Reducing extent of cracks and increasing time to failure of concrete gravity dams by optimization of properties of layers of concrete

A. Joghataie* and M.S. Dizaji

Department of Civil Engineering, Sharif University of Technology, Tehran, Iran.

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Abstract. The objective of this paper is to study the improvement in the seismic behavior of concrete gravity dams by optimization of concrete mechanical properties. The criteria to measure the improvement have been: 1) reduction in the extent of cracks and 2) increase in the amount of time dams are able to tolerate earthquakes before failure. The mechanical properties considered have included the density and modulus of elasticity of concrete. The Pine Flat Dam has been selected for this numerical study. During a high intensity earthquake, dams enter a nonlinear phase, where the cracks open and close repeatedly. A smeared crack model has been used for simulation of nonlinearity. For the purpose of optimization, the dam has been divided into horizontal layers, where the concrete is assumed to have the same properties at every point within each layer. The results of this study have shown that by using lower density concrete in upper layers and in the region of the crest of the Pine Flat Dam, it is possible to both reduce the extent of the induced cracks and increase the time to failure of the dam. The same methodology can be applied to other concrete gravity dams.

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1. Introduction

Concrete gravity dams experience cracks even at low service conditions, due to the low tensile strength of concrete. Small surface cracks do not pose a threat to the dam, but when the cracks propagate into the depth of the dam and connect to form a network, the cracks become detrimental and can cause problems. Also, during a severe earthquake, a concrete gravity dam might experience structural cracks, which repeatedly open and close. This opening and closing of cracks makes the response of the dam nonlinear.

Modeling nonlinear behavior of concrete gravity dams has been investigated by many authors in the past, including [1-25]. Because of their importance, including their social, economical, political and environmental effects, the failure of concrete gravity dams has been investigated from every angle, including their dynamic response to earthquakes [3,18,26-30].

The Köyna Dam, in India, suffered from a 6.5 Richter magnitude earthquake on December 11th, 1967, which caused severe cracks in the dam, especially at its crest. Since then, the dam has become a subject of study by researchers in civil engineering, including dam engineering [1,18,26,31,32].

The Sefidrud Dam in Iran was severely damaged during the 7.3 Richter Manjil earthquake on June 20th, 1990, which caused many cracks to appear in the body and crest of the dam. The most severe crack occurred right below the dam crest, propagated, and detached the crest from the rest of the dam body [17-28].
The above cases show the importance of designing a better concrete for the dams, to possibly reduce the level of damage.

Also, the Pine Flat Dam in the US has been a focus for researchers in dam engineering, though the dam has not experienced a noticeable crack during its lifetime. Many papers and reports have been published on its earthquake behavior and even tests on small scale models of the dam have been conducted [3,8,17,27,33-35]. Since there is a large amount of data available on this dam, it has been chosen as the sample case in this paper too.

Obviously, by enhancement of the properties of concrete, especially its tensile and flexural strength, it is possible to build concrete gravity dams with more desirable seismic response. Even with ordinary concrete, with which most dams have already been built (and are still being built), it might be possible to optimize its properties to improve the seismic behavior of concrete gravity dams.

In this paper, we wished to study the optimization of concrete properties for dam safety in more detail. Concrete strength directly depends on its density. On the other hand, although by increasing the density, it is expected to obtain a better performance from concrete, the mass of the dam increases, which is not desirable from the viewpoint of its seismic behavior. This numerical study and optimization of the dam is expected to provide more insight into the use of concrete in building concrete gravity dams with more desirable seismic performance.

In this paper, the authors have studied the Pine Flat Dam as an example and have provided details about the optimization algorithm and results.

In the following sections, first, the smeared crack model has been explained. Next, brief explanations about the Pine Flat Dam have been given, including its geometry, properties of the concrete used in construction, and the finite element mesh used in nonlinear dynamic analysis. The algorithm proposed for the optimization has been explained, followed by presentation of the results obtained from application of the algorithm to the Pine Flat Dam, and, finally, the conclusions.

2. Smeared crack model for nonlinear behavior of concrete gravity dams

Smeared crack is one of the concrete models widely used in the numerical simulation of concrete gravity dams. This model has also been used in this study to model the Pine Flat Dam. In this model, the main criterion for identification of the first crack is the damage energy denoted by $G_f$ [12-15]. The first step in the application of nonlinear damage mechanism models in the dynamic analysis of dams is to define the loading-unloading backbone curve. The model should be capable of modeling hysteretic behavior, which takes place during the dynamic response of the dam subjected to earthquakes.

