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ASYMPTOTIC BEHAVIOUR OF DEPTH OF COMPONENTS OF GRADED LOCAL COHOMOLOGY MODULES

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Abstract. Let $R = \bigoplus_{n\geq 0} R_n$ be a standard graded ring, $\mathfrak{a}_0$ an ideal of the base ring R_0 and let M be a non-zero finitely generated graded R-module. In this talk, we study the asymptotic behaviour of the sequence $(\text{grade}(\mathfrak{a}_0, H^i_{R_+}(M)_n))_{n\in\mathbb{Z}}$, of grades of components of the i-th graded local cohomology module, when $n \to -\infty$, in the following cases:

(i) $i = f_{R_+}(M)$;
(ii) $i = g_{R_+}(M)$;
(iii) $\dim(R_0) \leq 2$.

To this end we study Artinian and tameness properties of certain graded local cohomology modules.

1. Introduction

Throughout, $R = \bigoplus_{n\geq 0} R_n$ is a graded Noetherian ring where the base ring R_0 is a commutative Noetherian local ring with maximal ideal $\mathfrak{m}_0$ and R is generated, as an R_0-algebra, by finitely many elements of R_1. Moreover, we use $\mathfrak{a}_0$ to denote a proper ideal of R_0 and we set $R_+ = \bigoplus_{n>0} R_n$, the irrelevant ideal of R, $\mathfrak{a} = \mathfrak{a}_0 + R_+$, and $\mathfrak{m} = \mathfrak{m}_0 + R_+$. Also, we use M to denote a non-zero, finitely generated, graded R-module.

It is well known (cf [2, 15.1.5]) that for each $i \geq 0$, $H^i_{R_+}(M)_n$, the n-th graded component of the i-th local cohomology module $H^i_{R_+}(M)$, is finitely generated for all $n \in \mathbb{Z}$ and vanishes for all sufficiently large values of n. But we know not much about the properties of the R_0-modules $H^i_{R_+}(M)_n$ for sufficiently small

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values of n and the asymptotic behaviour of $H^i_{R_+}(M)_n$ when $n \to -\infty$; These attracts a lot of interest (see [1],[3], [4]).

For example, in [3], it has been shown that the set $\mathrm{Ass}_{R_0}(H^{f_{R_+}(M)}_{R_+}(M)_n)$ is asymptotically stable, as $n \to -\infty$, where $f_{R_+}(M)$, the finiteness dimension of M with respect to R_+, is the least non-negative integer i such that $H^i_{R_+}(M)$ is not finitely generated. It therefore follows that $\mathrm{grade}(\mathfrak{a}_0, H^{f_{R_+}(M)}_{R_+}(M)_n) < \infty$ for all $n \ll 0$. Recently, M. Brodmann raised the question whether the sequence $(\mathrm{grade}(\mathfrak{a}_0, H^{f_{R_+}(M)}_{R_+}(M)_n))_n$ of integers is stable, as $n \to -\infty$. One of the main proposes of this talk is to provide an affirmative answer for this question in certain cases.

A graded R-module $T = \bigoplus_{n\in\mathbb{Z}} T_n$ is said to be tame if either $T_n \neq 0$ for all $n \ll 0$ or else $T_n = 0$ for all $n \ll 0$. The tameness property of $H^j_{\mathfrak{m}_0 R}(H^i_{R_+}(M))$ plays an important role in the study of the asymptotic behaviour of the sequence $(\mathrm{depth}_{R_0}(H^i_{R_+}(M)_n))_{n\in\mathbb{Z}}$, as $n \to \infty$. Also, it is known that every graded Artinian R-module is tame. We therefore present some results on the Artinianess of the modules $H^j_{\mathfrak{m}_0 R}(H^i_{R_+}(M))$, too.

Throughout, we use $f^{R_+}_{\mathfrak{a}}(M)$ to denote the $\mathfrak{a}$-finiteness dimension of M relative to R_+ and $cd_{R_+}(M)$ to denote the cohomological dimension of M with respect to R_+, therefore

$$f^{R_+}_{\mathfrak{a}}(M) := \inf\{i \in \mathbb{N}_0 \mid R_+ \nsubseteq \sqrt{(0 :_R H^i_{\mathfrak{a}}(M))}\}$$

and

$$cd_{R_+}(M) := \sup\{i \in \mathbb{Z} \mid H^i_{R_+}(M) \neq 0\}.$$

2. Main results

Theorem 2.1. *Set* $\acute{f} = \inf\{i \in \mathbb{N} \mid H^{f+i}_{R_+}(M)$ *is not finitely generated*$\}$. *Then,*

$$\mathrm{grade}(\mathfrak{a}_0, H^{f_{R_+}(M)}_{R_+}(M)_n) \geq \min\{f^{R_+}_{\mathfrak{a}_0+R_+}(M) - f, \acute{f} + 1\}.$$

Lemma 2.2. *Let* $f^{R_+}_{\mathfrak{a}}(M) < \infty$. *Then* $H^{f^{R_+}_{\mathfrak{a}}(M)}_{\mathfrak{a}}(M)_n \neq 0$ *for all* $n \ll 0$.

