Pumping effect of bubble growth and collapse in microchannels: Thermo-hydraulic modeling

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Abstract In the past two decades, microfluidic systems have become more appealing due to their wide applications in many areas, such as electronics, biotechnology, medicine, etc. Recently, the advantages of using the bubble growth phenomenon as a robust actuator in microfluidic devices have directed research interests towards the investigation of various applications. In this research, a new transient thermo-hydraulic model has been developed for bubble growth in confined volumes. The present model has been used to describe the pumping effect produced by the bubble growth and collapse phenomenon in microchannels. The results show relatively good agreement with experimental data. This study is useful in getting a better understanding of the bubble growth mechanism in confined volumes, and its application as a reliable micro actuator.

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1. Introduction

The development of microfluidic devices for use in electronic cooling systems, biological and chemical analyzers and other applications requires simple, reliable and robust components, such as valves, pumps, actuators, etc. [1]. In recent years, the potential for utilizing the bubble growth phenomenon as a reliable actuator in microfluidic systems has motivated many researchers towards the investigation of various applications, such as actuating [2], pumping [3,4], mixing [5] and switching [6]. In designing such devices, it is important to predict bubble behavior in confined volumes.

Although there is widespread knowledge about bubble dynamics in unconfined spaces, little work has been done on the bubble growth process in confined volumes whose dimensions are comparable with bubble size. Although a limited number of research deals with boiling heat transfer in capillary tubes [7], microchannel heat sinks [8] and heat pipes [9], in all of them heat transfer occurs continuously from the entire surface rather than localized pulse heating, which is important in our case.

For the reliability of micro devices, bubble generation must be precisely repeatable. So, the usual non homogenous nucleation process, which is mainly affected by heating surface conditions, is not quite suitable for this purpose. On the other hand, the boiling phenomenon, under an extremely high heat flux, is different from the usual boiling mechanism in many aspects [10]. Due to the very high heat flux, the liquid temperature rises rapidly near the heating surface, whilst the remaining liquid remains at the initial temperature. In such a situation, the liquid becomes metastable, and spontaneous nucleation occurs before the gas or vapor nuclei, trapped in the surface cavity, grow. As a result of the very high initial pressure of the bubble, this boiling phenomenon is also known as explosive evaporation. As the bubble grows, the initial temperature and pressure of the bubble falls rapidly in several micro seconds, mainly due to the bubble expansion work.

The spontaneous boiling process is mainly governed by liquid properties instead of surface conditions, which are important in the non homogenous nucleation. So, the explosive boiling is more reproducible and reliable, and it is more convenient to work as a robust actuator in microfluidic systems. It must be noted that explosive evaporation is not only a theoretical or purely experimental phenomenon, but rather has been used successfully as a commercial actuator in thermal ink-jet printer heads [11]. This mechanism was used as an
In this research, a bubble growth process has been modeled as a set of ODEs, including all thermo-hydraulic aspects of the phenomenon, to describe bubble growth behavior in confined volumes. After verification, the present model is used to investigate the pumping effect of bubble growth and collapse in microchannels. The results give a better understanding of the bubble growth process in microchannels, and are useful in the analysis and design of various microfluidic actuators.

2. Analytical modeling

A sketch of the geometry considered in the model is shown in Figure 1. A microchannel connects two reservoirs, which are assumed to be large enough for their pressures to be almost constant. A micro heater situated on the microchannel substrate warms up the liquid locally. As a result, the liquid temperature rises rapidly and, if the heat flux is large enough, an explosive evaporation can occur. As mentioned previously, the bubble internal pressure falls down rapidly, and collapses after reaching its maximum volume. It is convenient to investigate the boiling process at separate stages including:

1. Pre-heating,
2. Vapor film growth,
3. Bubble growth,
4. Bubble collapse,
5. Final net flow.

At the first stage, named the pre-heating stage, the liquid near the micro heater surface warms up to the homogeneous nucleation temperature, after which explosive evaporation occurs. At this time, many tiny vapor bubbles nucleate and coalesce with each other to cover the entire heater surface, like a film. The vapor film grows rapidly until it occupies the whole microchannel cross section, named the vapor film growth stage. Afterwards, the bubble grows longitudinally, named the bubble growth stage. When the bubble reaches its maximum volume, it will collapse rapidly, named the bubble collapse stage. As the bubble collapses longitudinally, the governing equations of bubble collapse stage are similar to the bubble growth stage, and are modeled with it as a single stage. Finally, due to bubble collapse, the net flow can be produced under specific conditions.

