Experimental Investigation of Force Coefficients for Groups of Three and Four Circular Cylinders Subjected to a Cross-flow

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ABSTRACT
This paper has investigated the flow interference between three circular cylinders of equal diameter in an equilateral-triangular arrangement and also between four circular cylinders in a square arrangement when subjected to a cross-flow. Wind tunnel experiments were conducted to measure force coefficients for six spacing ratios (d/l) varying from 1.5 to 4 at subcritical Reynolds number of 41008.6××. The pressure distributions on the surface of the cylinders were measured, using pressure transducers. It was found that for three cylinders at 2>dl, the upstream cylinder experiences lower mean drag coefficient than that of the downstream ones. Also, the minimum drag coefficient values of the downstream cylinders occur at 5.1/dl and 2/dl.

Moreover, It was revealed that for four cylinders at 5.1/dl, due to severe flow interference between cylinders, there is a difference between lift coefficients for the upstream cylinders. Also, for 2/dl, the mean drag coefficients for downstream cylinders are negative. In addition, it was concluded that the variations in dl strongly affect the aerodynamic coefficients. Also, by decreasing dl, the effects of the flow interference between the cylinders increase.

Key Words: Multi-cylinder Arrays, Circular Cylinders, Flow Interference, Pressure Distribution

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1. Introduction

Cylinder-like structures can be often found both alone and in groups in the designs for heat exchangers tubes, offshore structures, skyscrapers, chemical reaction towers, chimneys, power lines, cooling systems for nuclear power plants, etc. In many of these engineering applications, fluid forces, Strouhal frequencies and flow configurations are major criteria for the design of the structures. It is possible to model the flows passing through these kinds of structures by multi-cylinder arrays in a cross-flow.

The steady and fluctuating fluid forces acting on each cylinder in multi-cylinder arrays are basically due to vortex shedding quality and characteristics of the flow pattern around the cylinders. When the flow interfere between the cylinders, vortex shedding from each cylinder, vortex shedding on the downstream cylinders and also near-wake, lead to high fluctuating forces on each cylinder [1]. The effects of mutual interference of shear layers, vortices and Karman vortex streets play a vital role in the flow interference [2-5]. In these cases, due to flow interference between cylinders, the fluid usually shows a complicated and surprising behavior. Major points in multiple-cylinder arrays studies are to find the effects of the flow interference on each cylinder, fluid-structure interactions and classification of flow pattern around cylinders; whereas, less well studied and understood have been reported the nature of the flow around multiple-cylinder arrays.

The flow around a circular cylinder in a cross-flow, which is a classical problem in fluid mechanics, due to very common and multiple usages, has been studied widely in experimental and numerical methods [6-7]; whereas, most of previous works have studied multi-cylinder arrays experimentally in high subcritical Reynolds numbers for industrial applications [8-10]. On the other hand, due to complexity of the flow in these cases, there have been relatively few numerical studies on the circular cylinders which mostly are based on just two cylinders at low Reynolds numbers [11-13].

Many experimental studies on flow interference between circular cylinders are on two circular cylinders in tandem, side-by-side and staggered arrangements [14-18]. Some studies have investigated the flow around three circular cylinders in an in-line arrangement [19-21]. Sayers [22] studied the drag and lift coefficients occurring on one cylinder in a group of three equispaced cylinders. He concluded that the angle of orientation to the free stream will strongly influence the force coefficients acting on each cylinder. Price and Paidoussis [23] investigated the mean lift and drag forces acting on two and three cylinders in staggered configurations over a wide range of $l/d$ and suggested that the principle of superposition could apply in determination of force coefficients. Moreover, Yunseok and Changkoon [24] reported the close relationship between aerodynamic responses and pressure distributions on three circular cylinders.

