Electromagnetic Analysis and Shielding Effectiveness of Rectangular Enclosures with Aperture Using Hybrid MoM/FEM

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Abstract—This paper presents a hybrid formulation in frequency domain, which combines the method of moment (MoM) with the edge-based vector finite element method (FEM) to solve electromagnetic field distribution inside a loaded enclosure with aperture. While the FEM is used to solve EM fields inside the enclosure, the MoM is used to solve the surface integrals related to the aperture field components using equivalent surface currents. Numerical results of shielding effectiveness (SE) and stored electrical energy (SEE), calculated by the hybrid method for an enclosure with aperture, are presented and validated by comparing with previous results in literature. Then, the hybrid method was applied to loaded enclosure and the SE, SEE and dissipated power (DP) in load were determined. In this work, SEE and DP were examined at two frequencies by changing the location of the dielectric slab inside the enclosure with aperture.

Index Terms—Shielding effectiveness, stored electrical energy, dissipated power, finite element method, method of moments.

I. INTRODUCTION

DEALING with EMC boundary considerations is mandatory in almost all electronic designs. To ensure electromagnetic protection, electronic systems should be placed in a shielding enclosure. On the surface of that enclosure, there may be apertures to couple the parts in the enclosure with outer environment. These apertures may serve as I/O connections, control panels, or ventilation devices. But the fields entering the apertures interfere with the circuits inside the enclosure. The shielding enclosure is so designed, therefore, to minimize the effects of apertures on shielding efficiency. To achieve this, it is necessary to predict the EM fields inside the enclosure.

Shielding effectiveness (SE) is an important parameter that quantifies the electromagnetic compatibility of devices. It is the ratio of the observed fields at some point within an enclosure to those at the same point outside the enclosure and is expressed in dB [1]. Such a reference could also be used to explain the application of the equivalence principle and to discuss in a general way the consolidated numerical techniques for the analysis of shielding problems. In EMC applications, it is very important to estimate SE during the designing stage. Another important parameter to characterize shielding efficiency is the ratio of stored electrical energy. For evaluating interference, all the EM fields inside the shielding enclosure should be calculated by numerical simulation or by analytical formulation. The analytical formulation for calculating the SE of an empty enclosure was presented by Robinson et al. [2], [3]. However, this formulation is applicable to only rectangular enclosures and to the fundamental mode of the enclosure. If the enclosure structures are complex, it is difficult to determine their SE analytically. In such cases, numerical methods are the only alternative.

In recent years, various methods are used to estimate the SE of metallic enclosures with apertures on their walls. Li et al. [4] determined the electromagnetic radiation from the apertures and gaps of shielding enclosures by experimental measurement or by calculation using the FDTD method. In [5], [6] the EM coupling of plane wave penetrating through apertures was examined with FDTD method and the electric field distribution inside the enclosure obtained. Olyssager [7] determined the shielding effectiveness of the shielding enclosures with an electromagnetic simulator based on the method of moment (MoM) as well as by experimental measurement. A Modal/MOM hybrid method that calculates the SE of rectangular enclosures with rectangular apertures was presented in [8]. The enclosure with thick apertures [9], as also the one with a perfectly conducting loading inside [10], was studied by MoM technique. Benhassine et al. [11] investigated the enclosure with aperture using finite element time domain with the mass lumping technique. In [12], the integral equation defined on aperture and obtained from expressing the EM fields in terms of free space Green's function and cavity Green's function are solved with MoM and shielding effectiveness of rectangular metallic enclosure with aperture is calculated. Wallen et al. used a native MoM technique that solves potential integral equation to obtain the shielding effectiveness of metallic rectangular enclosures [13]. In [14], the coupling of plane wave with a conducting wire inside a metallic cavity was examined using frequency and time domains FEM, and induced voltage on wire was computed. In [15], a hybrid technique, combining the finite difference method and the MoM, was proposed to compute the shielding effectiveness of rectangular enclosure with apertures.

In this paper, the method of moment (MoM)/finite element method (FEM) in the frequency domain was used. The interior and exterior regions of the enclosure were analyzed separately by employing the field equivalence principle. Internal electromagnetic fields were formulated...
by FEM, and the external fields by MOM. The hybrid technique takes advantage of FEM's versatility and MOM's high efficiency. This way, the FEM was applied only inside the metallic enclosure, and no absorbing boundary conditions (ABCs) were needed. Using FEM in evaluating the internal fields allows one to treat complex objects inside the enclosure as dielectric elements, conductors, printed circuit board (PCB), etc.