2.1. Pre-heating

As shown by Sajadi [20], if the heating time is small enough, the liquid and the substrate can be modeled as a semi infinite...
medium without significant error. So, the wall temperature becomes:

\[ T_{\text{wall}}^* = \frac{2 \dot{Q}}{F_c \frac{F_0 \tau}{\pi}}. \]  

where:

\[ F_0 = \frac{\alpha \cdot T^+}{D^2}. \]

\[ \dot{Q} = \frac{k_i}{k} \cdot \frac{\alpha}{\alpha_c} \cdot \frac{q^c D^2}{F_0 \tau (T_{sp} - T_{amb})}. \]  

The correction factor, \( F_c \), describes the heat transfer to the microchannel substrate, which is defined as:

\[ F_c = 1 + \frac{k_i}{k} \cdot \frac{\alpha}{\alpha_c} \cdot \frac{q^c D^2}{F_0 \tau (T_{sp} - T_{amb})}. \]  

As the heat flux is extremely large in this study, the nucleation time is very small, and can be predicted from Eq. (1). The spinodal temperature, at which the liquid becomes metastable, is nearly 0.9 of the critical temperature, and in the absence of experimental data, can be estimated by Lienhard’s equation [21]:

\[ T_{r,sp}^* = 0.905 + 0.095 T_{r,\text{sat}}^* \cdot \]  

2.2. Momentum equation

As already noted, when nucleation occurs, initial bubble pressure is extremely high, but after a few micro seconds, it rapidly falls. Previous investigations in both unconfined [22] and confined [23] volumes show that bubble pressure falls to vapor pressure at the ambient temperature, with an almost exponential behavior. Asai [11] suggested the following equation for bubble pressure:

\[ p_b = [p_{sp} - p_{sat}] \cdot \exp \left[-\left(\frac{t}{\tau_c}\right)^4\right] + p_{sat}, \]  

where the bubble growth time constant, \( \tau_c \), strongly depends on heating conditions, and the free parameter, \( \lambda \), is expected to be between 0.5 and 1 [11].

As the initial slope of Eq. (5) is very large, it is common to approximate it by an ideal impulse:

\[ p_b = P_0(t) + p_{sat}, \]  

where:

\[ P = \frac{P_{0f} \tau_c}{\lambda}. \]  

Based on our previous studies [17,24], the normalized governing momentum equations for the various stages are:

Vapor film growth stage:

\[ L_t \frac{d^2 V^*}{dt^2} + \frac{f}{ReSt} L_{t0} \frac{dV^*}{dt} = -\Delta p^*, \]

\[ V^*_i(0) = 0, \quad \frac{dV^*_i}{dt}(0) = \frac{p^*}{L_{t0}}. \]  

Bubble growth and collapse stages:

\[ (V_{i0}^* - V^*) \frac{d^2 V^*}{dt^2} + \frac{f}{ReSt} (V_{i0}^* - V^*) \frac{dV^*}{dt} = \Delta p^*, \]

\[ V^*_i(0) = V^*|_f, \quad \frac{dV^*_i}{dt}(0) = \frac{dV^*_i}{dt}|_f. \]  

Final net flow:

\[ \frac{d^2 V^*}{dt^2} + \frac{f}{ReSt} \frac{dV^*}{dt} = -\Delta p^*, \]

\[ V^*(0) = V^*|_c, \quad \frac{dV^*}{dt}(0) = \sum L_{i,c} \frac{dV^*_i}{dt}|_c. \]  

where \( i \) stands for \( r \) (right reservoir) and \( l \) (left reservoir), and \( f \) and \( c \) are abbreviations for filling time, end of film growth stage, collapse time and end of bubble collapse stage, respectively. In addition:

\[ L^* = \frac{l}{l_{mc}}, \quad V^* = \frac{V}{V_{mc}}, \]

\[ \tau = \frac{t}{\tau_c}, \quad p^* = \frac{p}{p_{mc}}, \]

\[ ReSt = \frac{D^2 \tau^+}{\nu}, \quad \Delta p^* = p_{sat}^* - p^*. \]

\[ p^* = \frac{p_{0f} \tau_c}{\lambda}. \]  