Figure 1 shows, schematically, how this model can be used in the nonlinear analysis of concrete gravity dams [14,36]. Figure 1(a) shows the Pine Flat Dam and the 2-dimensional finite element mesh designed for its nonlinear analysis under earthquake loading. Smeared cracks have occurred at the heel and neck of the crest of the dam. Figure 1(b) shows the backbone curve for loading followed by unloading. More details of loading, unloading and reloading cycles, and how the material eventually experiences fracture, have been provided in Figure 1(b). The hysteretic behavior of the material resulting from changing the load during an earthquake follows the stress-strain curve in this figure. Figure 1(c) shows the deformed shape of the dam at the instance of its failure. This figure is, in fact, similar to Figure 1(a), both of which show the dam at the instance of failure; however, Figure 1(c) shows the magnified deformed shape [14,15,36]. In Figure 1(d), $G_f$ has been used to denote the area under the loading-unloading curve.

Figure 1(e) represents a 2-dimensional 4-node element in the finite element mesh for the Pine Flat Dam [14,15].

Bazant and Gamberova [5] developed a nonlinear stress-strain model to explain the process of the opening and closing of cracks during the nonlinear response of the dams. Their model is schematically explained in Figure 2(a). Also, de Borst and Nauta [6] proposed a simple model, which is schematically shown in Figure 2(b). In a model proposed by Gamberova and Valente [5], tensile stresses are released and the stress-strain curve enters the compression phase (Figure 2(c)). Based on these studies, Dahlblom and Ottosen [10] proposed the following equation to explain the model, where Figure 2(b) and (c) visualizes the equation:

$$
\varepsilon = \frac{\sigma}{\sigma_{\text{max}}} [\lambda + (1 - \lambda)] \varepsilon_{\text{max}} \quad 0 < \lambda < 1
$$

where $\lambda$ is ratio of residual stress within the closed crack to the maximum strain which has taken place in the open crack, as in Figure 2(d). $\lambda = 0$ and 1 correspond to the models represented in Figure 2(b) and (c), respectively [37].

In this paper the stress-strain relationship shown in Figure 2(b), which corresponds to $\lambda=0$, has been used to model the hysteretic behavior of the damaged concrete in nonlinear dynamic analysis of the dams [6,12-15,37].

3. Pine Flat Dam

The dam was built in 1954 over the Kings River, 30 miles from Fresno in the USA, and its construction
Figure 1. Smear crack model: (a) Smear crack in pine flat concrete gravity dam after it has been subjected to an earthquake (drawn based on Bhattacharjee and Léger 1994); (b) hysteresis loading and unloading and resulting stress-strain curve (drawn based on Léger 2007); (c) deformed shape of dam at instance of its failure; (d) concept of fracture energy which is the area under load-unloading curve (drawn based on Léger 2007); and (e) 4-node isoparametric element in smear crack model (drawn based on Léger 2007).

Figure 2. Stress-strain relationship proposed for the concrete to use in nonlinear dynamic analysis of concrete gravity dams (drawn based on Bhattacharjee 1993): (a) Bazant and Gambarova model; (b) de Borst and Nauta model; (c) Gambarova and Valente model; and (d) the λ.

took 5 years. Figure 3 schematically shows the characteristics of the dam. The dam is made of 37 monoliths of 15.2 m width, and the length of its crest is 560 m. In this study, the tallest monolith has been simulated, which has a height of 122 m. According to a study of geological properties at the site of the dam, some slippage has occurred in the metamorphic rock. Table 1 contains basic information about the geometry of the dam [17, 27, 38].

Also, Table 2 contains information about properties of the concrete used in the numerical analysis of the dam, including its density, elastic modulus, Poisson’s ratio, tensile strength and fracture energy. A tensile
Table 1. Geometry of Pine Flat Dam according to Donlon and Hall [27], and Ghaemian and Gholbari [17].

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of crest (m)</td>
<td>500</td>
</tr>
<tr>
<td>Number of monoliths</td>
<td>37</td>
</tr>
<tr>
<td>Tallest monolith (m)</td>
<td>122</td>
</tr>
<tr>
<td>Crest width (m)</td>
<td>9.8</td>
</tr>
<tr>
<td>Bottom width (m)</td>
<td>96.8</td>
</tr>
<tr>
<td>The upstream slope</td>
<td>1.005</td>
</tr>
<tr>
<td>The downstream slope</td>
<td>1.078</td>
</tr>
</tbody>
</table>

Strength equal to 10% compressive strength has been assumed [27].

