A new approach for solving free boundary control problems based on mathematical programming techniques

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Abstract

A sample difficult free boundary elliptic problem with a boundary control function and the general functional performance criterion is considered. By use of some mathematical concepts, the optimal value is illustrated for a given domain from the result of a finite linear programming. Then the problem is converted into the minimization of a real valued function which is solved by applying some mathematical programming techniques. Some numerical examples are also given.

1) Introduction and problem

In spite of graphical workstations and modern software, finding the best shape for a given structure is still a very tedious and time consuming task. The main difficulty in the numerical realization of such problem is the presence of state constraint on a variable domain. The aim of this article is to present a method for solving free boundary problems based on the mathematical programming techniques. For this reason, as a sample , first a domain optimization problem for a system governed by an elliptic system with boundary control condition is introduced. Then, the method is explained and applied to obtain the optimal solution.

After a quick review, we define the main problem as follow:
Let ∂D be a bounded, closed and smooth curve which is made by two parts; a fixed given curve and a variable curve Γ, between two givne points A and B . Also suppose that D is the domain of ∂D and consider the elliptic control system as

$$Du(x)+f(x,u)=g(x)\ ,\ u\big|_{\partial D}=v \qquad (1)$$

where v is a bounded control function.

Our aim is to find the optimal pair (D, v) which belongs to a defined admissible set *D* and minimizing the following general functional performance:

$$I(D,v)=\int_D f_1(x,u)dx+\int_{\partial D} f_2(s,v(s))ds. \qquad (2)$$

2) The solution path

We try to find the solution by converting the problem into the optimization of an unrestricted real valued function, in two steps. First, for an arbitrary admissible given domain, we find the solution of (1). This fact is done by means of embedding method ([4]) and using the weak solution of (2) ([2]). In this manner, the value of I(D,v) is calculated uniquely from the solution of a finite linear programming problem.

In the second step, by idea of approximating a curve with finite number of segments, the unknown curve Γ is identified by m+2 corners, say, $x_0 = A$, $x_1, x_2, \ldots, x_m, x_{m+1} = B$ (for the specified m). Thus each admissible domain D is identified by the unique values of $x_1, x_2, \ldots, x_m$. Therefore, for a given D (given $x_1, x_2, \ldots, x_m$), the value of I(D,v) can be identified uniquely by the above explained method. Hence, one can define the function J: $D \longrightarrow \mathbf{R}$ that J(D,v)=I(D,v) in which it should be minimized on *D*. For this purpose, we use the Down-Hill simplex method introduce by

Nelder and Mead ([3]). Moreover, the necessary conditions are also employed by means of the penalty method ([2]). Also the convergence of the algorithm is also discussed.

3) Numerical examples

In the last section of the paper, some numerical examples, for the linear and nonlinear case of (1) and also for different cases of the domain, are given. The results show the abilities and advantages of the introduced method.

References

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