Mathematical Modeling of a Cross Flow Conveyor Belt Dryer

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The drying of sols in a cross flow conveyor belt dryer (continuous operation system), in which particles (thin-layer) move in a wire net conveyor, has been theoretically studied. A mathematical modeling that considers the influence of bed porosity and transient terms in the drying process was developed and the changes in bed porosity and product density with the moisture content of seeds have been considered. Also, to have a more accurate model, some changes have been made to the specific surface of the seed. The finite volume method was used to solve, numerically, the governing conservation equations. The results of the moisture content of the material (yellow corn kernel), residence time and conveyor length are presented and analyzed in order to modify the model, according to bed porosity, product density and the specific surface charges of the seed. This model can predict humidity ratio, the temperature of air, moisture content and the temperature of the material throughout the drying process.

INTRODUCTION

Grains are normally harvested while they are still fairly moist, therefore, they must be dried to minimize water content and to reduce spoilage problems due to the action of microorganisms [1]. In recent years, research on moving bed technology has been intensified, especially into application regarding agricultural dryers [2]. The technology of drying for the preservation of food and ingredients has been practiced for a long time, with some of the earliest references being from the 18th century. During this time, there has been a wide range of technologies developed, both for general and for specific applications [3]. Products are dried using two technologies: Fixed bed and continuous flow bed (concurrent, counter and cross-flow beds) [4]. One of the most common continuous drying flows for grains is the cross-flow dryer. Nowadays, it is necessary to study these dryers in more detail because of the high cost of the dryers, in particular, cross flow dryers, and their considerable stress and cracking on the grain kernel.

A large number of researchers have reported cross-flow dryer modeling. The models consider the void fraction and/or the transient terms [5-7]. Other researchers present numerical studies considering void fraction and/or the transient terms in the mathematical model [8]. Experimental studies have also been reported in the literature [1].

In a general way, the drying residence time of grain in the dryer has been obtained as a function of the following variables: The grain bed thickness layer inside the dryer along the air flow direction, initial and final moisture content, initial temperature, thermo-physical properties of grains, air flow rate, inlet temperature and the air humidity ratio [4, 9].

The aim of this work is to present a numerical model, using the finite volume method, to predict changes in air temperature and relative humidity, and in the solid temperature and moisture content during the drying process in a continuous cross flow belt conveyor, considering changes in product density and void fraction with the moisture content of the seed and in the modifying of the specific surface of the seed in the conservation equation.

The new approaches in this work are to gain a model with more accurate results, in comparison to the latest research reported in other references.

In fact, although some efforts have been made in this field, concerning the modeling of a cross flow conveyor belt dryer, the models were not adequately accurate and there was some deviation between the experimental data and numerical results.

Also, it should be taken into consideration that,
in the latest work in this area, the porosity and the specific surface of the material were assumed to be constant, whereas, in this work, these variables are modeled as functions of the moisture content of the bed.

**MATHEMATICAL MODELING**

In this work, two models have been studied. A primary model has been developed, assuming constant bed porosity and material density.

Also, in this model, the specific surface of the corn is according to the amount given by [8].

A modified model has been developed considering bed porosity and materials' density as a function of the moisture content of the material [10]. To develop a more accurate model, the corn's specific surface has been used as given by [11]. Initially, therefore, both mathematical models are introduced.

The results are given in the results and discussion sections.

**Primary Model**

The development of the conservation equation is based on the differential element, illustrated in Figure 1. The following assumptions, as a simplification of the model in a conveyor belt dryer, have been made:

a) The volume shrinkage is negligible;

b) The temperature and moisture content gradients within the individual particle are negligible;

c) Heat conduction among the particles is negligible;

d) Heat loss of the dryer to the surroundings is negligible;

e) Air and grain flows are plug-type.

According to the assumptions, the following partial differential equations are obtained to model the transient cross flow drying process in a belt conveyor dryer:

\[
\frac{\partial (\rho_a x)}{\partial t} + \frac{\partial}{\partial y} \left( \frac{\rho_a w_a}{\varepsilon_{solid}} x \right) = - \left( \frac{\rho_a}{\varepsilon_{solid}} \right) \frac{\partial \hat{M}}{\partial t}, \tag{1}
\]

Solid:

\[
\frac{\partial \hat{M}}{\partial t} = \text{Thin layer drying equation}, \tag{2}
\]

where \( \rho_a \) is the air density (kg/m³), \( x \) is the humidity ratio (kg/kg), \( w_a \) is the air velocity (M/s), \( \varepsilon \) is the porosity, \( \rho_p \) is the product density (kg/M³), \( M \) is the average moisture content (kg/kg), \( y \) is the Cartesian coordinate (M) and \( t \) is the time (s).