II. FORMULATION OF THE PROBLEM

The geometry of a rectangular enclosure with rectangular apertures is shown in Fig. 1. The width, length, and depth of the enclosure are A, B, and C, respectively. The width and length of the aperture are w and L, respectively. The walls of the enclosure are assumed to be very thin and perfectly conductive. A perpendicular yz-polarized planar wave was applied to the enclosure's wall containing the aperture.

If the enclosure wall containing the aperture is a perfectly conducting ground plane with an infinite width, the problem can be split into two regions as suggested by Schelnukoff's equivalence formula [16]. The first region is the inner volume of the enclosure, and the second region is free half-space, limited by the ground plane.

According to the equivalence formula, an aperture on a perfect conductor is equivalent to a magnetic current distribution given by

\[ \vec{J}_m = -\vec{n} \times \vec{E}_a \]

where \( \vec{E}_a \) is the electric field on the aperture. The electromagnetic radiation from the aperture, either to free space or to the inner part of the enclosure, is equivalent to the radiation from the above magnetic current source.

A. FEM Formulation

For calculating the electric field distribution inside the enclosure, the vector finite element method (FEM), also known as the edge element method (Whitney-1 Form), was used. The first step in the FEM formulation is constructing the vector wave equation given by

\[ \nabla \times \nabla \times \vec{E} - k_n^2 \vec{E} = 0 \]

where \( \vec{E} \) is the electric field distribution inside the enclosure, and \( k_n = \omega_0 \sqrt{\mu_r \epsilon_r} \) the wave number of free space. Using \( \vec{w} \) as the test function, Galerkin's method was applied to this equation and the following equation obtained

\[ \int \nabla \times \left( \nabla \times \vec{E} \right) \cdot \vec{w} \, dV - k_n^2 \int \vec{E} \cdot \vec{w} \, dV = 0 \]

By applying the primary vector Green's identity to (3), one gets the following

\[ \int \left( \nabla \times \vec{E} \right) \cdot \left( \nabla \times \vec{w} \right) \, dV - k_n^2 \int \vec{E} \cdot \vec{w} \, dV = \oint (\vec{n} \times \overrightarrow{H}) \cdot \vec{w} \, dS \]

The next step of the formulation was to discretize the problem domain. For this, the inner region of the enclosure was divided into tetrahedral elements. The electric field is expressed as

\[ \vec{E} = \sum_{i=1}^{N_e} e_i \vec{w}_i \]

where \( e_i \) and \( \vec{w}_i \) are unknown coefficients and basis functions, respectively, associated with the \( i \) th edge of the element and \( N \) the degrees of freedom. For Galerkin's method, the basis function must be identical to the test function.

In (4), the left hand term is used for appropriate boundary conditions. As the tangential electric field on the perfect conductor is zero, this term is considered zero on the enclosure walls. The value of the integral on the aperture must be computed. To do this, the tangential magnetic field on the aperture must be known. First, the aperture was divided into triangle elements and then, the tangential field on the aperture can be expressed as

\[ \vec{n} \times \overrightarrow{H} = \sum_{i=1}^{N_A} \alpha_i \vec{f}_i \]

where \( \alpha_i \) and \( \vec{f}_i \) are the unknown coefficients and basis functions, respectively, associated with the \( i \)th edge of the element and \( N_A \) is the total number of edges on the aperture.

Replacing (4) with (5) and (6), gives the following matrix equation

\[ \{ \vec{S} \} \{ \vec{e} \} = j \omega \mu \{ \vec{B} \} \alpha \]

where \( \{ \vec{S} \} \) is the tangential electric field on the aperture, \( \{ \vec{B} \} \) is the magnetic field on the aperture, and \( \{ \vec{e} \} \) is the total electric field on the aperture.

The next step was to further separate the coefficients \( \alpha_i \) by using the equivalence formula [16] and the problem was divided into two parts. The first part is composed of coefficients related to the edges on the aperture, and the second part to the edges inside the enclosure

\[ \alpha = \begin{bmatrix} 0 \\ \alpha_a \end{bmatrix} \quad \varepsilon = \begin{bmatrix} e_i \\ e_a \end{bmatrix} \]

where \( \{ \vec{A} \} = [\{ \vec{S} \} - k_n^2 \{ \vec{T} \}] \) and \( \{ \vec{B} \} \) are finite element matrices, and \( e \) and \( \alpha \) unknown coefficient vectors [17]. It is possible to split these coefficients into two parts. The first part is composed of coefficients related to the edges on the aperture, and the second part to the edges inside the enclosure.

