

Tarbiat Moallem University, 20th Seminar on Algebra,
2-3 Ordibehesht, 1388 (Apr. 22-23, 2009) pp 108-111

OBSTINATE PSEUDO-BCK IDEAL OF PSEUD-BCK ALGEBRAS

A. GHASEMI, A. KORDI, AND A. MOUSSAVI

^aDepartment Of Mathematics, Tarbiat Modares University
P.O.Box: 14115-170, Tehran, Iran
as_ghasemi@modares.ac.ir,
moussavi.a@modares.ac.ir

^bMathematics and Informatics Research Group, ACECR,
Tarbiat Modares University,
P. O. Box: 14115-343, Tehran, Iran.
kordi.ali@gmail.com

ABSTRACT. The notion of obstinate pseudo-BCK ideals is introduced and some related properties are investigated. We investigate the relations between this pseudo ideals and several pseudo-BCK ideals. We give the following characterization theorem of obstinate ideals in pseudo-BCK algebras: Let I be an pseudo ideal of pseudo-BCK algebra X . Then I is obstinate pseudo-BCK ideal if and only if I is pseudo maximal, pseudo irreducible, prime, positive implicative or implicative. In this talk we discuss about a relation between obstinate pseudo-BCK ideals and other above pseudo-BCK ideals.

1. INTRODUCTION

BCK-algebras are important logical algebras introduced by Iseki in 1966(see[3]). Since then this class of algebras has been intensively studied by many researchers. In particular, emphasis have been put on the ideal theory of BCK-algebras. In 1987, S.K.Goel and A.K.Arora[2] first introduced the concept of obstinate ideals in BCK-algebras. Georgescu and Iorgulescu [1] introduced the notion of pseudo-BCK algebras and studied their properties. In [4], Y. B. Jun, gave characterizations of pseudo-BCK ideals such as positive implicative pseudo-BCK ideals. In this talk we introduced the notion of obstinate pseudo-BCK ideals and show the relation between these ideals and (implicative, positive implicative, commutative, maximal, prime and irreducible) pseudo BCK-ideals of pseudo-BCK algebras. Recall that a pseudo-BCK algebra is a structure $(X, \leq, *, \diamond, 0)$, where \leq , $*$ and \diamond are binary operations on X and 0 is an element of X verifying the following axioms for all $x, y, z \in X$.

- 1): $(x * y) \diamond (x * z) \leq z * y$, $(x \diamond y) * (x \diamond z) \leq z \diamond y$,
- 2): $x * (x \diamond y) \leq y$, $x \diamond (x * y) \leq y$,

2000 Mathematics Subject Classification: 06F35, 03G25.

keywords and phrases: pseud-BCK algebras, (Obstinate, commutative, positive implicative) pseudo-BCK ideals.

- 3): $x \leq x$,
- 4): $0 \leq x$,
- 5): $x \leq y$ and $y \leq x$ imply $x = y$,
- 6): $x \leq y \Leftrightarrow x * y = 0 \Leftrightarrow x \diamond y = 0$.

Note that every pseudo-BCK algebra satisfying $x * y = x \diamond y$ for all $x, y \in X$ is a BCK-algebra. A pseudo-BCK algebra X is said to be \wedge -commutative (resp. \cap -commutative) if it satisfies for all x, y in X , $x \diamond (x * y) = y \diamond (y * x)$ i.e. $y \wedge x = x \wedge y$ (resp. $x * (x \diamond y) = y * (y \diamond x)$ i.e. $y \cap x = x \cap y$). A nonempty subset I of pseudo-BCK algebra X is said to be an pseudo-BCK ideal of X if it satisfies: (i) $0 \in I$, and (ii) $x * y, x \diamond y \in I$ and $y \in I$ imply $x \in I$.

2. CONCEPTS ON PSEUDO-BCK IDEALS

Definition 2.1. A pseudo-BCK algebra X is called to be *Positive implicative* if for all x, y and z in X , $(x * z) \diamond (y * z) = (x * y) \diamond z$, or for all x, y and z in X , $(x \diamond z) * (y \diamond z) = (x \diamond y) * z$.

Example 2.2. The $X = \{0, a, b, c\}$ equipped with the operations $*$, \diamond given by the following tables is a positive implicative pseudo-BCK algebra.

$*$	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	c	0

\diamond	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	c	a	0

Definition 2.3. Suppose $(X, *, \diamond, 0)$ is a pseudo-BCK algebra. A nonempty subset I of X is said to be an *positive implicative pseudo-BCK ideal*, if it satisfies the following conditions for all $x, y, z \in X$,

- (i) $0 \in I$,
- (ii) $(x * y) \diamond z, y \diamond z \in I \implies x \diamond z \in I$,
- (iii) $(x \diamond y) * z, y * z \in I \implies x * z \in I$.

Example 2.4. In example 2.2 the pseudo-BCK ideals of X are $\{0\}$, $\{0, a\}$, $\{0, b\}$ and X . $I = \{0, a\}$ is a positive implicative pseudo-BCK ideal. Also the other ideals of X are positive implicative pseudo-BCK ideal.

Definition 2.5. A pseudo-BCK algebra X is called *implicative pseudo-BCK algebra* if for all $x, y \in X$, $x = x \diamond (y * x) = x * (y \diamond x)$.

