v_i x_{ij} \leq 1 \qquad j = 1,...,n$$

$$u_r, v_i \geq 0$$

But, $\sum_{r=1}^{s} u_r y_{rj}$ and $\sum_{i=1}^{m} v_i x_{ij}$ are total revenue and total cost for jth DMU, respectively, which are calculated in an optimization problem. Therefore, EFF_p shows the relationship between total revenue and total cost in pth DMU (see [1]):

$$Total\ Revenue(TR_p) = Eff_p \times Total\ Cost(TC_p)$$

Definition: *The elasticity of input in the output*.
Relative change of an output quantity by one percent change of an input is the elasticity of that input in that output:

$$E_{ij} = \frac{\delta Y_i}{\delta X_j} \cdot \frac{X_j}{Y_i}$$

Where, E_{ij} is the elasticity quantity of input *j* in the output *i*.

Theorem. The elasticity of input *i* in the total revenue and the elasticity of output *j* in the total cost, respectively, in the CCR models is:

$$ex_{ip} = \frac{v_{ip} x_{ip}}{\sum_i v_{ip} x_{ip}} \quad where \quad \sum_i ex_{ip} = 1$$

$$ey_{jp} = \frac{u_{jp} y_{jp}}{\sum_j u_{jp} y_{jp}} \quad where \quad \sum_j ey_{jp} = 1$$

Therefore, relationship between the quantities of *i*th input elasticities and efficiency for each DMU is:

$$ex_{ip} = \frac{\Delta Eff}{\Delta x_i} \times \frac{x_i}{Eff} \quad \Leftrightarrow \quad \Delta x_i = \frac{x_i \times \Delta Eff}{Eff \times ex_i}$$

Similarly, relationship between the quantities of *j*th output elasticities and efficiency for each DMU is:

$$ey_{jp} = \frac{\Delta Eff}{\Delta y_j} \times \frac{y_j}{Eff} \quad \Leftrightarrow \quad \Delta y_j = \frac{y_j \times \Delta Eff}{Eff \times ey_{jp}}$$

Now, we can obtain effective factors on efficiency changes for each DMU using elasticities which are obtained by DEA models. Hence, above formulas present an efficiency improvement plan for each DMU privately. In other word, using elasticities, we can choose logical efficiency improvement for each DMU and obtain new inputs (outputs) to achieve the quantity of selective efficiency.

3. Conclusion

In this paper, an efficiency improvement approach was introduced. This method, presents efficiency improvement approaches which can be selective and gradual. Using elasticities which are obtained by DEA models, it presents effective factors on efficiency changes for each DMU, therefore, it is able to present efficiency improvement plan for each DMU privately.

References

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