Controlling of Absorption and Dispersion Spectrum via Electromagnetically Induced Transparency

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Abstract

In this paper we examine the absorption and dispersion properties of a weak probe field via Electromagnetically Induced Transparency (EIT) in a four-level system. It is shown that under certain condition, using this model, the absorption cancellation is appeared and the medium becomes transparent to the weak probe field. It will be shown that the controlling of absorption and dispersion spectrum depends on some atomic parameters. The effects of quantum interference on the absorption and dispersion spectrum are also investigated.

Keywords: Absorption; Dispersion; Electromagnetically Induced Transparency (EIT); Quantum interference

Introduction

Recently, quantum interference and quantum coherence in multi-level atomic system have attracted a lot of attention, since they can lead to a very novel phenomenon in quantum optics. It is well known that these phenomena have a key role in the absorption cancellation [1-3], Electromagnetically Induced Transparency (EIT) [4-6], lasing without inversion [7,8], coherent population trapping [9] and spontaneous emission reduction [10-15]. EIT is a quantum interference phenomenon that can make a normally opaque transition completely transparent to a probe beam due to coupling of a coherent pump field to a linked transition [16,17]. There is a close link between EIT and other atomic coherence phenomena such as coherent population trapping and other coherent adiabatic population transfer processes. The effect of EIT was first observed by Harris et al. [18], in which quantum interference was introduced by driving the upper two levels of a three-level atomic system with a strong coherent field. Under appropriate condition, the medium become effectively transparent (zero absorption) for a probe field. Since then the effect of EIT was investigated by many authors both experimentally and theoretically [19-21]. In many of these articles, three-level atomic systems are studied or at least systems that can be adequately reduced to three-levels when interaction with the pertinent electromagnetic fields are considered. In the typical configuration two upper levels are coupled by a strong driving field and a tunable probe field is introduced between the two-coupled levels and the lowest level. The transparency could be obtained for an appropriate chosen atomic parameter on the probe field resonance.

The absorption and dispersion properties of the medium in atomic system are related to the quantum interference and quantum coherence. The absorption properties of a weak probe beam in a two level atom driven by an intense pump field has been calculated by Mollow [22] and observed a few years later [23]. In the Autler-Townes effect, the transition of two levels,
connected by the strong field to a third level, is probed. The absorption spectrum exhibits two absorption components, known as the Autler-Townes components [24]. This Autler-Townes absorption doublet and the dispersion like behavior of the probe field can also be related to the dressed state [25]. Various experiments in gas [26] and solid-state systems [27] have confirmed the presence of such effects. Some authors investigated the absorption spectrum and its modification via quantum interference by using quantum regression theory [3,28,29]. An intensively studied example is the basic V system consisting of two close levels coupled by the same vacuum modes to another lower state. In this system, quantum interference results from cross coupling between two indistinguishable decay channels. The interference effects associate with this model lead to very narrow absorption spectrum or complete cancellation.

More recently, several schemes have been proposed for the investigating of the absorption and dispersion properties of the weak probe field via EIT. Agarwal et al. [30] predicted the possibility of making the medium transparent against two photons absorption in a four level atomic system, which was observed in sodium atom [31]. Harris et al. [32] introduced two photons absorption and some other nonlinear effects based on EIT. They showed that, four level-atomic systems can absorb two photons, but not one. In another related study, the effect of EIT was investigated in cascade schemes with N level and (N-1) fields [33]. It was shown that the transparency effects were presented when N was odd and that destruction of EIT was presented on line center when N was even.

Here we use another simple four-level atoms for investigating the absorption and dispersion properties of the weak probe field. A coherent field is coupled two upper levels \( |a_1\rangle \) and \( |a_2\rangle \) to the level \( |b\rangle \). The upper levels \( |a_1\rangle \), \( |a_2\rangle \) and \( |b\rangle \) decay to the lower level \( |c\rangle \) via interaction with the vacuum field with rates of \( \gamma_1 \), \( \gamma_2 \) and \( \gamma_3 \), respectively. A tunable weak probe field with frequency \( \nu_p \) and amplitude E is tuned between level \( |c\rangle \) and level \( |a_1\rangle \), whose dispersion and absorption are requested.