4. Loading

The loadings considered in this study, for which the dam has been analyzed, have included: the weight of the dam, hydrostatic pressure, earthquake and hydrodynamic loading.

In order to consider hydrostatic pressure, the height of water in the reservoir of the dam has been assumed to be 116.8 m. For earthquake loading, the horizontal component of El Centro (May 18, 1940), Park Field (Jun 28, 1966), San Fernando (February 9, 1971), Northridge (January 15, 1994), and White Noise have been used. Table 3 contains the basic information about the earthquakes. Figure 4(a)-(e) show the time history of ground acceleration for the horizontal components of the above earthquakes, where ground acceleration has been plotted versus time. Also, Figure 5(a)-(e) shows the frequency content of each earthquake, where ground acceleration has been plotted versus frequency.

Table 2. Properties of concrete according to Donlon and Hall [27].

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of concrete (ρ)</td>
<td>23520 N/m³</td>
</tr>
<tr>
<td>Modulus of elasticity (E)</td>
<td>27,580 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio (ν)</td>
<td>0.2</td>
</tr>
<tr>
<td>Tensile failure stress (f_t)</td>
<td>2.7 MPa</td>
</tr>
<tr>
<td>Compressive ultimate stress (f_u)</td>
<td>27 MPa</td>
</tr>
<tr>
<td>Fracture energy (G_f)</td>
<td>150 N/m</td>
</tr>
</tbody>
</table>

Table 3. Characteristics of earthquakes used in this study.

<table>
<thead>
<tr>
<th>Name of earthquake</th>
<th>Year</th>
<th>Magnitude</th>
<th>PGA (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro</td>
<td>1940</td>
<td>7.1</td>
<td>0.34</td>
</tr>
<tr>
<td>Park Field</td>
<td>1966</td>
<td>6.2</td>
<td>0.50</td>
</tr>
<tr>
<td>San Fernando</td>
<td>1971</td>
<td>6.6</td>
<td>0.67</td>
</tr>
<tr>
<td>Northridge</td>
<td>1994</td>
<td>6.9</td>
<td>0.41</td>
</tr>
<tr>
<td>White Noise</td>
<td>-</td>
<td>-</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Figure 4. Time history of earthquakes: (a) El Centro; (b) Park Field; (c) San Fernando; (d) Northridge; and (e) White Noise.

As seen in Figure 5, the selected earthquakes had different frequency content, so that the dam could be designed to withstand earthquakes of different characteristics. However, such designing of the dam seems too demanding and unnecessary. It seems sufficient to
Figure 5. Fourier spectrum of earthquakes: (a) El Centro; (b) Park Field; (c) San Fernando; (d) Northridge; and (e) White Noise.

only consider design earthquakes for the region where the dam is located.

5. Finite element model of the dam

For nonlinear dynamic analysis of the dam, a mesh with 1984 isoparametric 4-node elements has been used. Figure 6 shows the mesh. The meshing has been done, so that smaller elements can be placed at the dam crest and in its vicinity for more precision, since stress concentration and significant cracks are expected to occur at this point of the dam monolith. Similarly, a finer mesh has been used at the heel of the dam where cracks are induced [14-17-37]. A solid foundation has been assumed for analysis of the dam.

6. Layers of concrete

It is expected that higher stresses will be induced at the location of changes in the slope and curvature of
the dam’s upstream and downstream faces. Such high stresses might result in structural cracks.

There is a change in the slope of the dam at the 102.1 m level upstream and two changes in the curvature at 113.8 m and 98.05 m levels downstream. Also, detrimental cracks are expected to occur at the neck of the crest, which is located at about 95 m to 115 m level. Hence, it has been decided to optimize concrete properties in regions where cracks might occur. To this end, the region between the 80 m level and the top of the dam at 122 m, with a total height of 122 m - 80 m = 42 m, has been considered as the region to be optimized. 8 layers of concrete have been considered. The top layer, between levels 122 m to 115 m, is 7 m high, followed by 7 layers, each 5 m high. The remaining bottom part of the dam is thick enough and detrimental cracks are not expected to develop there. The cracks which appear at the heel of the dam are not serious enough to cause failure. However, one may decide to introduce more layers for optimization and/or to change the number or height of the layers.

Figure 7 shows the tallest monolith of the dam, which has been selected for optimization. The 8 layers of the dam are also shown in Figure 7.

7. Optimization algorithm

The complete algorithm for optimization of the concrete of the layers is explained in this section. Before explaining the algorithm steps for optimization, it is helpful to explain some definitions, terms and parameters that have been repeatedly used in the algorithm.