2.3. Energy equation

As mentioned previously, the time constant, \( \tau_c \), depends on heating conditions. So, the energy equation must be solved to close the governing equations. To estimate the time constant, the bubble growth process at initial times is studied in detail. At these times, the bubble volume can be neglected, but it must be noted that the volume change is large, and must be taken into account. It is also assumed that the heat flux has been turned off just after the bubble was generated. In other words, the bubble does not gain any heat flux from the heater during its growth process. When the heat flux is high, turning off of the heat flux is recommended to prevent damage of the heater [11].
which is a completely reasonable assumption. According to Asai's approach, it can be shown that:

$$S_0 q_c'' = \rho_s h \frac{dV_b}{dt}. \quad (12)$$

At initial times, by assuming $p_b \approx p_{sp}$, it can be shown that [24]:

$$\frac{dV_b}{dt} = \frac{p_{sp} A}{\rho_f} \left( \frac{1}{L_{i,0}} + \frac{1}{L_{r,0}} \right) t. \quad (13)$$

Combining Eqs. (12) and (13):

$$q_c'' = \frac{p_{sp} A}{S_0 \rho_f} \left( \frac{1}{L_{i,0}} + \frac{1}{L_{r,0}} \right) t. \quad (14)$$

As the bubble grows, its pressure falls rapidly. By a semi-infinite model, the bubble pressure is [11]:

$$p_b = p_{sp} \exp \left[ \left( 1 + \frac{L}{L_{i,0}} \right) \left( \frac{L}{L_{r,0}} \right)^{0.5} \right], \quad (15)$$

where:

$$t_1 = \frac{3 S_0 \rho_f q_b}{2 A \rho_s p_{sp} h} \left( \frac{1}{L_{i,0}} + \frac{1}{L_{r,0}} \right)^{-1},$$

$$t_2 = \frac{\pi (T_{sp} k \beta \gamma)^2}{4 L_{r,0}^2}, \quad \beta = 1 - \frac{\rho_s}{\rho_f},$$

$$\gamma = \frac{p_c}{p_{sp} h}.$$

Using Eq. (15) instead of $p_b \approx p_{sp}$, the more exact solution for $q_c$ can be extracted in a similar way to how Eq. (14) was obtained:

$$\dot{q}_c'' = \frac{p_{sp} h A \rho_c}{S_0 \rho_f} \left( \frac{1}{L_{i,0}} + \frac{1}{L_{r,0}} \right) \times \left[ 1 - \left( \delta + \delta' \frac{t}{t_1} \right) \left( \frac{t}{t_2} \right)^{0.5} \right] t, \quad (17)$$

where:

$$\delta = \frac{5}{4} \left( 1 - \beta \gamma + \frac{8}{15} \right) - \frac{\gamma}{4},$$

$$\delta' = \frac{7}{4} \left( 1 - \beta \gamma + \frac{8}{35} \right) - \frac{3 \gamma}{4}. \quad (18)$$

As supposed by Asai [11], we assume that Eq. (15) is valid until time $q_c$ reaches its maximum value. This time, $t_1$, can be calculated as:

$$t_3 = \frac{3 \delta}{5 \delta'} f \left[ \frac{2}{35} \left( \frac{5 \delta'}{35} \frac{t_2}{t_1} \right)^{0.5} \right]. \quad (19)$$

where $f$ is the following transcendental function:

$$(1 + f) \sqrt{f} = \chi. \quad (20)$$

Equalizing Eqs. (5) and (15) at $t_3$, with a reasonable assumption that $p_b \gg p_{sat}$, leads to:

$$t_3 = t_3 \left( 1 + \frac{t_3}{t_1} \right)^{-1/\chi} \left( \frac{t_3}{t_2} \right)^{-0.5/\chi}. \quad (21)$$

The above equations can be summarized as a normalized form:

$$\tau_\tau = t_3 \left( 1 + \frac{t_3}{t_1} \right)^{-1/\chi} \left( \frac{t_3}{t_1} \right)^{-0.5/\chi}.$$

Figure 2: Comparison between the results of the present model with the experimental data [13].

Eqs. (8)–(10), combined with Eq. (22), can be used to predict bubble growth dynamics and describe the resultant pumping effect.