Despite of numerous works on two cylinders arrangements, only a few studies have been reported on multi-cylinder arrays such as three and four cylinders arrays. Even among the studies on multi-cylinders many of them were on multi-cylinders in line, side-by-side or in tandem [25-26]. A three-cylinder array in an equilateral-triangular arrangement (or a four-cylinder array in a square arrangement) is a basic unit in many multi-cylinder arrays such as tube banks. The study of these arrays would be helpful to understand more complicated flow in much larger cylinder arrays, which are especially notable in shell and tube heat exchangers. These have also many practical applications in offshore structures. Tatsuno et al. [8] investigated the effects of the flow interference between the three cylinders in an equidistant triangular arrangement in a cross-flow. They found that the flow interference effects between three cylinders are severe in the case of low spacing ratio ($l/d$) and the flow interference effects weakens with higher $l/d$. In addition, Gu and Sun [9] identified four different affected regions i.e. small, transition, medium and large spacing ratios for the flow pattern on three cylinders in an equilateral-triangular arrangement at $Re = 1.4 \times 10^4$ with regards to different levels of interference of influence of spacing ratio ($l/d$). On the other hand, Sayers [27-28] studied the flow interference between four circular cylinders in a square arrangement and revealed that the magnitude of the mean lift and drag coefficients are strongly influenced by the orientation of array group to the free stream. Moreover, Lam et al. [10] have measured the mean and fluctuating force coefficients of four cylinders in square configuration for different $l/d$ and incidence angles ($\alpha$) at subcritical Reynolds numbers, using a piezo-electric load cell.

The present work has experimentally investigated force coefficients for groups of three and four circular cylinders subjected to a cross-flow with six spacing ratios ($l/d$) ranging from 1.5 to 4 at subcritical Reynolds number of $6.08 \times 10^4$. It is hoped that this study could provide a useful database and a better understanding of the flow.
interference phenomenon for multiple-cylinder arrays in a cross-flow.

2. Experimental Setup

The experiments were conducted in low-speed, open-circuit wind tunnel of department of aerospace engineering, K.N. Toosi University of Technology. The wind tunnel test-section is 1.2 m wide, 1 m high and 3 m long. The maximum velocity in the test-section is 60 m/s, under uniform flow conditions, the longitudinal free-stream turbulence intensity is less than 0.15% and the velocity non-uniformity across the test-section is ±0.5%.

Figure 1 shows a schematic diagram of the experimental setup in the wind tunnel. The configurations of three cylinders in an equilateral-triangular arrangement and four cylinders in a square arrangement are shown in Fig’s. 2 and 3. Six spacing ratios \( l/d \) vary from 1.5 to 4, where \( d \) is the cylinder diameter, \( l \) is the distance between the center of the adjacent cylinders and \( l/d \) is the spacing ratio between two cylinders. In Fig. 4, the position of a point on the surface of each cylinder is defined by the azimuthal angle \( \theta \), measured from the direction of the free-stream flow. Each cylinder of the group is a 92 cm long hollow aluminum tube of 38 mm external diameter, with a machine-finished surface. Thirty pressure taps of diameter 0.6 mm are provided every 12° in two parallel rows within 2 cm distance from each other around mid-span in a zigzag manner of each cylinder circumferentially. The aspect ratio is about 24 and the blocking ratio is 2.9% per cylinder. In addition, the total blockage ratio range for the three cylinders is about 7.2–8.7%. Also, the total blockage ratio of the four cylinders is 5.8%.

![Fig. (1): Schematic diagram of the experimental setup in the wind tunnel.](image)

![Fig. (2): The configuration of three circular cylinders in an equilateral-triangular arrangement.](image)

![Fig. (3): The configuration of four circular cylinders in a square arrangement.](image)

![Fig. (4): Angel \( \theta \): the position of a point on the surface of each cylinder.](image)

All tests were carried out at the Reynolds number of \( 6.08 \times 10^5 \), based on the diameter of a single cylinder and a free-stream velocity of 24 m/s. During the experiments, reference flow conditions were measured with a Pitot-static tube and a micro-manometer. The measurement system of the surface pressure was consisted of pressure transducers (Honeywell-DC005NDC4), a National Instruments (NI) PCI-6224 16-bit A/D board with 32 analogue input channels and a personal computer.

The estimated measurement uncertainties of the pressure, lift and drag coefficients are \( C_p \pm 0.01 \), \( C_L \pm 0.015 \) and \( C_D \pm 0.02 \), respectively. In this study, the pressure coefficient \( C_p \), the lift coefficient \( C_L \) and the drag coefficient \( C_D \) for each cylinder are defined as

\[
C_p = \frac{P - P_a}{0.5 \rho V_e^2}
\]
\( C_L = \frac{L}{0.5 \rho V_w^2 d} \) & \( C_D = \frac{D}{0.5 \rho V_w^2 d} \) respectively, where \( P \) is the mean static pressure on the surface of the cylinder, \( P_\infty \), the static pressure of the free-stream flow, \( L \), the lift force, \( D \), the drag force, \( \rho \), the air density, \( V_w \) and the free-stream velocity.

