Analyzing simple birth – death markov queuing systems with interval parameters

A.A. Noora [a] , B.Rahmani Pergekolaei [b] , A. Payan [c,*]

[a] Department of Mathematics, Sistan and Baluchestan University, Zahedan, Iran

[b] Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran

[c] Department of Mathematics, Islamic Azad University, Zahedan, Iran

Abstract. Arrival rate and service rate in queuing systems are acquired from statistical information. Although, these parameters are determined, they are imprecise. Also these parameters change with time. A suitable way for studying these states is consider them as interval numbers. In this paper, we introduce simple birth – death markov queuing systems with interval parameters and specially discuss the queuing systems M/M/1/∞ and M/M/1/K when arrival rate and service rate are interval and is determined the performance measures as interval.

Methodology

For compare two intervals A=[a^l,a^u] and B=[b^l,b^u] we define a function as

$$g(A \prec B) = \begin{cases} 1 & a^u \le b^l \\ 0 & b^u \le a^l \\ \dfrac{b^u - a^l}{L_A - L_B} & 0/w \end{cases} \qquad (1)$$

That $L_A = a^u - a^l$, $L_B = b^u - b^l$. When B is a real number, we have :

$$g(A \prec B) = \begin{cases} 1 & a^u \le b \\ 0 & b \le a^l \\ \dfrac{b - a^l}{L_A} & a^l < b < a^u \end{cases} \qquad (2)$$

Because $0 < g(A \prec B) < 1,$ if we have $g(A \prec B) = \alpha$ and $0 < \alpha < 1$ then, $\dfrac{b - a^l}{L_A} = \alpha$ and so $b = a^l + (a^u - a^l)\alpha$ that

* Corresponding author.

E-mail addresses: alip654@yahoo.com

$a^l < b < a^u$. There for, for each $0 < \alpha < 1$ we have $a^l < b < a^u$, then can be consider as a selection of interval $\left[a^l, a^u\right]$. We use this for interpretation interval systems, specially queuing systems.

Consider a simple birth – death markov queuing system when is in state n. Consider λ_n^l, λ_n^u are minimum and maximum arrival rate and μ_n^l, μ_n^l are minimum and maximum service rate. So arrival rate and service rate in state n are in intervals $\left[\lambda_n^l, \lambda_n^u\right], \left[\mu_n^l, \mu_n^u\right]$. In this system we want to find performance measures as interval in stationary state. If $g([\lambda_n^l, \lambda_n^u] \prec \lambda_n) = \alpha_n (n \geq 0)$ and $g([\mu_n^l, \mu_n^u] \prec \mu_n) = \beta_n \ (n \geq 1)$ then $\lambda_n = \lambda_n^l + \alpha_n(\lambda_n^u - \lambda_n^l) \quad (n \geq 0)$ and $\mu_n = \mu_n^l + \beta_n(\mu_n^u - \mu_n^l) \quad (n \geq 1)$ are arrival rates and service rates that are selected. With these selections difference equations can be written as:

$$\begin{cases} 0 = & -(\lambda_n^l + \alpha_n(\lambda_n^u - \lambda_n^l) + \mu_n^l + \beta_n(\mu_n^u - \mu_n^l))p_n + (\mu_{n+1}^l + \beta_{n+1}(\mu_{n+1}^u - \mu_{n+1}^l))p_{n+1} \\ & +(\lambda_{n-1}^l + \alpha_{n-1}(\lambda_{n-1}^u - \lambda_{n-1}^l))p_{n-1} \qquad n \geq 1 \quad (3) \\ 0 = & -(\lambda_0^l + \alpha_0(\lambda_0^u - \lambda_0^l))p_0 + (\mu_1^l + \beta_1(\mu_1^u - \mu_1^l))p_1 \end{cases}$$

and so $$p_n = p_0 \prod_{i=1}^{n} \frac{\lambda_{i-1}^l + \alpha_i(\lambda_{i-1}^u - \lambda_{i-1}^l)}{\mu_i^l + \beta_i(\mu_i^u - \mu_i^l)} \qquad (4)$$

and therefore necessary and sufficient conditions for stationary state is convergencey of:

$$\sum_n P_n = \sum_n \prod_{i=1}^{n} \frac{\lambda_{i-1}^l + \alpha_i(\lambda_{i-1}^u - \lambda_{i-1}^l)}{\mu_i^l + \beta_i(\mu_i^u - \mu_i^l)} \qquad (5)$$

If now consider $\lambda_n^l = \lambda^l, \lambda_n^u = \lambda^u, \mu_n^l = \mu^l, \mu_n^u = \mu^u$, we have a M/M/1/∞ queue with interval parameters and for each (α,β) that 0<α<1,0<β<1, by $g([\lambda^l, \lambda^u] \prec \lambda) = \alpha,\ g([\mu^l, \mu^u] \prec \mu) = \beta$, we have a classic M/M/1/∞ with parameters as $\lambda = \lambda^l + \alpha(\lambda^u - \lambda^l),\ \mu = \mu^l + \beta(\mu^u - \mu^l)$. We show that length of a queuing system M/M/1/∞ with interval parameters is as

$$\left[L^l, L^u\right] = \left[\underset{\alpha,\beta \in [0,1]}{Min} \frac{\rho(\alpha,\beta)}{1-\rho(\alpha,\beta)}, \underset{\alpha,\beta \in [0,1]}{Max} \frac{\rho(\alpha,\beta)}{1-\rho(\alpha,\beta)}\right]$$

where $\rho(\alpha,\beta) = \dfrac{\lambda^l + \alpha(\lambda^u - \lambda^l)}{\mu^l + \beta(\mu^u - \mu^l)} < 1.$ we acquire other performance measures similarly as interval.

Conclusion .In this paper, we analyzed simple birth – death markov queuing systems with interval parameters. In M/M/1/∞ and M/M/1/K with interval parameters, we determined performance measures as interval. These intervals give to DM important information about queue systems that are not available when we analyze queuing systems with crisp parameters.

References

[1]. C. Kao, C. C. Li, S.P.Chen, Parametric programming to the analysis of fuzzy queues, Fuzzy sets and systems, 107 (1999) 93-100.

[2]. S.P.Chen, A bulk arrival queuing model with fuzzy parameters and varying batch size, Applied mathematical modeling, 30 (2006) 920-929.