3. Result and discussion

To investigate the pumping effect of the bubble growth and collapse process in the microchannels, a finite difference algorithm has been developed to solve the governing equations numerically. The free non-dimensional parameter, $\lambda$, has been set as 0.5 in the calculations [25]. Although the present model is one dimensional, Figure 2 shows that the predicted results are in fairly good agreement, at least at the trends, with the experimental data [13]. As a result, the present model can be used successfully to describe bubble growth dynamics and its resultant pumping effect.

The main reason for deviation in theoretical results is the assumption taken in the derivation of Eq. (12). In experimental work [13], the heating time is constant in all cases. So, the bubble gains heat during the growth process, which causes some differences between analytical and experimental results.

The present model can be used to describe the pumping effect due to bubble growth and collapse in microchannels. Consider geometry similar to Figure 1, where the heater is
located in the left half of the microchannel. In Figure 3, the initial pressure impulse is too low for the maximum bubble volume to occupy the entire microchannel cross section. As a result, only the vapor film growth occurs, and the bubble does not enter the longitudinally growth stage. As the inertia of the left liquid column is less than that of the right, its liquid is pushed out of the channel faster, due to the initial pressure impulse. But, as the bubble does not grow longitudinally, bubble conditions are similar during its growth and collapse stages. Consequently, the bubble collapses in the same manner as it grows, and finally there is no net flow production, as depicted in Figure 3. As shown in Figure 4, where there is a condition in which the bubble grows longitudinally, i.e. where the bubble growth stage occurs, variation in bubble volume is completely different.

In addition to Figures 4 and 5 can help in getting a better understanding of the bubble growth and collapse process in this situation. As depicted in Figure 5(a), the initial length and inertia of the left liquid column is less than that of the right. During the vapor film growth stage (hatched area in Figure 4), the left liquid column exits out more rapidly than the other, due to its lower initial inertia (Figure 5(a)). This trend continues at initial times of the bubble growth stage, where momentum nonlinear effects are not considerable (Figure 5(b)). As the bubble grows, the length of the left liquid column, which gains a greater velocity from the previous stage, reduces more rapidly than does the right one. Consequently, its momentum falls down rapidly and the flow direction reverses before the right one, due to the lack of momentum and pressure of the left reservoir (Figure 5(c)). This occurs in a normalized time,

Figure 3: Conditions in which the bubble cannot grow longitudinally: no net flow production. (a) Variation of liquid column volume; and (b) variation of liquid column velocity.

Figure 4: Conditions in which the bubble can grow longitudinally: net flow production. (a) Variation of liquid column volume; and (b) variation of liquid column velocity.

Figure 5: A sketch of net flow production process. (a) Vapor film growth does not produce any net flow; (b) left column discharges more rapidly than the right one; (c) left column reverses before the right one; (d) finally, the right liquid column reverses; and (e) the bubble collapse position goes over to the right, which creates net flow production.
about 0.022, in Figure 4. Finally, the right column also loses its momentum and begins to reverse (Figure 5(d)). Because of this asymmetry in the motion of liquid columns, the bubble collapses at a position which shifts over to the right. This indicates the production of net flow (Figure 5(e)). The net flow can be clearly recognized in Figure 4(a), where the final volume is positive.

Regarding the above discussions, it can be concluded that the main reason for the pumping effect of the bubble growth and collapse process in confined volumes is the asymmetric initial length of the liquid columns, which leads to the asymmetric momentum variation of the liquid columns during bubble growth and collapse stages. So, if the bubble is generated at the middle of the channel, no net flow can be predicted. This result is confirmed by previous experimental work [13]. Applying the present model, the bubble growth process in confined volumes and its pumping effect are modeled in a simple robust manner. The results can be used to study the effect of various parameters on the operation of bubble microactuators.

4. Concluding remarks

The bubble growth process in microchannels was modeled theoretically. Based on the extracted set of thermo-hydraulic equations, the pumping effect of this phenomenon was investigated. The results showed that the present model can predict generated flow with an acceptable accuracy. Consequently, the model was used to physically describe the pumping effect, due to the bubble growth-collapse sequence, in microchannels. The results are quite useful in obtaining a physical discernment of the bubble growth process in confined volumes. The present model can be used to study the effect of various parameters on the produced net flow and to optimize construction of a bubble microactuator, including its operating conditions.

References