A New Analytical Maclaurin Series Based $\lambda$-Logic Algorithm to Solve the Non-Convex Economic Dispatch Problem Considering Valve-Point Effect

S. M. Sajjadi and R. Kazemzadeh

Abstract—To solve the power system economic dispatch (ED) problem, in this paper a new analytical approach using Maclaurin series based $\lambda$-logic algorithm, considering valve loading and transmission losses effect is presented. In current research, first the non-convex generation cost functions due to valve point effect are expanded by Maclaurin series and the expanded equations are solved by $\lambda$-logic algorithm at the presence of the transmission loses. The proposed method is an analytical optimization algorithm which minimizes the total cost of the power plants and gives optimum generation schedule subject to the operating constrains of the system. Three machines 6-bus system, IEEE 5-machines 14-bus system, and IEEE 6-machines 30-bus system are tested successfully for giving legal authority of our approach. Results are compared with the results produced from Genetic Algorithm (GA) and Hybrid Genetic Algorithm (HGA).

Index Terms—Economic dispatch (ED), lambda logic, valve-point effect, Maclaurin series.

I. INTRODUCTION

The main objective of economic dispatch (ED) problem is to determine the low cost operation to provide the power demand on the system satisfying all operational constraints. Already several techniques have been applied to attain better solutions included iteration, gradient [1], Dynamic Programming (DP) [2]-[5], and base-point participation factor method [6], [7]. In these algorithms, the ED problem is assumed to be a convex or smooth and continues quadratic form. In the convex economic dispatch, the generators input-output curves are perused to be monotonically increasing. In real power systems operation, the generation units cost function may not usually be monotonicity increasing and generators cost function represents as non-convex form. Non-convex ED problem cannot be solved using traditional optimization algorithms. Normally, heuristic search algorithms algorithms are used for solution of non-convex ED problem such as Evolutionary Programming (EP) [8], Genetic Algorithm (GA) [9], Simulated Annealing (SA) [10], Tabu Search (TS) [11], and Differential Evolution (DE) [12]. The EP can be a quite powerful evolutionary approach, but it is rather slow converging to a near optimum for some problems [13]. Sometimes GA lacks a strong capacity of producing better offspring and causes slow convergence near global optimum, sometimes may be trapped into local optimum [14]. The SA is very time consuming and cannot be utilized easily to tune the control parameters of the annealing schedule. The TS is difficult in defining effective memory structures and strategies which are problem dependent. Differential Evolution (DE) is a very powerful algorithm with low doubt, but its greedy updating principle and intrinsic differential property usually leads to the computing process to be trapped at local optima [15]. The main problem of heuristic search techniques is that they usually cannot guarantee an optimal or near optimal solution in a single run [2], i.e., and each time we run program, a different response is usually acquired. However, in analytical methods this problem can be solved.

There are few analytical methods for solving the non-convex generation cost functions in economic dispatch problem. Our approach is a new analytical method and fixes all the disadvantages of heuristic approach. This method combines Maclaurin series and the $\lambda$-logic algorithm for solving the economic dispatch problem considering valve loading and transmission losses effect simultaneously for non-convex cost function. In this paper, first the non-convex generation cost function (due to sine term of this function) is expanded by Maclaurin series and the expanded equation is solved using $\lambda$-logic algorithm. The proposed method is an optimization algorithm which minimizes the total generation cost of the plants and gives optimum generation schedule subject to the operating constrains of the system. This algorithm uses pre-prepared power demand data (PPD) to solve ED problem. Comparing to the prevalent methods of non-convex ED problem, our method results in a unique answer, while the others usually do not have a unique answer and each time program runs, a different response is acquired. Our proposed approach has been tested on 3-machines 6-bus [7], IEEE 5-machines 14-bus, and IEEE 6-machines 30-bus [16]. Test systems are used and results from these tests are compared with the results gathered from Genetic Algorithm (GA) and Hybrid Genetic Algorithm (HGA) [17].

II. ECONOMIC DISPATCH FORMULATION

The mathematical form of each type of ED problem should be described in the following subsections.

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shows ripple [17]. The ED cost objective function considering the valve-point effects is described as superposition of sinusoidal function and quadratic function mathematically [17]. This non-convex curve is represented as [2]

$$F_f = \sum_{i=1}^{n} A_i + B_i P_i + C_i P_i^2 + |e_i \times \sin(f_i \times (P_{i_{\text{min}}} - P_i))|$$  (5)

where $A_i$, $B_i$, and $C_i$ are the same as (1) and $e_i$ and $f_i$ are the cost coefficients for the $i$th generator reflecting the valve-point effects.