Optimization earthquake: The first step is selection of the earthquake to use in the optimization of the dam.

This earthquake has been called the “optimization earthquake” and has been denoted by $E_{opt}$.

**Test earthquakes:** After its optimization, both the original and optimized dams are subjected to other earthquakes, to study if the optimization has helped improve the performance of the dam generally. The earthquakes used for the evaluation have been called “test earthquakes”, denoted by $E_t$. In this paper, the Pine Flat Dam has been the dam under study and the El Centro (1940) earthquake has arbitrarily been selected as the optimization earthquake, though other earthquakes could have been selected. The test earthquakes have included: White Noise, Park Field and San Fernando earthquakes.

**Layer properties:** After preparing the dam finite element mesh for analysis and defining the n layers for the dam, the density ($\rho$) and elastic modulus ($E$) in each layer have been used as the variables, so that they could be modified during the optimization. The layers have been identified by $L_i$ to $L_n$, with $L_1$ as the top layer, where their corresponding density and modulus of elasticity have been denoted by $\rho_i$ and $E_i$, respectively, $i = 1, 2, \ldots, n$.

At the beginning of optimization, $\rho_i = \rho_0$ and $E_i = E_0$. 

where $\rho_0$ and $E_0$ are the properties of the original dam. Determination of $\rho_i$ has been sufficient for the determination of $E_i$, $i = 1, 2, \ldots, n$ according to the following equation [30]:

$$E_i = 0.043\rho_i^{1.5} \sqrt{f'_c}, \quad i = 1, 2, \ldots, n,$$

where $E$, $\rho$ and $f'_c$ are modules of elasticity (MPa), density of concrete (kg/m$^3$) and 28-day cylindrical compressive strength (MPa), respectively.

**Scaled optimization earthquakes:** Denoting the scale factor by $\alpha\%$, the optimization earthquake is multiplied by different values of $0 < \alpha \leq 100$, resulting in weaker earthquakes of acceleration $E_\alpha$, which are called “scaled optimization earthquakes”, where:

$$E_\alpha = \alpha\%E_{opt}.$$  

The values of $\alpha$ can be specified in different ways. One method used in this paper is to select the $\alpha$ values at equal intervals. To this end, if the desired number of intervals of change in the $\alpha$ values is denoted by $n_a$, then, starting from $\alpha = 0$, the $n_a + 1$ values of $\alpha$ are simply calculated from:

$$\alpha = 100(i - 1)/n_a, \quad i = 1, 2, \ldots, n_a + 1,$$

which can be expressed simply as:

$$\alpha = 0, 100/n_a, 200/n_a, \ldots, 100.$$  

**Failure time:** Is defined as the time at which the dam
fails under a scaled earthquake, $E_a$, and is denoted by $t_a$.

**Failure time curve for an earthquake:** Is the plot of $t_a$ against $\alpha$ for the given earthquake. Figure 8 shows the failure time curve for the example problem, which will be discussed later in the paper.

**Cutting time of failure:** The designer specifies a cutting time of failure to stop the analysis for cases when the scaled earthquake is not strong enough to cause failure in the dam. This time depends on factors such as the duration and frequency content of the optimization earthquake, as well as the experience and judgment of the designer. Denoted by $t_c$, the cutting time should be long enough to assume the dam has remained safe during the earthquake. Nonlinear analysis of a dam is time consuming, and defining appropriate $t_c$ is necessary. The maximum value of $\alpha$, for which the dam fails at $t_c$, has been called the “cutting scale factor” and has been shown by $\alpha_c$.

$t_a - \alpha$ Area: Area under $t_a - \alpha$ curve, denoted by $A_a$.

**A complete analysis:** The dam is analyzed for all the scaled optimization earthquakes.

$t_a$ corresponding to each scaled earthquake is determined. $t_a$ is plotted against $\alpha$. By increasing $\alpha$, $t_a$ reduces.

The elements which have experienced cracking are identified and their total number is determined. The finite element mesh, containing the crack profile at $t_a =$ time of failure, is recorded and plotted for further study. If the time to failure has been longer than the cutting failure time, $t_c$, the latter should be used.

Figure 8 shows the above definition for the example, which will be discussed later in the paper.

**Minimum and maximum $\rho$ values:** Since the properties of concrete change depending on its density, it is necessary to define the type of concrete and the lower and upper bounds of density in each of the layers. The lower and upper bounds are denoted by $\rho_l$ and $\rho_u$, respectively.

