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An interactive method resolution for Linear programming with fuzzy random variables

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Abstract

This paper proposes a method for solving linear programming where all the coefficients are, in general, fuzzy random variables. We use a fuzzy random ranking method to rank the fuzzy random objective values and to deal with the inequality relation on constraints. It allows us to work with the concept of satisfaction degree of constraints. The bigger the feasibility degree is, the worst the objective value. We suggest the decision-maker (DM)the optimal solution for several different degree of feasibility. With this information the DM is able to establish an optimistic (pessimistic) optimal solution, and a fuzzy goal. By using Belmman-Zadeh min operator, we build fuzzy set in the decision space whose membership function represents the balance between feasibility degree of constraints and satisfaction degree of the goal. The best solution is obtained by this fuzzy set. Finally, a numerical example is solved to clarify our method.

Key words: Linear programming; Fuzzy stochastic theory; Ranking fuzzy random variables

1 Introduction

Linear programming is the optimization technique most frequently applied in real-world problems and therefore it is very important to introduce new tools in the approach that allows the model to fit into the real-world as much as possible.
When a probabilistic description of unknown elements is at hand, one is naturally lead to stochastic linear programming (Kall (1976); Kall and Wallace (1994); Liu (1996); Tapiero (1994); Vajda (1972)). In the presence of intrinsic or informational imprecision (Zadeh (1978)), one has to resort to fuzzy linear optimization (Buckley (1988); Lai and Hwang (1992); Liu (2000); Liu and Iwamura (2001); Luhandjula (1989); Sakawa (1993); Slowinski (1986); Verdegay (1984)).
Any linear programming model representing real word situation involves a lot of parameters whose values have both possibilistic imprecise and probabilistic uncertain. In these parameters, linguistic and frequent imprecision nature coexists, and in the conventional approach, they are required to fix an ex- act value to the aforementioned parameters.

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However, both experts and DM do not know the value of those parameters. They have to base decisions on information which is both fuzzily imprecise and probabilistically uncertain (Czogala (1988); Czogala (1990); Kacprzyck and Fedrizzi (1988); Luhandjula and Gupta (1996)).

This difficulty mainly comes from the uncertainty of many factors. If exact values are suggested these are only statistical inference from past data, their stability is doubtful. Thus, it is useful to consider the fuzziness and randomness of the parameters as fuzzy random variables. A fuzzy random variable is a random variable whose actual value is a fuzzy number. The concept of fuzzy random variables was first introduced by Kwakernaak (1978) and then by Puri and Ralescu (1986).

This paper considers LP problems whose parameters are fuzzy random variables but whose decision variables are crisp. A new ranking method is defined for fuzzy random variables. The aim of this paper is to introduce a resolution method for this type of problems by using this ranking method. Then we will obtain balanced solution for original problem.

Two key questions may be found in these kinds of problems: how to handel the relationship between the fuzzy random left and the fuzzy random right hand side of the constraints, and how to find the optimal value for the fuzzy random objective function. The answer is related to the problem of ranking fuzzy random variables.

A variety of methods for comparing fuzzy numbers have been reported in the literature (Wang and Kerre (1996)) and ranking method do not always agree with each other. Different properties have been applied to justify ranking methods, such as: distinguishability (Bortolan and Degani (1985)), rationality (Nakamura (1986)), fuzzy or linguistic presentation (Delgado et al. (1988);Tong and Bonissone (1980)) and robustness (Yager (1979)). We can extend these methods for ranking fuzzy random variables. In this paper we developed a method (Jimenez (1996)) that verifies all the above properties and that, besides, is computationally efficient to solve an Linear Programming problem, because it preserves its linearity.

Ranking method can be classified into two approaches. One of them associates each fuzzy random variable to a single point of the real line and a total crisp order relationship between fuzzy random variables is established such as Er-expected value of fuzzy random variable(Eshghi and Nematian (2005))and scalar expected value of fuzzy random variable(Liu et al. (2003)). The other approach ranks fuzzy random variables by means of a fuzzy relationship. It allows DM to present his/her preference in a gradual way, which in an LP problem allows it to be handle with different degree of satisfaction of constraints. In section 3 we show how we use our method to rank fuzzy random variables in order to define the feasibility degree of the decision vector and to define the accepted optimal solution.

Obviously the higher the feasibility degree is, the smaller feasible solution set and, consequently, the objective optimal value is worse. So, the DM has to find a balanced solution between two object in conflict: to improve the objective function value and to improve the degree of feasibility. In section 4 we show how we can operate in an interactive way in order to evaluate the two aforementioned conflicting factors. Finally in section 5 we solve a numerical example.

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