III. PROPOSED METHOD AND ITS FORMULATION

In the proposed method, non-convex cost function is approximated using Maclaurin sine series expansion. The Maclaurin series is generally used for approximating functions [2]. The Maclaurin expansion for sine function is as follows

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$  (6)

the fuel cost function (5) of unit $i$ is represented as in (7)

$$F_f = \sum_{i=1}^{n} A_i + B_i P_i + C_i P_i^2 + |e_i \times \sin(x_i)|$$  (7)

$$x_i = f_i \times (P_{i_{\text{min}}} - P_i)$$  (8)

by considering the first two terms of the expansion of Maclaurin sine series given in (9) [2], we have

$$\sin x = x - \frac{x^3}{3!}$$  (9)

by substituting $x_i$ in (9) and then putting the approximated sine term, $\sin(x_i)$ in (7), the cost equation for the $i$th unit will be as (10). The derivative of (10) which is the incremental cost function is given in (11)

$$\frac{dF_f(P_i)}{dP_i} = B_i + 2C_iP_i + e_i f_i \left(\frac{f_i^2}{2}P_i^2 - f_i^2 P_{i_{\text{min}}} P_i + \left(\frac{f_i^2 P_{i_{\text{min}}}}{2} - 1\right)P_i + P_{i_{\text{min}}} - P_i\right)$$  (11)

by rearranging the terms, (11) can be presented in terms of $x_i$ as

$$\frac{dF_f(P_i)}{dP_i} = B_i + 2C_iP_i + e_i f_i \left(1 - \frac{x_i^2}{2!} + \frac{x_i^4}{4!}\right)$$  (12)

As the sine term is approximated by Maclaurin series, there will be an approximation error and the solution may not converge to an optimal value. An initialization factor, $y_i$ given in (13) is multiplied to the right hand side of $x_i$ in (8) to compensate the approximation made using Maclaurin sine series expansion, and also to minimize the error due to approximation [2]

$$y_i = \text{real}\left(\frac{\cos^{-1}(1 - \frac{x_i^2}{2!} + \frac{x_i^4}{4!})}{x_i}\right)$$  (13)

When each steam valve starts to open, the valve-point

$$F_i = A_i + B_i P_i + C_i P_i^2 + e_i f_i \left(\frac{f_i^2}{6} P_i^3 - \frac{f_i^2}{2} P_{i_{\text{min}}} P_i^2 + \left(\frac{f_i^2 P_{i_{\text{min}}}}{2} - 1\right)P_i + P_{i_{\text{min}}} - P_i\right)$$  (10)
where \( x_i \) is given in (8). The generated power, \( P_i \) is initially unknown and it can be chosen using

\[
P_i = \frac{P_{\text{max}} + P_{\text{min}}}{\varepsilon}
\]

where \( \varepsilon \) is a normalizing factor that normalizes the value of \( x_i \) between 0 and 1. Selecting an appropriate value for \( \varepsilon \) minimizes the error and vouch the best solution for the problem. If the \( \varepsilon \) value is fixed fittingly for any type of system, then optimal or near optimal solution can be attained by the method. Parameter \( \varepsilon \) is calculated by trial and error [2]. The initialization factor \( y_i \) multiplication to the right hand side of (8) and then replacing in (12) gives

\[
\frac{dF(P_i)}{dP_i} = B_i + 2C_iP_i + e_i f_i \left(1 - \frac{(y_i f_i (P_{\text{min}} - P_i))^2}{2!}\right)
\]

Substituting (16) in (15), the modified incremental cost equation will be as

\[
\frac{dF(P_i)}{dP_i} = \lambda_i
\]

For the case the term inside the vertical bars of (17) is positive

\[
\lambda_{\text{min}} = \frac{dF_i}{dP_i} \quad \text{at} \quad P_i = P_{\text{min}}
\]

\[
\lambda_{\text{max}} = \frac{dF_i}{dP_i} \quad \text{at} \quad P_i = P_{\text{max}}
\]

For the case the term inside the vertical bars of (17) is negative

\[
\lambda_{\text{min}} = \frac{dF_i}{dP_i} \quad \text{at} \quad P_i = P_{\text{min}}
\]

\[
\lambda_{\text{max}} = \frac{dF_i}{dP_i} \quad \text{at} \quad P_i = P_{\text{max}}
\]

- Step (a): For the case that the term inside the vertical bars of (17) is positive

- Step (b): Arrange \( \lambda \)-vector in ascending order based on (18) and descending order based on (19).