**An optimization cycle:** A complete updating of density in all the $n$ layers of the dam.

**Increment of change in $\rho$:** Different classical and modern optimization methods for updating the design parameters could be used [40]. A simple practical method, which is expected to provide a suitable answer, is proposed here. Once the direction of change in a design parameter is determined, i.e., increasing or decreasing, a constant increment of change is added to or subtracted from the current value of the parameter, respectively. Since the design parameters are the densities of the layers of the dam, the increment of change is denoted by $\Delta \rho$, which has been specified by the designer. Obviously, a smaller $\Delta \rho$ value means more precision, but higher computation cost. In this application, as the optimization has proceeded, a smaller $\Delta \rho$ has been introduced to achieve more precision at the final cycles of the updating of variables.

The value of $\Delta \rho$ in the 7 cycles of optimization has been:

$$\Delta \rho = 100, 100, 50, 50, 20, 10 \text{ kg/m}^3.$$  \hspace{1cm} (6)

**Direction of change in $\rho$:** For each of the design variables, which have been the layer densities in this paper, a direction of change has been defined as $S_i = 1, i = 1, 2, \ldots, n$. The density, $\rho_i$, has been updated as follows:

$$\rho_i = \rho_i + S_i \Delta \rho, \quad i = 1, 2, \ldots, n.$$ \hspace{1cm} (7)

**The criterion for feasibility of change:** A change is considered as feasible if the variable remains feasible after the change. In this paper, the criterion to evaluate a change as feasible is:

Feasibility criterion 1: After the change the variable remains within its feasibility upper and lower bounds.

**The criteria of usability of change:** A change in a design variable is considered as usable if it results in improvement of the conditions. In this paper, the criteria to evaluate a change as usable are:

Usability criterion 1: $\alpha_c$ has increased;
Usability criterion 2: $A_a =$ area under the $t_a - \alpha$ curve, has increased.

**Termination criteria:** The algorithm is brought to an end if at least one of the following criteria has been satisfied:

Termination criterion 1: Number of optimization cycles $\geq$ max-cycles;
Termination criterion 2: Increase in \( \alpha_c \) after an optimization cycle \(< \min - \Delta \alpha_c \);

Termination criterion 3: Increase in \( A_i \) area under \( t_i - \alpha \) curve, after an optimization cycle \(< \min - \Delta A_i \);

Termination criterion 4: Change in \( \rho_i \) after an optimization cycle \(< \min - \Delta \rho_i \), \( i = 1, 2, ..., n \) that is the change in the density of all the layers has been less than a minimum,

where \( \text{max-cycles} = \text{maximum number of cycles allowed}, \min - \Delta \alpha_c, \min - \Delta A_i \) and \( \min - \Delta \rho \) represent the minimum acceptable improvement after one complete optimization cycle in the values of \( \alpha_c, A_i \) and \( \rho \), respectively.

7.1. Optimization steps

Based on the above definitions, the steps of optimization are now explained as follows:

**Step 0.** Select \( E_{\text{opt}} \) = optimization earthquake; \( E_{\text{test}} \) = test earthquakes; \( n = \) number of layers; \( n = \) number of intervals of \( \alpha \); \( t_i \) = cutting failure time; \( \Delta \rho \) = increment of change in density; \( \min \Delta \alpha_c \) = minimum improvement in \( \alpha_c \); \( \min \Delta A_i \) = minimum improvement in \( A_i \) and \( \rho \) and \( \rho_c \) = lower and upper bounds on concrete density.

The finite element mesh for nonlinear dynamic analysis of the dam is defined. The levels of the \( n \) layers are determined. At the beginning of optimization, \( \rho_i = \rho_0 \) and \( E_i = E_0 \), \( i = 1, 2, ..., n \).

The directions of change are set to \( S_i = -1, i = 1, 2, ..., n \). Hence, the default direction is the reduction in the densities.

Set \( k = \) layer number to be optimized \( = 0 \).

**Step 1.** If \( k = n \) and if the termination criteria are satisfied, then stop the optimization, the answer has been obtained.

\( k = k + 1 \). If \( k > n \), then \( k = 1 \).

Perform a complete analysis.

**Step 2.** Set \( \rho = \rho_k \) and \( \rho_i = \rho_k + S_k \delta \rho \).

If \( \rho_k \) is not feasible, then \( \rho_k = \rho \) and \( S_k = -S_k \). Go to Step 4.

Perform a complete analysis.