3. Results and Discussion
The pressure distributions on the surface of the cylinders were measured by the pressure transducers. In Fig. 5, distribution of the pressure coefficient on the surface of a single cylinder is presented. In this study, the pressure distribution method has been employed for measuring mean force coefficients in which lift and drag coefficients are only calculated by pressure integration around the cylinders at mid-span. As depicted in Fig. 5, for a single cylinder at \( \text{Re} = 6.08 \times 10^4 \), it is found that \( C_L = 0 \) and \( C_D = 1.20 \), which shows a good agreement with the results reported by Lam et al. [10] for a single cylinder which has been measured at \( \text{Re} = 6 \times 10^5 \) using a piezo-electric load cell (\( C_D = 1.217 \) and \( C_L = 0 \)).

3.1. Three Cylinders
Figures 6 to 11 show the pressure coefficient distributions on the surface of the three circular cylinders in equilateral-triangular arrangement for different values of \( l/d = 1.5 \) to \( 4 \). In addition, Figs. 12 and 13 show variations of the lift and drag coefficients with respect to \( l/d \) at \( \text{Re} = 6.08 \times 10^4 \).

Fig. (5): The pressure coefficient distribution on the surface of a single cylinder.

Fig. (6): The pressure coefficient distributions of the three cylinders for \( l/d = 1.5 \).

Fig. (7): The pressure coefficient distributions of the three cylinders for \( l/d = 2 \).

Fig. (8): The pressure coefficient distributions of the three cylinders for \( l/d = 2.5 \).
Fig. (9): The pressure coefficient distributions of the three cylinders for $l/d = 3$.

Fig. (10): The pressure coefficient distributions of the three cylinders for $l/d = 3.5$.

Fig. (11): The pressure coefficient distributions of the three cylinders for $l/d = 4$.

Fig. (12): Variation of the mean lift coefficients with $l/d$ (for three cylinders).

Fig. (13): Variation of the mean drag coefficients with $l/d$ (for three cylinders).

In Fig. 12, it is evident that when three cylinders are in equilateral-triangular arrangement, there is an attractive force between the two cylinders B and C. Furthermore, according to the values of $C_L$ for the downstream cylinders, it is clear that for slight variations of $l/d$, lift coefficients of the downstream cylinders change considerably which indicates substantial changes in the flow pattern around the cylinders. On the other hand, for $l/d > 2$, the upstream cylinder experiences lower mean drag force ($C_D$) than that of the downstream cylinders (see Fig. 13). The minimum drag coefficient values for the downstream cylinders occur at $l/d = 1.5$ and $l/d = 2$, but for $l/d$ higher than 2 the drag coefficients rapidly increase. This is a general trend that by increasing $l/d$, mean drag coefficients for all cylinders have almost been increased.

According to Fig. 12, it is revealed that for $l/d = 1.5$, there is a difference between lift
coefficients for two cylinders B and C in side-by-side arrangement as a result of severe flow interference between the cylinders. The same phenomenon has been found for three cylinders in an equilateral-triangular arrangement which has been reported by Tatsuno et al. [8]. From the other viewpoint, by increasing the distance between the cylinders for \( l/d = 4 \) and so weakening the flow interference between the cylinders, it is found that the lift and drag coefficients for cylinder A is almost similar to that of a single cylinder. Thus, it is concluded that the variations in \( l/d \), strongly affect the aerodynamic coefficients and also by decreasing the \( l/d \), the effects of the flow interference between the cylinders increase.

### 3.2. Four Cylinders

Figures 14–19 show the pressure coefficient distributions on the surface of the four circular cylinders in square arrangement for different values of \( l/d = 1.5 \) to 4. Moreover, Figs. 20-21 show variations of the lift and drag coefficients with respect to \( l/d \) at \( \text{Re} = 6.08 \times 10^3 \).