- Step (c): Calculate total power demand at each \( \lambda \) of the \( \lambda_{\text{asc}} \)-vector (\( \lambda_{\text{desc}} \)-vector). For this, the following fundamental observation of ED condition is:

For the case that the term inside the vertical bars is positive

\[
\lambda_i \leq \lambda_{\text{min}} \quad \text{then} \quad P_i = P_{\text{min}}
\]

\[
\lambda_i \geq \lambda_{\text{max}} \quad \text{then} \quad P_i = P_{\text{max}}
\]

For the case that the term inside the vertical bars is negative

\[
\lambda_i \leq \lambda_{\text{min}} \quad \text{then} \quad P_i = P_{\text{max}}
\]

\[
\lambda_i \geq \lambda_{\text{max}} \quad \text{then} \quad P_i = P_{\text{min}}
\]

where \( \lambda_i \) is the value of \( \lambda \) for each unit in \( \lambda_{\text{asc}} \)-vector (\( \lambda_{\text{desc}} \)-vector). This infers that the graph between any two entries of \( \lambda_{\text{asc}} \) (\( \lambda_{\text{desc}} \))-vector and PPD \( J \) is supposed to be a quadratic nature. The slope between any two intervals can be calculated and the \( \lambda_{\text{asc}} \) related to \( P_{\text{new}} \), which lies between PPD \( J \) and PPD \( J+1 \) is given by [18]:

For the case that the term inside the vertical bars is positive

\[
\lambda_{\text{asc}} = (m(\Delta P_i) + \lambda_{\text{asc}}(J))
\]

For the case that the term inside the vertical bars is negative

\[
\lambda_{\text{asc}} = (m(\Delta P_i) + \lambda_{\text{desc}}(J))
\]

and

\[
m = \frac{\lambda(j+1) - \lambda(j)}{\text{PPD}(j+1) - \text{PPD}(j)}
\]

Where slope between any two intervals is presented as \( m \).

- Step (d): Using \( \lambda_{\text{asc}} \) calculated from (28) and (29) and considering (22) to (25), the values of \( P_i \) is calculated. If any mismatch occurs between \( P_{\text{gen}} \) and \( P_i \), then we alter \( y_i \) value and then repeat the similar steps with the quadratic interpolation. The flowchart for the proposed algorithm is shown in Fig. 2.
IV. NUMERICAL RESULTS AND DISCUSSION

Three test systems namely 3-machines 6-bus system [6], IEEE 5-machines 14-bus system and IEEE 6-machines 30-bus system [16] have been inspected for the exhibition of the impressiveness of the suggested analytical approach.

The results have been taken on the Pentium IV 1.70 GHz machine. The generators data of these systems are shown in Table I. The B-coefficients for the systems in Table II, and value \( \lambda \) calculated corresponding to the optimum generation cost is shown in Table III. Minimum cost that shown are related to the optimum lambda values for one
TABLE II
TRANSMISSION LOSSES COEFFICIENTS [14]

<table>
<thead>
<tr>
<th>Case</th>
<th>Test system</th>
<th>B-Coefficient</th>
</tr>
</thead>
</table>
| 1    | 6-Bus 3-machines system | \[
B = \begin{bmatrix}
0.0552 & 0.0062 & -0.0046 \\
0.0062 & 0.0253 & 0.0064 \\
-0.0046 & 0.0064 & 0.0286
\end{bmatrix}
\] |

| 2    | IEEE 14-Bus 5-machines system | \[
B = \begin{bmatrix}
0.0212 & 0.0085 & -0.0009 & 0.0021 & 0.0007 \\
0.0103 & 0.0158 & 0.0010 & -0.0074 & 0.0007 & 0.0024 \\
0.0016 & 0.0010 & 0.0474 & -0.0687 & -0.0060 & -0.0350 \\
-0.0053 & -0.0074 & -0.0687 & 0.3464 & 0.0105 & 0.0534 \\
0.0009 & 0.0007 & -0.0056 & 0.0105 & 0.0149 & 0.0007 \\
-0.0013 & 0.0024 & -0.0350 & 0.0534 & 0.0007 & 0.2353
\end{bmatrix}
\] |