Here, the subscripts \( a \) and \( i \) refer to the aperture and the inner region of the finite element volume, respectively. Then, the \( \{ \vec{A} \} \) and \( \{ \vec{B} \} \) matrix elements in (8) can be separated into inner and aperture elements

\[ \begin{bmatrix} A_1 & A_{1a} \\ A_{2a} & A_{2} \end{bmatrix} \begin{bmatrix} e_i \\ e_a \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & B_{sa} \end{bmatrix} \begin{bmatrix} \alpha_i \\ \alpha_a \end{bmatrix} \]
B. Method of Moments (MoM)

The FEM formulation requires knowledge of the magnetic field on the aperture. The tangential field on the aperture is found by applying the boundary conditions, and it should be continuous. Then, the boundary condition on the aperture can be expressed as

$$\hat{n} \times \vec{H}^i + \hat{n} \times \vec{H}^e = \hat{n} \times \vec{H}^m = \hat{n} \times \vec{H}^a$$  \hspace{1cm} (11)

where $\vec{H}^i$ is the magnetic field of the applied planar wave, $\vec{H}^m$ the magnetic field of the radiation to the outer environment by $J_a$, which formed in the aperture (this field can be computed using the free space Green's function), $\vec{H}^e$ the value of the field on the aperture that is radiated into enclosure by $J_a$ and is equal to the magnetic field ($\vec{H}^a$) of the aperture, and $\vec{H}^m$ the magnetic field required by the FEM formulation, which is important for formulation, as it provides for hybridization.

For determining the fields radiated from the aperture to free space, the dyadic Green's function of the electrical vector potential $\vec{F}$ can be used. The electrical vector potential $\vec{F}$ was obtained by solving the following integral expression

$$\vec{F} = \varepsilon_0 \int \overline{G(r/r')} \cdot \vec{J}_a(r') \, dS'$$  \hspace{1cm} (12)

where $G(r/r')$ is the dyadic Green's function of free space electrical vector potential, provided that $r'$ is the source point, $r$ the field point, and $S_a$ the area of the aperture. Rearranging (12) by using the magnetic current intensity given in (1) and the free space Green's function, one obtains

$$\vec{F}(r) = \varepsilon_0 \int \overline{G(r/r')} \cdot \vec{J}_a(r') \, dS'$$  \hspace{1cm} (13)

where $E_a$ is the electric field on the aperture and equal to the value of the electric field inside the enclosure computed by the FEM on the aperture. Hence, in (5), one can obtain the electrical field with the use of coefficients.

With the vector potential in (13), the magnetic field radiating into the free half space is defined as

$$\vec{H}^m(r) = \frac{1}{j\omega \mu_0 \varepsilon_0} [k_0^2 \vec{F}(r) + \nabla \times \vec{F}(r)]$$  \hspace{1cm} (14)

where $\vec{H}^m$ is the value on the aperture of the tangential component of the magnetic current inside the enclosure and is expressed in (6). The integral equation achieved by applying Galerkin's method to (11) can be converted into matrix form as shown below

$$[k^m + \{Y^m\} \{e_s\}] = \{Y^m\} \{\alpha_s\}$$  \hspace{1cm} (15)

Here, $\{\alpha_s\}$ is the vector of unknown coefficients of the electric field on the aperture, and $[k^m]$, $[Y^m]$, and $[Y^e]$ are matrices representing the inner products of magnetic fields on the aperture. If the aperture and inner edge separation are taken into account, one can obtain the following from (15)

$$\begin{bmatrix} 0 \\ \{\alpha_s\} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \{0\} & \{0\} \end{bmatrix} \begin{bmatrix} 0 \\ \{Y^m\} \{Y^e\} \{e_s\} \{\{Y^m\} - \{Y^e\}\} \{\{Y^m\} - \{Y^e\}\} \end{bmatrix}$$  \hspace{1cm} (16)

As both (8) and (15) (the former from FEM formulation and the latter from the MoM formulation) have common unknowns, they can be combined using these common unknowns. By rearranging (15) to get $\alpha_s$ and $e_s$ and substituting it in (10), one obtains the following

$$\begin{bmatrix} \hat{A}_s & \hat{A}_m \\ \hat{A}_m & \hat{A}_e \end{bmatrix} \begin{bmatrix} \{e_s\} \\ \{\alpha_s\} \end{bmatrix} = \begin{bmatrix} 0 \\ \{B^e\} \end{bmatrix}$$  \hspace{1cm} (17)

where $\{B^e\} = \{B\} - \{B^{int}\} \{Y^{int}\}$ and $\{B^{int}\} = \{B\} - \{B^{m}\}$.