**Step 3.** Compare the results from Steps 1 and 2.

Check the criteria of usability.

If the usability criteria are satisfied, then go to Step 2.

If the usability criteria are not satisfied, set \( \rho_k = \rho \) and \( S_k = -S_k \).

**Step 4.** \( \rho = \rho_k \) and \( \rho_i = \rho_k + S_k \delta \rho \).

If \( \rho_k \) is not feasible, then \( \rho_k = \rho \). Go to Step 1.

Perform a complete analysis.

**Step 5.** Compare the results from Steps 1 and 4.

Check the criteria of usability.

If the criteria are satisfied, then go to Step 4.

If the criteria are not satisfied, then go to Step 1.

8. Dam optimization example: Pine Flat Dam

Pine Flat concrete gravity dam has been optimized using the algorithm presented in the above sections. Explanation about the geometry and material properties of the dam have been provided in the previous sections too. The parameters which should be defined and specified to be used with the optimization algorithm have been as follow: optimization earthquake \( E_{\text{opt}} = \) El Centro; test earthquakes \( E_{\text{test}} = \) Park Field, San Fernando, Northridge and White Noise; number of layers \( n = 8 \); number of intervals of \( \alpha = n_a = 10 \); cutting failure time \( t_i = 10 \) sec; increment of change in density \( \Delta \rho = 100, 100, 50, 50, 50, 10 \) kg/m\(^3\); minimum improvement in \( \alpha_i = \) \( \min - \Delta \alpha_i \) = 2%; minimum improvement in \( A_i = \) \( \min - \Delta A_i \) = 5%, sec; lower and upper bounds on concrete density \( \rho_l = 1400 \) kg/m\(^3\) and \( \rho_u = 2400 \) kg/m\(^3\).

8.1. Analysis of dam before optimization

Figure 8 shows the \( t_i - \alpha \) curve at the beginning of optimization where the dam has been subjected to the scaled El Centro (1940) earthquakes. The maximum scale factor at which the dam has remained safe after 10 seconds of vibration, has been \( \alpha_c = 45\% \). The area under this curve has been \( A_0 = 651.38\% \) sec.

Figure 9 shows the crack profiles for the original and optimized dams corresponding to different values of \( \alpha = 50\%, 60\%, 70\%, 80\%, 90\%, 100\% \), where the figures in each row correspond to the same value. Also in each row, the left figure denoted by (a) shows the crack profile at \( t_c = \) the time of failure of the original dam before it has been optimized. To visualize how the optimization has improved the crack distribution, Figure 9(b) shows the crack profile for the optimized dam but at time of failure of the original dam, and Figure 9(c) shows the cracks at time of failure of the optimized dam. By comparing Figure 9(a) and (b) for each \( \alpha \) value, it is obvious that the crack extent has been limited after optimization.

Figure 10 shows the peak absolute value of the dam crest displacement before the dam has failed, as a function of \( \alpha \) both for the original and optimized dams. The values corresponding to the data points in Figure 10 have also been tabulated in Table 4 for better presentation of results. Clearly, there have been a direct relationship between \( \alpha \) and peak crest displacement both before and after optimization.

However, the peak displacement has considerably increased after the dam has been optimized. This indicates the dam has become more ductile and flexible.
Figure 9. Crack profiles for different $\alpha$ values: (a) Before optimization and at time of failure; (b) after optimization and at time of failure of original dam; and (c) after optimization and at time of failure of optimized dam.

Table 4. Peak absolute value of dam crest displacement in original and optimized dam for different $\alpha$ values.

<table>
<thead>
<tr>
<th>Dam</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original dam</td>
<td>3.3 cm</td>
<td>3.5 cm</td>
<td>4.1 cm</td>
<td>4.4 cm</td>
<td>4.42 cm</td>
</tr>
<tr>
<td>Optimized dam</td>
<td>9.46 cm</td>
<td>10.4 cm</td>
<td>12 cm</td>
<td>12.1 cm</td>
<td>12.9 cm</td>
</tr>
</tbody>
</table>

Figure 10. Peak absolute value of dam crest displacement versus $\alpha$ before and after optimization.

Figure 11. Number of elements which have experienced damage during scaled El Centro earthquake versus scale factor, $\alpha$, before and after optimization.

The weight of a 1 m wide section of the dam per unit of its height, which has been calculated as the concrete density before optimization $\times$ the thickness of the dam at any given level, when the density of concrete has been constant anywhere in the dam. Obviously the shape of this curve is similar to the cross section of the dam.
the exact values of the points shown in Figure 13(a) and (b).