**Energy**

Air:

\[
\frac{\partial}{\partial t} (\rho_a T) + \frac{\partial}{\partial y} \left( \rho_a w_a T \right) = - \frac{A^* h_p (T - \bar{T})}{c_p + c_w M} \varepsilon_{solid} (c_a + 2c_t) \tag{3}
\]

where \( T \) is the air temperature (°C or °K), \( A^* \) is the surface area of the solid per volume unit of the bed (M²/M³), \( h_p \) is the convective heat transfer coefficient (W/m² °K), \( \bar{T} \) is the average temperature of the solid (°C or °K) and \( c_a \) and \( c_t \) are the specific heat of the air and vapor (kJ/kg, °K), respectively.

Solid:

\[
\frac{\partial}{\partial t} (\rho_p \bar{T}) = - \frac{A^* h_p (T - \bar{T})}{c_p + c_w M} \rho_p \varepsilon_{solid} (c_a + 2c_t) \frac{\partial \hat{M}}{\partial t}, \tag{4}
\]

![Figure 1. Schematic representation of the continuous cross flow belt conveyor dryer differential element.](www.SID.ir)
where \( h_{f_0} \) is the heat of vaporization of the product (kJ/kg) and \( c_{wv} \) is the specific heat of water (kJ/kg·°C) [8].

Brooker et al. [8] report the following thin-layer drying equation to describe the drying rate:

\[
\frac{dM}{dt} = \frac{M_e - M}{3600 (A^2 + \frac{M}{B})^{1/2}} \tag{5a}
\]

\[
A = -1.70548246 + 0.00879170, \tag{5b}
\]

\[
B = 148.60862 x \exp (-0.0594186), \tag{5c}
\]

where \( t \) is time (s) and \( M_e \) is the equilibrium moisture content (kg/kg).

Initial and boundary conditions were used as follows:

\[
M(y, t = 0) = M_0, \tag{6a}
\]

\[
\frac{\partial M}{\partial y}(y, t = 0) = 0, \tag{6b}
\]

\[
T(y = 0, t) = T_0, \tag{6c}
\]

\[
x(y = 0, t) = x_0. \tag{6d}
\]

The heat of vaporization \( (h_{f_0}) \) equilibrium moisture content \( (M_e) \), specific surface area \( (A^x) \), dry solid density \( (\rho_d) \), and specific heat of the corn grain \( (C_p) \) and the void fraction of the bed \( (\varepsilon) \) are given by [8]:

\[
h_{f_0} = (2502.2 - 2.39T)(1 + 1.2925 \exp [4.81(M_e)]) \text{ kJ/kg,} \tag{7a}
\]

\[
M_e = \frac{1}{100} \left( \frac{\ln(1 - x)}{8.6541 x 10^{-5} (T + 49.81)} \right) \text{ kg/kg,} \tag{7b}
\]

\[
C_p = 1.361 + 3.97 \frac{M}{(1 + M)} \text{ kJ/kg.K,} \tag{7c}
\]

\[
\rho_d = 650 \text{ kg/M}^3, \tag{7d}
\]

\[
\varepsilon = 0.44, \tag{7e}
\]

\[
A^x = 784 \text{ M}^2/\text{M}^3. \tag{7f}
\]