By solving this equation, one can obtain $e_s$ and $\alpha_s$ coefficients. Hence, the electric field distribution in the enclosure, as also the magnetic current intensity on the aperture, can be found.

III. NUMERICAL RESULTS

In this section, several numerical results are presented using the hybrid MoM/FEM technique introduced above. To demonstrate the performance of the method, an empty enclosure with an aperture was used. The width (A), height (B), and length (C) of the enclosure are 30 cm, 12 cm, and 30 cm, respectively. The aperture, which had a length (L) of 10 cm and width (W) of 0.5 cm, was placed on the $z = 0$ plane with the coordinates $x_a = 15$ cm and $y_a = 6$ cm. A perpendicular y-polarized planar wave was applied to the surface of the enclosure containing the aperture. The enclosure was split into 7, 3, and 6 hexahedra in the $x$, $y$, and $z$ dimensions, respectively. Then each hexahedron was divided into five tetrahedral elements. The number of unknowns in the enclosure was 1105. Using the MoM/FEM hybrid model, the electric field distribution was calculated. To obtain SE, the following equation was used

$$SE (dB) = -20 \log \left( \frac{\sqrt{\vec{E}_{total}^2}}{\vec{E}_{inc}} \right)$$  \hspace{1cm} (18)

where $\vec{E}_{total}$ is the electric field value at the center of the enclosure, calculated by MoM/FEM method and $\vec{E}_{inc}$ is the electric field value of the incident plane wave at the same point without the enclosure. The change in the SE with frequency was obtained and compared with the results of Robinson et al. [2] (Fig. 2). It is seen that the results obtained by the presented method and those given by Robinson et al. [2] are compatible. In Fig. 2, a notch can be seen at approximately 700 MHz. This notch represents the resonance frequency of the enclosure’s TE$_{101}$ mode.
To show the effect of aperture’s length on the SE, two different lengths, \( L = 10 \) cm and \( L = 20 \) cm, were selected with a fixed width \( w = 0.5 \) cm. As shown in Fig. 3, in the first case (\( L = 10 \) cm), a notch representing the resonance mode occurred at about 700 MHz, and in the second case (\( L = 20 \) cm), two notches occurred at about 650 MHz and 800 MHz. The second notch formed by interaction between the aperture resonance and enclosure resonance. For aperture dimensions of 10\( \times \)0.5 cm, no interaction was observed, because the resonance frequency of this aperture was far away from that of the enclosure. It is evident that the SE decreases with increase in the length of the aperture.

The relationship between width of aperture and shielding effectiveness is shown in Fig. 4. With increase in aperture width, the shielding effectiveness changes, but insignificantly. The polarization of applied planar wave is effective for such a result. Especially near the resonance frequency, both apertures have the same SE value. Farther from that frequency, the shielding effectiveness increases.

To demonstrate the effect of dimensions of the enclosure on SE, three enclosures of different dimensions are discussed. The dimensions of these enclosures are 30\( \times \)12\( \times \)25 cm, 30\( \times \)12\( \times \)30 cm, and 30\( \times \)12\( \times \)40 cm. The aperture measures 10 cm in length (L), and 0.5 cm in width (w), and was placed on the plane at the coordinates \( x_0 = 15 \) and \( y_0 = 6 \), for each enclosure. A perpendicular y-polarized planar wave was applied to the surface of the enclosure with the aperture. The unknown quantities in the enclosure for three situations were 858, 1105, and 1329, respectively. The center points of the enclosures were taken to be the points where the SE would be calculated.

The effect of enclosure dimensions on the SE is shown in Fig. 5. The resonance frequencies change with changing enclosure dimensions. The notches in the SE indicate these resonance frequencies. As can be seen from the Fig., the SE changes with enclosure dimensions. So, during the designing stage, the enclosure dimensions can be adjusted to produce the optimal SE.

