Multi-period efficiency analysis in data envelopment analysis

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Abstract
This paper discusses the problem of multi-period data envelopment analysis (DEA). We consider the performance of a production unit in the course of *T* periods. The proposed DEA models not only measure the efficiency across all periods, but also they provide the efficiency measures for each of the periods. The applicability of the models developed is illustrated in the context of the analysis of gas companies performance.

Keywords: Data envelopment analysis, technical efficiency, overall efficiency.

1 Multi-period Technical and overall Efficiency Measurement

Standard DEA assumes that the assessed units are homogeneous and the DEA models presented, are designed to obtain a single measure of efficiency for each decision making unit *(DMU)* (see [1]). There are two types of efficiency measures: technical and overall efficiency. The technical efficiency measures the *DMU*'s success in producing maximum outputs from a given set of inputs. On the other hand, the overall efficiency or price efficiency measures the *DMU*'s success in choosing an optimal set of inputs with a given set of input prices.
In their previous paper, Nemoto and Goto [3] extended DEA to a dynamic framework. Their research incorporates two different types of inputs (variable inputs and quasi-fixed inputs) in to dynamic DEA. In 2005, Sueyoshi and Sekitani [4] proposed a new type of DEA efficiency measure within a framework of dynamic DEA. They extended the dynamic DEA of Nemoto and Goto in a manner that the concept of returns to scale (RTS) is incorporated into the DEA dynamic. Those authors are not explicitly interested in obtaining measures of efficiency at each period, but rather are concerned with the efficiency across all periods. Also, all of these studies are designed to obtain a single measure of overall efficiency.
We assume there are *n* decision making units and their production activities are examined in the course of *T* periods $(t = 1, \dots , T)$. In period *t*, each DMU_o uses two different types of inputs: $y_{t,o}$ and $k_{t-1,o}$ to yield two different types of outputs: $y_{t,o}$ and $k_{t,o}$. The inputs $k_{t-1,o}$ comes from period *(t-1)* and $k_{t,o}$ is used as inputs to the later period. This, essentially defines the DEA as *pseudo-Markov chain* in the sense that each period *t* depends upon the previous one, *(t-1)*, and affects the subsequent one, *(t+1)*. However, as a matter of fact, it is not a stochastic process and the periods are not random variables. Here, we present a DEA-like model for deriving an aggregate efficiency across all periods with accompanying period measures, such that a convex combination of these

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period measures make up the aggregate efficiency score. As far as we are aware, there is no DEA-work considering this issue and the only papers given previously in the literature, considering dynamic DEA are Färe and Grosskopf [2], Sueyoshi et al.[4] and Nemoto et al. [3]. Our objective here is twofold: first, we consider a multi-period efficiency model involving technical efficiency across all periods with accompanying period measures. Second, we extend the method for deriving overall efficiency across all periods.

2 An empirical study

This section illustrates the multi-period efficiency measure discussed in this paper with the analysis of gas companies activities. The data set consists of 25 gas companies located in 24 regions in Iran. The data for this analysis are derived from operations during 2004 and 2005. We use seven variables from the data set as inputs and outputs. Inputs include capital(x_1), number of staff(x_2) and operational costs(excluding staff costs) (x_3), and outputs include number of subscribers (y_1), amount of pipe-laying (y_2), length of gas network (y_3) and the revenue of sold-out gas in current period (y_4). The last output (y_4) is used as input in later period. The companies sell out their products and so the revenue of sold-out gas is an output. They can use this revenue as input (as budget) to later period. We have seen that efficient companies in each period are also CCR-efficient. As the results indicated, ten companies (#2, #3, #4, #5, #8, #10, #12, #20, #23 and #25 are technically efficient in aggregate sense. However, only one company (#10) is overally efficient in aggregate sense. We have evaluated the companies activity separately in each period using the CCR model. Efficient companies in each period using our proposed model, are efficient if we evaluate them, separately, in each period using CCR-model. In this study, we used the GAMS software to solve the empirical example.

3 Conclusions

Models of DEA have been generalized here in multi-period version, under the condition of constant returns to scale. In this study, we obtain measures of technical and overall efficiencies of a *DMU* across all periods with accompanying period measures. It is shown that a convex combination of these period measures make up the efficiency across all periods. The applicability of the models developed is illustrated in the context of the analysis of gas companies performance.

4 Reference

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