The specific heat \( (C_a) \), density \( (\rho_a) \), air relative humidity \( (UR) \), absolute temperature \( (T_{atm}) \), universal constant \( (R) \) applied to the air, the saturation pressure of the vapor and local atmospheric pressure \( (P_a) \) are given by [4]:

\[
c_a = 1.00926 - 44.04033.10^8 l + 6.17596.10^7 l^2 - 4.6972 x 10^{-6} l^3 \text{ kJ/kg.K,} \tag{8a}
\]

\[
p_a = \frac{P_{atm} M_a}{R T_{atm}} \text{ Pa,} \tag{8b}
\]

\[
T_{abs} = T_a + 273.15 \text{ K,} \tag{8c}
\]

\[
R = 8314.34 \text{ J/kg.K,} \tag{8d}
\]

\[
P_{atm} = 101325 \text{ Pa,} \tag{8e}
\]

\[
UR = \frac{P_{atm} X_a}{(X_a + 0.0622) \mu S} \%(\%), \tag{8f}
\]

\[
R_{ah} = 2.21059492.25 \exp \{ -27465.53 + 97.5413 T_{abs} \}
- 0.146244 T^4_{abs} + 0.1255 x 10^{-9} x T^3_{atm, s} \]

\[
- 0.48502 x 10^{-4} T^4_{abs}/1.43493 T_{abs} \]

\[
- 0.39381 x 10^{-2} T^2_{abs}, \text{ Pa,} \tag{8g}
\]

\[
C_v = 1.8830 - 0.16737 x 10^{-3} T_{atm}
+ 0.84386 x 10^{-6} x T^2_{atm, s} \]

\[
- 0.26966 x 10^{-1} T^3_{atm, s} \text{ KJ/kg.K.} \tag{8h}
\]

The heat transfer coefficient was obtained by using the following equations [8]:

\[
h_{tc} = \begin{cases} 
101.4(\rho_a u_a)^{0.59} & \text{if } \rho_a u_a \geq 0.68 \text{ W/m}^2\text{K} \\
99.6(\rho_a u_a)^{0.47} & \text{if } \rho_a u_a < 0.68 
\end{cases} \tag{9}
\]

After calculating \( \overline{M}, \overline{\theta}, T \) and \( x \) at each position inside the bed and the drying time, the relative humidity is obtained. If this value is greater than 1, saturation or super saturation is assumed and condensation is modeled. The condensation of water may occur when a large amount of water vapor is transported by the air and cooled when it passes through cool grains. The following procedure was used to model the condensation:

a) \( \overline{M}, \overline{\theta}, T \) and \( x \) are calculated at a new location inside the bed, and then \( \rho_{ps} \) and \( UR \) are obtained;

b) If \( UR > 1 \), a new value of the absolute humidity \( x = x - Ax \) is assumed, so go to passes c. If \( UR < 1 \), stop the condensation and go to a new y location;

c) Using the new value \( x \), the new values of \( \overline{M}, \overline{\theta} \) and \( T \) are calculated;
d) Using the new value of \( T \), we calculate \( \rho_{e} \) and \( UR \) and return to passes b.

The new value of \( \overline{M} \) is given by:

\[
\overline{M} = \overline{M}_{bc} + \left( \frac{\rho_{e} u_{a} \Delta \tau}{\rho_{e} u_{b} \Delta y} \right) (x_{bc} - x).
\] (10)

The new value of \( T \) is given by:

\[
T = \frac{\varepsilon_{w} u_{a} \Delta \tau (c_{w} + c_{1} (x_{bc} - x)) T_{0} I_{bc} + \rho_{b} \Delta y (c_{p} + c_{w} \overline{M}_{bc}) \bar{b}_{bc}}{\rho_{e} u_{a} \Delta \tau (c_{w} + c_{1} x)}
\] (11)

The subscript "bc" refers to values before condensation [4,12].

**Modified Model**

All assumptions and equations in this model are as the same as in the primary model, but the following modifications (Relations 12 and 13 are by Subramanyam [10] and Relation 14 is by Curtiss [11]) have been considered to develop a more accurate model:

- \( \rho_{e} = 1353 - 179.4 \ M + 78.4 \ M^{2} \)
- \( 0.05 < M < 0.85 \ \text{kg}/\text{M}^{3} \),
- \( \varepsilon = 0.513 - 0.11 \ M + 0.48 \ M^{2} - 0.56 \ M^{3} \)
- \( 0.05 < M < 0.85 \),
- \( A^{*} = 729 \ M^{2}/\text{M}^{3} \),

where \( \rho_{e} \) is the product density, \( \varepsilon \) is material porosity and \( A^{*} \) is specific surface area.

**NUMERICAL SOLUTION**

In this work, the finite-volume method was used to discretize the basic equations integrating under the control volume and time, as illustrated in Figure 2.