The total Hamiltonian for the model in the rotating wave approximation is given by:

\[
H = H_0 + H_1
\]

Figure 1. Energy scheme used in this paper.
Where

\[ H_0 = \hbar \omega_0 \left| a_1 \right\rangle \langle a_1 | + \hbar \omega_2 | a_2 \rangle \langle a_2 | + \hbar \omega_c | c \rangle \langle c | \]

\[ H_1 = -\frac{\hbar}{2} \left[ (\Omega_e e^{-i\omega} | a_1 \rangle \langle b | + \Omega_e e^{-i\omega} | a_2 \rangle \langle b | + E \varphi_{ac} e^{-i\omega} | a_1 \rangle \langle c | + H.c. \right] \]

Here, \( \omega_0 \) corresponds to the energy of state \( |i\rangle \), and \( \varphi_{ac} \) is the dipole moment of the atomic transition from level \( |c\rangle \) to level \( |a_1\rangle \). \( \Omega_1 = \Omega_2 = \Omega \) denotes the Rabi-frequency of the driving between levels \( |a_1\rangle - |b\rangle \) and \( |a_2\rangle - |b\rangle \) to have a carrying phase \( \varphi \), i.e., \( \Omega = |\Omega| e^{-i\varphi} \). The master equation for the density operator \( \rho \) of the atom takes the form \([34,35]\).

\[ \dot{\rho} = -i\hbar[H, \rho] - \frac{1}{2} (\Gamma, \rho), \]

(4)

Where \( (\Gamma, \rho) = \Gamma \rho + \rho \Gamma^* \). Here the decay rates are incorporated into the equation through the decay matrix \( \Gamma \), which is defined by \( \langle i | \Gamma | k \rangle = \gamma_{ik} \delta_{ik} \). The off-diagonal density matrix elements for the atomic variables take form.

\[ \dot{\rho}_{ac} = -(i \omega_{ac} + \gamma_1 + \gamma_2) \rho_{ac} + \frac{i \Omega}{2} e^{-i\omega} \rho_{bc} \]

\[ -\frac{i \Omega^*}{2} e^{i\omega} \rho_{bc} + \frac{i E}{2 \hbar} \varphi_{ac} e^{i\omega} \rho_{ab} \]

\[ \dot{\rho}_{ab} = -(i \omega_{ab} + (\gamma_1 + \gamma_2)) \rho_{ab} - \frac{i \Omega}{2} e^{-i\omega} (\rho_{a_1} - \rho_{b_1}) \]

\[ -\frac{i \Omega^*}{2} e^{i\omega} (\rho_{a_2} - \rho_{b_2}) + \frac{i E}{2 \hbar} \varphi_{ac} e^{i\omega} \rho_{ab} \]

\[ \dot{\rho}_{ac} = -(i \omega_{ac} + \gamma_1) \rho_{ac} + \frac{i \Omega}{2} e^{-i\omega} \rho_{bc} \]

\[ -\frac{i E \varphi_{ac} e^{-i\omega}}{2 \hbar} (\rho_{a_1} - \rho_{c_1}) \]

\[ \dot{\rho}_{bc} = -(i \omega_{bc} + \gamma_2) \rho_{bc} + \frac{i \Omega}{2} e^{i\omega} \rho_{ab} \]

\[ -\frac{i E \varphi_{ac} e^{i\omega}}{2 \hbar} \rho_{abc}. \]

(5)

Similarly, one can obtain the equation for the level populations, i.e., the diagonal density matrix elements. Here \( \omega_{ik} = \omega_i - \omega_k \) corresponds to the energy difference between the level \( |i\rangle \) and \( |k\rangle \). By using change of variables like,

\[ \rho_{ac} = \tilde{\rho}_{ac} e^{i\omega_1}, \rho_{bc} = \tilde{\rho}_{bc} e^{i\omega_2}, \rho_{ab} = \tilde{\rho}_{ab} e^{-i\omega}, \rho_{bc} = \tilde{\rho}_{bc} e^{i(\omega_2 - \omega_1)}, \]