| 3    | IEEE 30-Bus 6-machines system | \[
B = \begin{bmatrix}
0.00085357 \\
0.00224 & 0.0103 & 0.0016 & -0.0053 & 0.0009 & -0.0013 \\
0.0103 & 0.0158 & 0.0010 & -0.0074 & 0.0007 & 0.0024 \\
0.0016 & 0.0010 & 0.0474 & -0.0687 & -0.0060 & -0.0350 \\
-0.0053 & -0.0074 & -0.0687 & 0.3464 & 0.0105 & 0.0534 \\
0.0009 & 0.0007 & -0.0056 & 0.0105 & 0.0149 & 0.0007 \\
-0.0013 & 0.0024 & -0.0350 & 0.0534 & 0.0007 & 0.2353
\end{bmatrix}
\] |

TABLE III
CALCULATED CORRESPONDING LAMBDA TO THREE TEST SYSTEM

<table>
<thead>
<tr>
<th>Case</th>
<th>Test system</th>
<th>Optimum lambda corresponding to minimum cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6-Bus 3-machines system</td>
<td>16.1435</td>
</tr>
<tr>
<td>2</td>
<td>IEEE 14-Bus 5-machines system</td>
<td>3.4270</td>
</tr>
<tr>
<td>3</td>
<td>IEEE 30-Bus 6-machines system</td>
<td>3.8302</td>
</tr>
</tbody>
</table>

TABLE IV
RESULTS FOR 3-MACHINES 6-BUS TEST SYSTEM FOR PD = 210 MW.

<table>
<thead>
<tr>
<th>Case</th>
<th>Test system</th>
<th>Maximum cost</th>
<th>Minimum cost</th>
<th>Maximum cost</th>
<th>Minimum cost</th>
<th>Minimum cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6-Bus 3-machines system</td>
<td>77.8905</td>
<td>53.2604</td>
<td>61.6467</td>
<td>54.4574</td>
<td>98.0372</td>
</tr>
<tr>
<td>2</td>
<td>IEEE 14-Bus 5-machines system</td>
<td>674723</td>
<td>88.9645</td>
<td>95.1632</td>
<td>115.6889</td>
<td>37.7016</td>
</tr>
<tr>
<td>3</td>
<td>IEEE 30-Bus 6-machines system</td>
<td>72.153</td>
<td>74.7693</td>
<td>60.54</td>
<td>47.58</td>
<td>81.5641</td>
</tr>
</tbody>
</table>

run, but the results of three test systems for Genetic Algorithm (GA) and Hybrid Genetic Algorithm (HGA) approaches are during 50 runs corresponding to the maximum and minimum costs. The GA parameters for the three test systems are adopted from [17]. The results of the three mentioned test systems for both the approaches corresponding to the maximum and minimum cost are shown in Tables IV-VI.

From above tables it is clear that the total generation cost concerned to our algorithm is better than the other methods and this is a proof of the superiority of this new derived algorithm. Another advantage of the proposed method compared to others is that unique and accurate answer is acquired, whenever the program is run. While in other approaches, maximum and minimum cost is shown during 50 runs [17]. In the case one, 3-machines 6-bus the optimal generation schedule is obtained in 10 iterations satisfying the system constraints. The execution time is 0.047 s. In the case two, IEEE 5-machines 14-bus system, the optimal generation schedule is obtained in 15 iterations satisfying the system constraints. The execution time is 0.05 s. In the case three, IEEE 6-machines 30-bus system the optimal generation schedule is obtained in 20 iterations satisfying the system constraints. The execution time is 0.064 s.

V. CONCLUSIONS

This paper has proposed a new approach for the application of analytical methods for solving the economic dispatch problem considering generator constraints and transmission losses simultaneously for non-convex cost function using the combination of Maclaurin series and the $\lambda$-logic algorithm. For this kind of problem, the most of
render methods are heuristic. Our method results in a unique answer, while the others usually do not have a unique answer and each time program runs, different responses are acquired. The capability of the proposed method has been illustrated on 3-machines 6-bus system, IEEE 5-machines 14-bus system, and IEEE 6-machines 30-bus system. Our results have been compared with the results gathered from Hybrid Genetic Algorithm (HGA) and Genetic Algorithm (GA). The comparison of the results with other methods reported in the literature confirms the feasibility of the proposed method for solving non-convex ED problems in power systems. The performance of this approach is better in exhibiting the consistency of reaching the global optimal and guaranteeing on obtained solution quality without violating the constraints. The proposed method is a new generalized analytical approach that can be extended to any test system.

REFERENCES


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