The result of the integration is a set of linear equations in the discretized form as follows:

**Grain:**

- **Energy:**
  \[
  A_{e} \overline{\bar{q}}_{p} = A_{e} \bar{q}_{p}^{w} + S_{e}^{\bar{q}}.
  \] (15)

- **Mass:**
  \[
  A_{e} \overline{\bar{m}}_{p} = A_{e} \bar{m}_{p}^{w} + S_{e}^{\bar{m}}.
  \] (16)

**RESULTS AND DISCUSSION**

Table 1 presents all drying conditions used in the work. The numerical results of the average moisture content of grain are compared with experimental data for the fixed bed drying, in order to validate the methodology [13]. The comparison is possible because: \( u_{p} < u_{a} (u_{p} = 0.005 \ \text{m/s}) \). Table 2 shows this data.

As shown in Tables 3 and 4, the residence time and length of the conveyor and the moisture content of the material are presented along the drying process. These tables also show the variation of the numerical data in comparison to the experimental data. As illustrated, the results of the modified model show fewer errors and smaller errors.

Table 3 shows that most of the errors are more than 10% and the \( \Sigma [t \text{ error}] \) is about 107.62. But, according to Table 4, most of the errors of the modified model are fewer than 10% and the \( \Sigma [t \text{ error}] \) is about 64.77. Also, the \( z \) error of the modified model is -11.4%, whereas this is -4.7% for the modified model.
Table 1. Air and grain conditions used in this work, final moisture content, total drying time and length at the dryer.

<table>
<thead>
<tr>
<th>No.</th>
<th>$M_0$ (kg/kg)</th>
<th>$H$ (m)</th>
<th>$\theta$ (°C)</th>
<th>$M_f^*$ (kg/kg)</th>
<th>$X_0$ (kg/kg)</th>
<th>$W_0$ (m/s)</th>
<th>$T_0$ (°C)</th>
<th>RH (%)</th>
<th>$t$ (s)</th>
<th>$T$ (m)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3</td>
<td>0.1</td>
<td>24</td>
<td>0.180</td>
<td>0.01134</td>
<td>1.6</td>
<td>75</td>
<td>4.7</td>
<td>4907</td>
<td>24.5</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
<td>0.2</td>
<td>25</td>
<td>0.231</td>
<td>0.00262</td>
<td>1.5</td>
<td>30</td>
<td>10.0</td>
<td>8000</td>
<td>40</td>
</tr>
<tr>
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<td>0.3</td>
<td>0.2</td>
<td>25</td>
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<td>0.00458</td>
<td>1.5</td>
<td>40</td>
<td>10.0</td>
<td>8000</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>0.3</td>
<td>0.2</td>
<td>25</td>
<td>0.277</td>
<td>0.01251</td>
<td>1.5</td>
<td>60</td>
<td>10.0</td>
<td>8000</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td>0.2</td>
<td>25</td>
<td>0.158</td>
<td>0.01977</td>
<td>1.5</td>
<td>70</td>
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<td>8000</td>
<td>40</td>
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<td>0.2</td>
<td>25</td>
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<td>0.00076</td>
<td>1.5</td>
<td>50</td>
<td>10.0</td>
<td>8000</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
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<td>0.2</td>
<td>25</td>
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<td>0.00282</td>
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<td>5.00</td>
<td>8000</td>
<td>40</td>
</tr>
<tr>
<td>8</td>
<td>0.3</td>
<td>0.2</td>
<td>25</td>
<td>0.166</td>
<td>0.00769</td>
<td>1.5</td>
<td>50</td>
<td>10.0</td>
<td>8000</td>
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<tr>
<td>9</td>
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<td>0.2</td>
<td>25</td>
<td>0.168</td>
<td>0.01556</td>
<td>1.5</td>
<td>50</td>
<td>20.0</td>
<td>8000</td>
<td>40</td>
</tr>
</tbody>
</table>

Table 2. Experimental data: When $T = 75^\circ$C, $\theta = 24^\circ$C, $M_f = 0.18$ kg/kg, $X_0 = 0.01134$ kg/kg, $H = 0.1$ m, $W_0 = 1.63$ m/s.