(6)

the equation of motion in the rotating frame can be written as:

\[ \tilde{\dot{\rho}}_{ac} = -(i \omega_{ac} + \gamma_1 + \gamma_2) \tilde{\rho}_{ac} + \frac{i \Omega}{2} e^{-i\omega} \tilde{\rho}_{bc} \]

\[ -\frac{i \Omega^*}{2} e^{i\omega} \tilde{\rho}_{bc} + \frac{i E}{2 \hbar} \varphi_{ac} e^{i\omega} \tilde{\rho}_{ab} \]

\[ \tilde{\dot{\rho}}_{ab} = -(i \omega_{ab} + (\gamma_1 + \gamma_2)) \tilde{\rho}_{ab} - \frac{i \Omega}{2} e^{-i\omega} (\tilde{\rho}_{a_1} - \tilde{\rho}_{b_1}) \]

\[ -\frac{i \Omega^*}{2} e^{i\omega} (\tilde{\rho}_{a_2} - \tilde{\rho}_{b_2}) + \frac{i E}{2 \hbar} \varphi_{ac} e^{i\omega} \tilde{\rho}_{ab} \]

\[ \tilde{\dot{\rho}}_{ac} = -(i \omega_{ac} + \gamma_1) \tilde{\rho}_{ac} + \frac{i \Omega}{2} e^{-i\omega} \tilde{\rho}_{bc} \]

\[ -\frac{i E \varphi_{ac} e^{-i\omega}}{2 \hbar} (\tilde{\rho}_{a_1} - \tilde{\rho}_{c_1}) \]

\[ \tilde{\dot{\rho}}_{bc} = -(i \omega_{bc} + \gamma_2) \tilde{\rho}_{bc} + \frac{i \Omega}{2} e^{i\omega} \tilde{\rho}_{ab} \]

\[ + \frac{i \Omega^*}{2} e^{i\omega} \tilde{\rho}_{bc} - \frac{i E \varphi_{ac} e^{i\omega}}{2 \hbar} \tilde{\rho}_{abc}. \]
\[ \hat{\rho}_{bc} = -[i(\delta - \Delta_1 + \gamma_3)]\hat{\rho}_{bc} + \frac{i\Omega^*}{2} \hat{\rho}_{af} + \frac{iE\psi_{ae}}{2h} \hat{\rho}_{ba} + \frac{i\Omega}{2}\hat{\rho}_{af} - \frac{iE\psi_{ae}}{2h} \hat{\rho}_{ba} \]  
\[ (7) \]

Where \( \Delta_1 = \omega_{ab} - \nu \), \( \Delta_2 = \omega_{ab} - \nu \) and \( \delta = \omega_{ae} - \nu_p \) are detuning. Here the decay rates from levels \( |a_1\rangle \) and \( |a_2\rangle \) to level \( |b\rangle \) are ignored.

The absorption and dispersion are determined by \( \rho_{ae}^{(0)} \) and we need to calculate the polarization to lowest order in \( E \). However, the coherent field coupling of the level \( |a_1\rangle \), \( |a_2\rangle \) and \( |b\rangle \) is large and we must treat this part of the problem exactly, keeping \( \Omega \) to all orders. As the atom is initially in the level \( |c\rangle \),

\[ \rho_{cc}^{(0)} = 1 \], \( \rho_{a_1b}^{(0)} = \rho_{a_2b}^{(0)} = \rho_{b_1a}^{(0)} = \rho_{b_2a}^{(0)} = 0 \].  
\[ (8) \]

the necessary equation of motion required calculation of the susceptibility can be written as:

\[ \frac{\hat{\rho}_{ae}}{\hat{\rho}_{bc}} = \frac{\hat{\rho}_{ae}}{\hat{\rho}_{bc}} = \frac{\hat{\rho}_{ae}}{\hat{\rho}_{bc}} = \frac{i\Omega}{2}\hat{\rho}_{af} + \frac{iE\psi_{ae}}{2h} \hat{\rho}_{ba} + \frac{i\Omega}{2}\hat{\rho}_{af} - \frac{iE\psi_{ae}}{2h} \hat{\rho}_{ba} \]