While the SE produces point results for the design and optimization aspects, the stored electrical energy and dissipated power, which represent the electric field distribution inside the whole enclosure, must be taken into account. Relative stored electrical energy (SEE) is given by [18]

\[
\text{SEE} = 10 \log \left( \frac{\epsilon_r \epsilon_0 |\vec{E}_\text{in}|^2 |dV|}{\epsilon_r \epsilon_0 |\vec{E}_\text{in}|^2 |dV|} \right)
\]

Dissipated power in the dielectric layer (DP) is

\[
\text{Dissipated Power (dBm)} = \frac{\int \int \int \sigma |\vec{E}_\text{in}|^2 |dV|}{1 \text{mW}}
\]

\[\text{(19)}\]
Fig. 7. The affect of the change of the aperture width on SEE.

Fig. 8. Geometry of loaded rectangular enclosure with aperture.

Fig. 9. Variation of SEE and DP with slab position for 700 MHz frequency.

Fig. 10. Variation of SEE and DP with slab position for 900 MHz frequency.

where \((\vec{E}_{\text{total}})\) refers to the total fields computed at the calculation point inside the enclosure and \((\vec{E}_{\text{inc}})\) to the incident field computed at the same location in the absence of the enclosure [19], [20].

The ratio between stored electrical energy and frequency inside empty enclosure was examined. The enclosure size was \(30 \times 12 \times 30\) cm. The aperture with \(20 \times 1\) cm dimensions was located on the front surface of the enclosure. The change of SEE with frequency, in an empty enclosure with aperture, is shown in Fig. 6. It is observed that the results obtained by hybrid MoM/FEM method are in good agreement with the data of Siah et al. [18].

SEE results obtained from (11) under different aperture widths are shown in Fig. 7. It can be seen that the electrical energy stored in the enclosure volume is less when the aperture is narrower. Wider aperture decreases the shielding performance, but increases SEE in the enclosure.

Finally the effect of the presence of dielectric slab inside the enclosure was examined. A \(30 \times 12 \times 30\) cm enclosure filled with a \(30 \times 1 \times 30\) cm dielectric slab, characterized by \(\varepsilon_r = 2.65, \sigma = 0.22\), was considered. The aperture was...
Fig. 11. The change of SE with frequency in an empty and loaded enclosure with aperture; (a) 20×1 cm and z = 23 cm and (b) 20×0.5 cm and z = 25 cm.

The effect of the changes in the location of the dielectric slab along the z axis on the stored energy and dissipated power is shown in Figs. 9 and 10. From these figures, it can be said that determining the slab location is particularly important in designing and optimization. The interference is less at 700 MHz as can be seen from the figures. The SEE and DP difference with respect to the difference between the calculated locations of the dielectric slab for two different aperture widths were investigated at frequencies 700 and 900 MHz, respectively. It can be seen from these Figs. that, with increase in aperture width, interference also changes depending on the location of the dielectric slab. For optimization, the location of the dielectric slab in the enclosure, aperture width and the amount of interference should be determined. From the Figs., it can be seen that the interference is least when the aperture is narrower and the regions are near the center.

The position of the dielectric slab affects simultaneously the SE and the properties of the SEE and DP. Fig. 11 shows the change in SE with frequency regarding to the position of the slab with the maximum dissipated power for two different aperture widths. In both cases, the survey point was taken as the geometrical center. With increasing aperture width, relatively the same levels are observed near 700 MHz, but an increased one near 700 MHz.

IV. CONCLUSION

Hybrid MoM/FEM formulation was developed for computing the electric field distribution inside an enclosure with an aperture. The formulation has significant advantages when applied to EMC problems. Both the inner and outer regions were modeled with a single software solution. The outer region of the enclosure was modeled using the moment method without widening the solution space, and therefore, the computation time was reduced. This hybrid method, with fewer unknowns than the finite element method, ensures that less memory will be used. The electric field distribution of a known enclosure was computed using this method, and the changes in the shielding effectiveness of the enclosure with frequency were obtained and compared with the results in the literature. The performance of the proposed method is presented, and it is shown that the results obtained by this method are appropriate when compared to those of previous studies. For EMC system design and optimization, the characteristics of shielding effectiveness due to changes in the dimensions of the aperture and enclosure were obtained. The effect of changes in aperture's width on shielding effectiveness behavior versus frequency was insignificant; however, the effect of changes in aperture's length on shielding effectiveness was remarkable. With the inclusion of aperture's resonance frequency, the shielding effectiveness decreased. Changes in the enclosure's dimensions change the resonance frequency of the excited mode in the enclosure; therefore, the frequency with minimum shielding effectiveness changes. By adjusting the aperture dimensions, cavity dimensions, dielectric properties and the location of dielectric slab inside the enclosure, SE, SEE and DP can be controlled. Then the minimum required configuration was obtained for the design.

REFERENCES


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