The monotonic increases in $\alpha_c$ and $A_n$ show that the optimization of the dam is an effective way to increase the dynamic characteristics of the dam to withstand earthquakes for longer duration.

Figure 14 shows the curves for the peak absolute value of dam crest displacement during optimization cycles for different $\alpha$ values, where each curve belongs to a specific $\alpha$. This figure also shows that the displacement for a given $\alpha$ has increased as a result of optimization.

Figure 8 shows the $t_d - \alpha$ curve after optimization has been completed, when the dam has been subjected to scaled El Centro (1940) earthquakes. In Figure 8, the maximum time to failure $t_{d,\text{max}} = 10$ sec corresponds to $\alpha_c = 80\%$, which is about twice that for the original dam, when $\alpha_c = 45\%$. So, optimization has doubled the intensity of the earthquake which can cause failure.

Figure 15 shows how the number of cracked elements has reduced during optimization, where each figure belongs to a given $\alpha$. Also, in the same figure, the time to failure of the dam during optimization has been plotted for each $\alpha$ value. Without exception, for all the $\alpha$ values, the time to failure has increased and the number of cracked elements has decreased, monotonically but gradually, during the process of optimization. In this figure, the number of cracked elements has been multiplied by 0.1, so that it could be shown together with $t_d$ on the same figure. For example, for $\alpha = 70\%$, after optimization cycle 2, the number of cracked elements $= 7.9 \times 10^{-1} = 79$ at the time of failure $= 4.5$ sec.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimization cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_c$, (%)</td>
<td>1 2 3 4 5 6 7</td>
</tr>
<tr>
<td>$A_n$, (% sec)</td>
<td>705.8 757.15 776.7 842.9 8523 9045 9255</td>
</tr>
</tbody>
</table>

Table 5: Exact values of $\alpha_c$ and $A_n$ of points shown in Figure 13.
Figure 15. Obtained for scaled El Centro earthquakes. Monotonic gradual decrease in number of cracked elements and increase in duration of vibration of dam before failure, during cycles of optimization where each figure belongs to a specific $\alpha$ value.

Figure 16. Profile of weight per unit height of a 1 m wide section of dam before and after each cycle of optimization.

The profile of weight per unit height of the dam after its optimization has been plotted in Figure 16. This figure shows how the inertia of the dam has been modified to become optimized gradually.

Figure 17. For El Centro earthquake. Curves drawn after each cycle of optimization has completed where each curve is for a specific optimization cycle, showing the time to failure as a function of $\alpha$.

El Centro earthquakes. Since the final optimization result has been obtained after 7 cycles, there are 7 curves in this figure, corresponding to these 7 cycles. After each cycle of optimization, the curve has slightly shifted to the right and $\alpha_c$ has increased too. At the beginning of optimization, the cutting scale factor was
\( \alpha_c = 45\% \), while, at the end of the 7th cycle, the cutting scale factor increased to about \( \alpha_c = 80\% \). For example, at \( \alpha = 70\% \), \( t_a \) has increased monotonically after each additional optimization cycle, where \( t_a = 2.531 \) sec before optimization and \( t_a = 3.45, 5.2 \) sec after the first, second, and third optimization cycles, respectively, while, after the fourth cycle, \( t_a = 10 \) sec.

Figure 18(a)-(c) also shows how the time to failure has increased and the number of cracked elements has decreased after optimization has been completed, where each figure belongs to a specific \( \alpha \) value. For example, in Figure 18(a), which belongs to \( \alpha = 60\% \), the original dam before optimization has been analyzed for 60% El Centro earthquake, where the time to failure has been 5.105 sec. The number of elements damaged during the analysis has been recorded and plotted versus time. Then, the optimized dam was also analyzed for 60% El Centro earthquake, where the dam has not failed. Also, the time history of the number of cracked elements has been recorded and plotted in the same figure. These figures also show that not only has the time to failure increased, but the number of damaged elements has reduced considerably.

Table 6 shows how the concrete density has changed in each layer after each optimization cycle.