Theorem 2.3. *Let* $f^{R_+}_{\mathfrak{a}}(M) < \infty$ *and suppose that* $f := f_{R_+}(M) = cd(R_+, M)$. *Then,* $\mathrm{grade}(\mathfrak{a}_0, H^f_{R_+}(M)_n) = f^{R_+}_{\mathfrak{a}}(M) - f$ *for all* $n \ll 0$.

Theorem 2.4. *Let* $g(M) < \infty$. *Then* $H^i_{\mathfrak{m}_0 R}(H^j_{R_+}(M))$ *is an Artinian* R-*module for* $i = 0, 1$ *and* $j \leq g(M)$.

Corollary 2.5. *Let* $g(M) < \infty$ *and let* $j \leq g(M)$. *Then, just one of the following statements hold:*
(i) $\mathrm{depth}_{R_0}(H^j_{R_+}(M)_n) = 0$ *for all* $n \ll 0$*;*

(ii) $\operatorname{depth}_{R_0}(H^j_{R_+}(M)_n) = 1$ *for all* $n \ll 0$;
(iii) $\operatorname{depth}_{R_0}(H^j_{R_+}(M)_n) \geq 2$ *for all* $n \ll 0$.

Theorem 2.6. *Let* $i \in \mathbb{N}_0$ *and let* $cd(R_+, M) = f_{R_+}(M) + 1$. *Then* $H^i_{\mathfrak{m}_0 R}(H^{cd(R_+,M)}_{R_+}(M))$ *is an Artinian* R*-module if and only if* $H^{i+2}_{\mathfrak{m}_0 R}(H^{f_{R_+}(M)}_{R_+}(M))$ *is an Artinian* R*-module.*

Theorem 2.7. *Let* $\dim(R_0) \leq 1$. *Then for all* $i \in \mathbb{N}_0$ *the sequence* $(\operatorname{depth}_{R_0}(H^i_{R_+}(M)_n))_{n\in\mathbb{Z}}$ *is asymptotically stable, for* $n \to -\infty$.

Lemma 2.8. *Let* $\dim(R_0) \leq 2$. *Then* $H^1_{\mathfrak{m}_0 R}(H^j_{R_+}(M))$ *is an Artinian* R*-module for all* $j \in \mathbb{N}_0$

Theorem 2.9. *Let* $\dim(R_0) \leq 2$ *and* $i \in \mathbb{N}_0$. *Assume that the sequence* $(\operatorname{Ass}_{R_0}(H^i_{R_+}(M)_n))_{n\in\mathbb{Z}}$ *is asymptotically stable, for* $n \to -\infty$. *Then so is the sequence* $(\operatorname{depth}_{R_0}(H^i_{R_+}(M)_n))_{n\in\mathbb{Z}}$.

Corollary 2.10. *Let* $\dim(R_0) \leq 2$ *and assume that* R_0 *is a finite integral extension of a domain or essentally of finite type over a field. Then the sequence* $(\operatorname{depth}_{R_0}(H^i_{R_+}(M)_n))_{n\in\mathbb{Z}}$ *is asymptotically stable, for* $n \to -\infty$.

Theorem 2.11. *Set* $c = cd(R_+, M)$ *and* $d = dim(R_0)$. *Then the following statements hold.*
(i) $H^d_{\mathfrak{m}_0 R}(H^c_{R_+}(M))$ *and* $H^{d-1}_{\mathfrak{m}_0 R}(H^c_{R_+}(M))$ *are Artinian* R*-modules.*
(ii) *If* $H^{d-3}_{\mathfrak{m}_0 R}(H^c_{R_+}(M))$ *and* $H^{d-2}_{\mathfrak{m}_0 R}(H^{c-1}_{R_+}(M))$ *are Artinian, then* $H^d_{\mathfrak{m}_0 R}(H^{c-2}_{R_+}(M))$ *and* $H^{d-1}_{\mathfrak{m}_0 R}(H^{c-1}_{R_+}(M))$ *are Artinian.*
(iii) *If* $H^{d-3}_{\mathfrak{m}_0 R}(H^c_{R_+}(M))$ *is Artinian, then* $H^{d-1}_{\mathfrak{m}_0 R}(H^{c-1}_{R_+}(M))$ *is Artinian.*
(iv) *If* $H^d_{\mathfrak{m}_0 R}(H^{c-2}_{R_+}(M))$ *and* $H^{d-1}_{\mathfrak{m}_0 R}(H^{c-1}_{R_+}(M))$ *are Artinian, then* $H^{d-3}_{\mathfrak{m}_0 R}(H^c_{R_+}(M))$ *is Artinian.*
(v) $H^{d-2}_{\mathfrak{m}_0 R}(H^c_{R_+}(M))$ *is Artinian if and only if* $H^d_{\mathfrak{m}_0 R}(H^{c-1}_{R_+}(M))$ *is Artinian.*

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