This set of equations can be solved, by writing in the matrix form,

\[ \hat{R}(t) = -MR(t) + C \]  
\[ (10) \]

and then integrating it:

\[ R(t) = M^{-1}C \]  
\[ (11) \]

Now the coherent part of the matrix elements, i.e. \( \rho_{ae} \), is given by:

\[ \rho_{ae} = \frac{E\psi_{ae}e^{-i\nu_p t}}{2hY} \left[ i[\Delta_1 + \omega_{ae} + \gamma_3] + \frac{\Omega^*}{2} \right] \]

\[ \left[ \frac{-\Omega^2}{4} \right] \left[ i[\gamma_2(\delta - \Delta_1) + \gamma_3(\delta - \omega_{ae})] \right] \]

\[ Y = A + iB \] , and

\[ A = -\delta^2(\gamma_1 + \gamma_2 + \gamma_3) + \delta_1(\gamma_1 + \gamma_2 + \gamma_3) \]

\[ + \gamma_1(\delta - \Delta_2) + \frac{\Omega^2}{4} \left( \gamma_1 + \gamma_2 + \gamma_3 - \Delta_2 \right) \]

\[ B = \frac{-\Omega^2}{4} \left( 2\delta - \omega_{ae} \right) + \gamma_1 \gamma_2 \left( \delta - \Delta_2 \right) \]

The complex susceptibility of the system is defined as,

\[ \chi = 2N \frac{\varphi_{ae}}{\hbar \psi e} \rho_{ae} e^{i\nu_p t} \]  
\[ (15) \]

Where \( N \) is the atom number density and \( \chi = \chi' + i\chi'' \). Therefore the real and imaginary parts of susceptibility can be written as:

\[ \chi' = \frac{N \varphi_{ae}}{\hbar \psi e} \left[ \left[ \gamma_1(\delta - \Delta_1) + \gamma_3(\delta - \omega_{ae}) \right] A \right] \]

\[ + \left[ \delta_1(\Delta_1 + \omega_{ae}) + \gamma_2 \gamma_3 - \delta^2 + \frac{\Omega^2}{4} \right] B \]

\[ \chi'' = \frac{N \varphi_{ae}}{\hbar \psi e} \left[ \left[ \gamma_1(\delta - \Delta_1) + \gamma_3(\delta - \omega_{ae}) \right] B \right] \]

\[ + \left[ \delta_1(\Delta_1 + \omega_{ae}) + \gamma_2 \gamma_3 - \delta^2 + \frac{\Omega^2}{4} \right] A \]

Where, \( Z = YY^* \).

### III) Results and Discussion

Expressions (16) and (17) are our basic results for the real and imaginary parts of susceptibility, that they depend on the atomic parameters i.e., probe and driving field detuning, \( \delta \) and \( \Delta_1 \), Rabi frequency \( \Omega \), and decay rates \( \gamma_1 \), \( \gamma_2 \), and \( \gamma_3 \). In the following we summarize our results for the dependence of the real and imaginary parts of the susceptibility \( \chi \) (i.e., \( \chi' \) and \( \chi'' \)) on the intensity of the control field. In particular, we will see that the absorption spectrum can be controlled by the appropriate chosen of the atomic parameters. A simple explanation of the peak elimination and cancellation of the absorption can be given within the dressed-state picture. Since we discussed in the previous sections, the controlling of probe field absorption and dispersion are related to the quantum interference and coherence. As a first step we
examine our results for \( \gamma_1 = \gamma_2 = \gamma_3 = \gamma \), \( \Omega = 0 \), \( \omega_{\text{det}} = 0.1\gamma \) and \( \Delta_1 = \gamma \). It is clear that in this case our system reduces to the usual form of two level scheme (Fig. 2). As we expected from our model, in the probe absorption we get three peaks associated with three-dressed state (Fig. 3).