Definition 2.6. Suppose $(X, *, \diamond, 0)$ is a pseudo-BCK algebra. A nonempty subset I of X is said to be an *implicative pseudo-BCK ideal* if it satisfies the following conditions, for all $x, y, z \in X$,

- (i) $0 \in I$,
- (ii) $(x * (y \diamond x)) * z, z \in I \implies x \in I$,
- (iii) $(x \diamond (y * x)) \diamond z, z \in I \implies x \in I$.

A pseudo-BCK algebra X is called *with condition (K)*, if $(x * z) \diamond (y * z) \leq x * y$ and $(x \diamond z) * (y \diamond z) \leq x \diamond y$ for all $x, y \in X$.

Theorem 2.7. *An implicative pseudo-BCK ideal of pseudo-BCK algebra with condition (K) is a positive implicative pseudo-BCK ideal, but the invers is not true.*

Definition 2.8. A pseudo-BCK algebra X is called to be *commutative pseudo-BCK algebra* if it satisfies for all x, y and z in X ,
 (a) $x \diamond (x * y) = y \diamond (y * x)$ and (b) $x * (x \diamond y) = y * (y \diamond x)$.

Definition 2.9. A nonempty subset I of X is called *commutative pseudo-BCK ideal* if it satisfies, for all x, y, z in X , the following conditions

- (i): $0 \in I$,
- (ii): $(x * y) \diamond z \in I$ and $z \in I \implies x * (y \diamond (y * x)) \in I$,
- (iii): $(x \diamond y) * z \in I$ and $z \in I \implies x \diamond (y * (y \diamond x)) \in I$,

Theorem 2.10. *Let X be a pseudo BCK-algebra with condition (K), then a nonempty subset I of X is an implicative pseudo-BCK ideal if and only if, it is both a commutative pseudo-BCK ideal and positive implicative pseudo-BCK ideal.*

3. OBSTINATE PSEUDO-BCK IDEAL

Definition 3.1. A proper pseudo-BCK ideal I of X is called *obstinate pseudo-BCK ideal* if for any $x, y \in X$,

- (a): $x, y \notin I \implies x * y \in I$ and $y * x \in I$,
- (b): $x, y \notin I \implies x \diamond y \in I$ and $y \diamond x \in I$.

Theorem 3.2. *Let I be a pseudo-BCK ideal of X . Then the following are equivalent:*

- (a): I is obstinate pseudo-BCK ideal,
- (b): I is positive pseudo-BCK ideal and maximal pseudo-BCK ideal,
- (c): I is implicative pseudo-BCK ideal and maximal pseudo-BCK ideal.

Definition 3.3. A pseudo-BCK ideal I of a pseudo-BCK algebra X is called *maximal pseudo-BCK ideal* if it is proper and no pseudo-BCK ideal of X strictly contains I , i.e., for each pseudo-BCK ideal $J \neq I$, if $I \subseteq J$ then $J = X$.

Definition 3.4. A proper pseudo-BCK ideal I of a pseudo-BCK algebra X is called *irreducible pseudo-BCK ideal*, if $I = I_1 \cap I_2$ implies $I = I_1$ or $I = I_2$ for any pseudo ideals I_1 and I_2 of X .

Definition 3.5. Let I be a proper pseudo-BCK ideal of a pseudo-BCK algebra X . Then I is called *prime pseudo-BCK ideal*, if $x \wedge y \in I$ implies $x \in I$ or $y \in I$ for any $x, y \in X$. ($x \wedge y := y \diamond (y * x)$)

Theorem 3.6. *For a pseudo ideal I of a pseudo-BCK algebra X , the following are equivalent.*

- (a): I is obstinate pseudo-BCK ideal.
- (b): I is maximal pseudo-BCK ideal.
- (c): I is prime pseudo-BCK ideal.
- (d): I is irreducible pseudo-BCK ideal.

Definition 3.7. Let X and Y be pseudo-BCK algebras. A mapping $f : X \rightarrow Y$ is called a *pseudo homomorphism* if $f(x * y) = f(x) * f(y)$ and $f(x \diamond y) = f(x) \diamond f(y)$ for all x, y in X .

Theorem 3.8. Let I be a proper pseudo ideal of a pseudo-BCK algebra X . Then for any pseudo-BCK algebra X' there exists $f \in \text{Hom}(X, X')$ such that $\ker(f) = I$, if and only if, I is obstinate pseudo-BCK ideal.

Theorem 3.9. Let X, Y and Z are pseudo-BCK algebras. Let $h : X \rightarrow Y$ be an epimorphism and $g \in \text{Hom}(X, Z)$. If $\ker(h) \subset \ker(g)$ then there exists a unique homomorphism $f : Y \rightarrow Z$ such that $f \circ h = g$.

REFERENCES

- [1] G. Georgescu and A. Iorgulescu, *Pseudo-BCK algebras: an extension of BCK algebras*, Springer Ser. Discrete Math. Theor. Comput. Sci, (2001), 97-114.
- [2] S. K. Goel and A. K. Arora, *obstinate ideals in BCK-algebras*, Math. Japon. **32** (1987), 559-561.
- [3] K. Iseki, *An algebra related with a propositional calculus*, Proc. Jap. Acad. **48** (1966), 26-29.
- [4] Y. B. Jun, M. Kondo and K. H. Kim, *Pseudo-ideals of pseudo-BCK algebras*, Sci. Math. Jap. **58** (2003), 9397.