**Fig. (14):** The pressure coefficient distributions of the four cylinders for \( l/d = 1.5 \).

**Fig. (15):** The pressure coefficient distributions of the four cylinders for \( l/d = 2 \).

**Fig. (16):** The pressure coefficient distributions of the four cylinders for \( l/d = 2.5 \).

**Fig. (17):** The pressure coefficient distributions of the four cylinders for \( l/d = 3 \).

**Fig. (18):** The pressure coefficient distributions of the four cylinders for \( l/d = 3.5 \).
Experimental Investigation of Flow Interference between Circular Cylinders

In Fig. 19, it is evident that when four cylinders are in square arrangement, there is a repulsive force between the upstream cylinders A and B which is weakened by increasing \( l/d \). Furthermore, it can be seen in Fig. 21 that the downstream cylinders experience lower mean drag coefficients \( (C_D) \) than that of the upstream cylinders and the minimum drag coefficient values for the upstream cylinders occur at \( l/d = 3 \). By increasing \( l/d \), the mean drag coefficients for downstream cylinders are increased. It is notable that for \( l/d \leq 2 \) these coefficients are negative.

According to Fig. 20, it is clear that for \( l/d = 1.5 \), there is a difference between lift coefficients for the upstream cylinders A and B as a result of severe flow interference between the cylinders. On the other hand, by increasing the distance between the cylinders for \( l/d = 4 \) and thus weakening the flow interference between the cylinders, it is revealed that the lift and drag coefficients for the upstream cylinders, are almost similar to that of a single cylinder. So, it is found that the variations in \( l/d \), strongly affect the aerodynamic coefficients and also by decreasing the \( l/d \), the effects of the flow interference between the cylinders increase.

4. Conclusions

Flow interference between three circular cylinders in an equilateral-triangular arrangement and also between four circular cylinders in a square arrangement was investigated when subjected to a cross-flow. For this purpose, force coefficients for six spacing ratios \( (l/d) \) varying from 1.5 to 4 were measured experimentally at subcritical Reynolds number of \( 6.08 \times 10^5 \). Some useful conclusions are obtained and they are summarized as follows:

1-When the three cylinders are arranged in the current configuration, two cylinders B and C exert attractive force on each other. Moreover, for \( l/d = 1.5 \), due to severe flow interference between cylinders, there is a difference between lift coefficients for downstream cylinders B and C.

2-In the case of three cylinders, it is found that for \( l/d > 2 \), the upstream cylinder experiences lower mean drag coefficient than that of the downstream cylinders. Also, the minimum drag coefficients of the downstream cylinders occur at \( l/d = 1.5 \) and \( l/d = 2 \). Moreover, by increasing \( l/d \), mean drag coefficients for all cylinders, nearly increase.

3-When the four cylinders are arranged in the current configuration, upstream cylinders A and B exert repulsive force on each other which is
weakening by increasing \( l/d \). In addition, for \( l/d = 1.5 \), due to severe flow interference between cylinders, there is a difference between lift coefficients for upstream cylinders A and B.

4-In the case of four cylinders, the downstream cylinders experience lower mean drag coefficients than that of the upstream cylinders. Moreover, the minimum drag coefficient values for the upstream cylinders occur at \( l/d = 3 \). On the other hand, by increasing \( l/d \), the mean drag coefficients for downstream cylinders are increased. It is notable that for \( l/d \leq 2 \) these coefficients are negative.

5-It is revealed that the effects of variations in \( l/d \), on the aerodynamic coefficients are strong and significant. Furthermore, by increasing the \( l/d \), the effects of the flow interference between the cylinders decrease. For instance, in the case of three cylinders, at \( l/d = 4 \) flow behavior around cylinder A to be somehow the same as that of around a single cylinder. Also, for four cylinders, at \( l/d = 4 \) flow behavior around each upstream cylinder to be somehow the same as that of around a single cylinder.

Finally, it is suggested that the future studies should focus on the effects of Reynolds number in supercritical and low Reynolds numbers flow regimes, high free-stream turbulence intensity, incidence angle, presence of thermal gradients around the cylinders, and the effects of three-dimensional cylinders with finite span, on the flow pattern and the aerodynamic coefficients, when the flow interfere between three or four circular cylinders.

5. References