We consider the effects of the dynamical variables, namely, the amplitude or more precisely the Rabi frequency, detuning parameter and decay rates on the probe absorption and dispersion. The variation of these parameters influences the real and imaginary part of susceptibility efficiently. In the probe absorption, central peak is significant when \( \gamma_1 = \gamma_2 = \gamma_3 = \gamma \), \( \omega_{\text{det}} = 0.1\gamma \), \( \Delta_1 = 0.5\gamma \) and \( \Omega = 5\gamma \). The plot for these values shows an extremely suppression of two side peaks and enhancement of the central peak (Fig. 4). For the case \( \gamma_1 = 2\gamma \), \( \gamma_2 = 20\gamma \), \( \gamma_3 = 0 \), \( \Delta_1 = 0 \), \( \omega_{\text{det}} = 0.1\gamma \) and \( \Omega = 2\gamma \), the central peak is suppressed while two side peaks are enhanced (Fig. 5).

This was the basic result, which we expected from the EIT. In this case the level \( |b\rangle \) is metastable i.e., \( \gamma_3 = 0 \), and for the driving field resonance, \( \Delta_1 = 0 \), the medium becomes transparent for the probe field. As a usual EIT scheme when the driving field is very strong and probe field is very weak, the interference effects will be important due to driving field. Alternatively, if the EIT process is viewed within the atomic bare-state basis (rather than the dressed state), the so-called 'coherence' can be seen being the quantities pertinent to the interference. This coherence can be thought of, in a semi-classical picture, as associated with the oscillating electric dipoles driven by the coupling fields applied between pairs of quantum states of the system, i.e. \( |i\rangle - |k\rangle \). Strong excitations of this dipole occur whenever electromagnetic field is applied close to resonance with an electric dipole transition between two states. If there are several way to excite the oscillating dipole associated with \( |i\rangle - |k\rangle \), then it is possible for interference to arise between the various contribution to this dipole, and these must be summed to give the total electric dipole oscillation between \( |i\rangle - |k\rangle \). This is directly analogous to the Fano [1] effect in atomization. The perfect absorption cancellation depends on the metastability of level \( |b\rangle \), any radiative or collisional decay of this state will lead to finite absorption even at zero detuning of driving and probe fields (\( \delta = \Delta_1 = 0 \)). EIT will manifest itself in the value of the density matrix element whose real and imaginary parts should vanish at zero detuning (i.e. the coherence is canceled both by the interference of the pathways that can excite it).

IV) Conclusion

EIT effect has been proposed in a four-level system driven by a strong coherent field. The effects of atomic parameters on the real and imaginary parts of susceptibility, which leads to narrowing, and elimination of absorption spectrum, are considered. The effects of amplitude, \( \Omega \), driving and probe field detuning and decay rate on the absorption and dispersion are discussed. Note that quantum coherence and interference have the key role on controlling the absorption and dispersion spectrum in our system.

![Figure 2](image2.png)

**Figure 2.** Real (solid) and imaginary (dash) parts of susceptibility at probe frequency in the presence of driving field vs. detuning for \( \gamma_1 = \gamma_2 = \gamma_3 = \gamma \), \( \omega_{\text{det}} = 0.1\gamma \), \( \Delta_1 = \gamma \), \( \Omega = 0 \).

![Figure 3](image3.png)

**Figure 3.** Real (solid) and imaginary (dash) parts of susceptibility at probe frequency in the presence of driving field vs. detuning for \( \gamma_1 = \gamma_2 = \gamma_3 = \gamma \), \( \omega_{\text{det}} = 0.1\gamma \), \( \Delta_1 = 0.5\gamma \), \( \Omega = 5\gamma \).
Figure 4. Real (solid) and imaginary (dash) parts of susceptibility at probe frequency in the presence of driving field vs. detuning for $\gamma_1 = \gamma_2 = \gamma$, $\Omega = 0.5\gamma$, $\omega_{ap} = 0.1\gamma$, $\Delta_1 = 0.5\gamma$.

Figure 5. Real (solid) and imaginary (dash) parts of susceptibility at probe frequency in the presence of driving field vs. detuning for $\gamma_1 = 2\gamma$, $\gamma_3 = 0$, $\gamma_2 = 2\gamma$, $\Omega = 2\gamma$, $\omega_{ap} = 0.1\gamma$, $\Delta_1 = 0$.

References