<table>
<thead>
<tr>
<th>No.</th>
<th>Moisture Content (kg/kg)</th>
<th>Time (s)</th>
<th>Zt: Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3</td>
<td>0</td>
<td>C</td>
</tr>
<tr>
<td>2</td>
<td>0.274</td>
<td>450</td>
<td></td>
</tr>
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<td>3</td>
<td>0.277</td>
<td>900</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.247</td>
<td>1350</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.235</td>
<td>1800</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.226</td>
<td>2300</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.216</td>
<td>2700</td>
<td></td>
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<td>0.208</td>
<td>3150</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.200</td>
<td>3600</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.190</td>
<td>4050</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.185</td>
<td>4500</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.180</td>
<td>4907.96</td>
<td>24.54</td>
</tr>
</tbody>
</table>

Figure 3 illustrates the comparison between the experimental data, the primary model and the modified model during the drying process. As shown in this figure, the drying rate curve for the modified model is closer to the experimental one. In other words, although the deviation of the primary model is not very large, the curve of the modified model shows the least deviation, in comparison to the others.

According to the results and the data that are included in the tables, the deviation of results from the experimental data is very small. This shows that this model can predict the behavior of a cross flow conveyor belt dryer exactly.

This model can also predict the material's temperature, moisture content and air temperature, and humidity ratio and UR. The modified prediction of these variables is given in Table 5, and Figures 4 to 6 show the variation of these variables versus time.

Figure 4 shows the variation in material temperature during the drying process in a cross flow conveyor belt dryer. As shown, the temperature of the material increases rapidly, reaching a constant rate that is maintained near to the initial air temperature. In fact, this increase occurs very rapidly and the grain reaches the inlet air temperature in few seconds of drying.

Figure 5 shows the variation of air humidity ratio during the drying process. As shown in this figure, at first, the humidity ratio of the air increases and then reduces. It seems that the increase in the air humidity ratio is because of air contact with the moisture bed. However, during the process and with a reduction
in the moisture of the material, the humidity ratio decreases and almost reaches a constant amount.

Figure 6 illustrates the air temperature behavior and shows its variation during the process. As shown, the reduction of air temperature is not large and is about just 0.5°C. In fact, the air temperature is nearly constant and only some decrease (about 0.5-1°C) occurs that, according to the thin layer bed assumption, is acceptable.

**CONCLUSION**

The following conclusions can be summarized:

1. The finite-volume method can be used to simulate the drying process in a cross flow dryer, due to the
Table 5. Numerical data: modified model prediction of material and air moisture and temperature.

<table>
<thead>
<tr>
<th>No.</th>
<th>M (kg/kg)</th>
<th>(\theta) (°C)</th>
<th>T (°C)</th>
<th>X (kg/kg)</th>
<th>t (s)</th>
<th>UR</th>
</tr>
</thead>
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<td>0.1685</td>
<td>597.95</td>
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<td>0.0659</td>
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<td>3.410</td>
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</tbody>
</table>

Figure 4. Material temperature prediction versus time.

Figure 5. Air humidity ratio prediction versus time.

good agreement obtained by comparison between numerical and experimental data;
2. The conveyor belt drying process is strongly related to porosity, and consideration of the porosity variation by the moisture content of the grain makes more accurate results for the mathematical modeling of a cross-flow conveyor belt drying process;
3. In recent works, usually, material density was modeled as a constant amount. However, it is predictable that material density changes with time, because of change in the moisture content of the material, so the constant material density assumption is not true and modification of the model, according to material density variation with moisture content, is necessary. The results of modeling confirm this and more accurate results have been gained by this consideration;
4. The specific surface of grains is one of the physical parameters of seeds that affect the results of the drying process modeling. By modification at the specific surface, more accurate results were obtained and the t and s errors were reduced;
5. The grain reaches the inlet air temperature in few seconds of drying;
6. The air temperature is nearly constant and only some decrease (about 0.5-1°C) occurs that, according to the thin layer assumption, is acceptable;
7. The humidity ratio of air changes during the process in two periods. At first, there is an increase and then there is a reduction in the ratio of the air
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Figure 6. Air temperature prediction versus time.

humidity. It seems that an increase in air humidity ratio is because of air contact with the moisture bed, but during the process and reduction in the moisture of the material, the humidity ratio is decreased and almost reaches a constant amount.

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