### Table 6. Concrete density (kg/m³) in each layer at the end of each cycle of optimization.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Optimization cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1400 1400 1400 1400 1400 1400 1400</td>
</tr>
<tr>
<td>2</td>
<td>1500 1400 1400 1400 1400 1400 1400</td>
</tr>
<tr>
<td>3</td>
<td>1700 1600 1550 1500 1400 1400 1400</td>
</tr>
<tr>
<td>4</td>
<td>1700 1600 1550 1500 1450 1410 1410</td>
</tr>
<tr>
<td>5</td>
<td>1800 1700 1650 1600 1500 1420 1410</td>
</tr>
<tr>
<td>6</td>
<td>1800 1700 1650 1600 1500 1440 1420</td>
</tr>
<tr>
<td>7</td>
<td>1900 1800 1700 1600 1550 1470 1420</td>
</tr>
<tr>
<td>8</td>
<td>1900 1800 1700 1600 1550 1470 1420</td>
</tr>
</tbody>
</table>

The first column shows the layer number, from 1 at the top of the dam to 8. Each row is for a specific layer, and each column is for a specific optimization cycle.

Table 7 shows the increment of density, \( \Delta \rho \), considered for optimization during the optimization cycles.

### 8.3. Test on other earthquakes

The original and optimized Pine Flat Dams have been tested on other earthquakes, including artificial, far and near field earthquakes. To this end, the following
Table 7. $\Delta \rho \ (\text{kg/m}^3)$ used for optimization at each optimization cycle.

<table>
<thead>
<tr>
<th>Parameter $\Delta \rho \ (\text{kg/m}^3)$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

earthquakes have been utilized and the performance of the dams, both before and after optimization, has been evaluated. The earthquakes include: Park field, San Fernando, Northridge and White Noise.

Figure 19(a)-(d) shows the $t_a - \alpha$ curve for the original and optimized dams, under different earthquakes. The figures complement the previous discussions in the paper and provide a better understanding of the improvements obtained in the performance of the dam, as a result of optimization. In these figures, it can be seen that even for test earthquakes, for which the dam has not been designed, the performance of the dam has been much better that the original dam.

Table 8 also shows the value of $\alpha_c$ for the original and optimized dams under test earthquakes. The cutting scale factor almost doubled after optimization.

<table>
<thead>
<tr>
<th>Dam</th>
<th>Test earthquakes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Park Field</td>
</tr>
<tr>
<td>Original dam</td>
<td>30%</td>
</tr>
<tr>
<td>Optimized dam</td>
<td>60%</td>
</tr>
</tbody>
</table>

9. Conclusion

In this paper, improving the dynamic response of concrete gravity dams to earthquakes by optimizing their material properties, including the density, modulus of elasticity and strength of concrete, has been studied. On the one hand, it is desirable to reduce the mass of a dam in order to reduce the shear force induced in the dam from ground acceleration. On the other hand, by reducing concrete density, its modulus of elasticity and strength also reduce, causing the cracks to widen and distribute further in the dam; hence, making it more vulnerable to ground shaking. Creating a balance between a decrease in dam concrete density and its associated undesirable effects has been the subject of this paper. To this end, an algorithm has been developed to optimize the concrete density of concrete gravity dams. The objectives of the optimization have been: 1) increasing the time a dam can withstand earthquakes until it fails and 2) strengthening the dam to withstand higher intensity earthquakes.

The general algorithm, proposed in this paper to achieve the above goals, has been applied to the Pine Flat Dam in the US, which has been studied
extensively by other researchers in the past, regarding its dynamic response. First, the Pine Flat Dam was divided into 8 layers of different concrete density, which have been optimized to satisfy the optimization criteria as much as possible. The optimization earthquake is the El Centro (1940) earthquake, and the dam has been tested on other earthquakes also, including Park Field, San Fernando, Northridge and a White Noise earthquake. The obtained results have shown that for all the test earthquakes, including the optimization earthquake, the intensity of the earthquakes causing failure has doubled and the time to failure has increased significantly too.

Since concrete gravity dams exhibit a nonlinear response against earthquakes close to failure, because of the occurrence of cracks and their repeated opening and closing, the simulation of the Pine Flat Dam response has been done using nonlinear finite element analysis, where the smeared crack model has been utilized to model the nonlinear behavior of the concrete. The loads considered in this study have included: the weight of the dam, water pressure on its upstream face, hydrodynamic pressure and earthquake loading.

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References

Biographies

Abdolreza Joghataie is Faculty Member in the Civil Engineering Department of Sharif University of Technology, Tehran, Iran. His research interests include application of neural networks in different areas of structural engineering including nonlinear dynamic analysis of dams.

Mehrdad Shafiei Dizaji obtained his BS degree in Civil Engineering from Tabriz University, Tabriz, Iran, and his MS degree in Hydraulic Structures from Sharif University of Technology, Tehran, Iran. His research interests include application of artificial neural networks in dynamic analysis of concrete gravity dams.