Parameter Identification of a DC Motor via Moments Method

M. Hadef, A. Bourouina, and M. R. Mekideche

Abstract—The identification process consists of estimating the unknown parameters of system dynamics. Consequently, determination of the assumed system structure is of great importance in the process of system identification. Time moments have been introduced in automatic control because of the analogy between the impulse response of a linear system and a probability function. This basic idea has generated applications in identification, model order reduction and controller design. In this paper, a newly developed identification algorithm, called moments method, is introduced and applied to the parameter identification of a dc motor. The simulation and experimental results are presented and compared.

Index Terms—Identification, moments method, dc motor.

I. INTRODUCTION

DC MOTORS have long been widely used in many industrial applications. A dc motor can be considered as a single input, single output (SISO) system having torque-speed characteristics compatible with most mechanical loads. This makes a dc motor controllable over a wide range of speeds by proper adjustments of its terminal voltage. Mathematical modeling is one of the most important and often the most difficult step towards understanding a physical system.

Although models of some systems can be constructed from the physical laws, most often systems are too complex to be modeled this way. A model is usually constructed from a set of input-output data which is obtained experimentally and is represented in the form of a table or a graph [1], [2]. In modeling a dc motor, the aim is to find the governing differential equations that relate the applied voltage with the produced torque or speed of the rotor, the second part of the modeling is to determine the parameters of the model [1]-[3].

System identification is the subject which deals with the problem of building mathematical models of dynamical systems based on the observed system data [2]-[4]. The actual identification of models from data involves decision making on the part of the person in search of models, as well as fairly demanding computations to furnish bases for these decisions. System identification of dc motors is a topic of great practical importance, because for almost every servo control design a mathematical model is needed [1], [2], [5]. There are situations when identification model is available. For example, the motor parameters might be subject to some time variations [6]. In these cases, a mathematical model that is accurate at the time of the design may not be accurate at a later time. Moreover, a mathematical model is never a complete description of a given system, this is because a model that represents a system well over a range of frequencies, may not represent the system as well as over a different range of frequencies. Therefore, accuracy and adequacy are two major modeling issues that always have to be dealt with. Additionally, identification methods are useful for the validation (or invalidation) of an existing model [7]. On a broader sense, system identification is often the only means of obtaining mathematical models of most physical systems; this is because most systems are usually so complex that, unlike dc motors, there is no easy way to derive their models based on the physical laws.

Time moments have been introduced in automatic control, because of the analogy between the impulse response of a linear system, and a probability function. Thus, an impulse response is characterized by infinity of moments, practically, only the first ones are necessary, as for a probability density function. This basic idea has generated applications in identification, model order reduction and controller design, known as the method of moments [8], [9]. This characterization principle is well adapted to the case of a periodic response systems, whose impulse responses are strongly analogous to probability density functions. Controller design gives satisfactory results for this kind of systems. On the other hand, there are severe drawbacks for systems having singularities (non minimum phase) or for the oscillatory type. In these two cases, a good characterization of impulse response needs to take into account a larger number of moments, which is the main cause of failure of initial methodology.

This paper is structured as follows. Section II, describes the dynamic of the separately excited dc motor. Section III, describes the method of moments and analyzes the developed identification algorithm, applied to the parameter identification of a dc motor. In section IV, experimental results are illustrated and compared to the simulation results. Finally, conclusions of the paper are summarized in section V.

II. DC MOTOR MODEL

The block diagram of the dc motor used in this study is shown in Fig. 1. The dynamic of the separately excited dc motor may be expressed by the following equations

$$K\alpha(t) = -R_a i_a(t) - L_a \frac{di_a(t)}{dt} + U_a(t) \quad (1)$$
\[ K_i(t) = J \frac{d\omega(t)}{dt} + f \omega(t) + T_L(t) \] (2)

where \( K, Ra, La, J \) and \( f \) are respectively, the torque and back-EMF constant, the armature resistance, the armature inductance, the rotor mass moment of inertia and the viscous friction coefficient. \( \omega(t), i_s(t), U_s(t) \) and \( T_L(t) \) respectively denote the rotor angular speed, the armature current, the terminal voltage and the load torque.

### III. Method of Moments

The moments constitute the basis for a non classical representation of linear systems. The characterization of an impulse response by its moments is equivalent to the moment characterization of a probability density function [9]. Impulse response moments are system invariants. Like for a probability density function, it is not necessary to compute infinity of moments to characterize with a good approximation the shape of the impulse response only the first ones are necessary to perform this characterization.

#### A. Temporal Moment of a Function

Let us consider a stable linear system, characterized by its impulse \( h(t) \) then,

\[ H(s) = \frac{B(s)}{A(s)} \] (3)

\( H(s) \) can be expanded in Taylor series in the vicinity of \( s=j \omega \)

\[ H(s) = \sum_{n=0}^{\infty} (-1)^n (s-j \omega)^n \tilde{A}_{n,\omega} \] (4)

where

\[ \tilde{A}_{n,\omega} = \frac{1}{n!} \int_0^\infty h(t)e^{-jwt}dt \]

is the \( n \)th order frequency moment of \( h(t) \) for \( \omega=\omega_0 \), notice that \( \tilde{A}_{n,\omega_0} \) is complex. In the particular case \( \omega_0 = 0 \), frequency moments correspond to classical time moments

\[ A_s(h) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{n!} h(t)dt \] (5)

they permit the characterization of \( H(\omega) \) around \( \omega_0=0 \), as well as that of the impulse response \( h(t) \). \( A_s(h) \) is the area of \( h(t) \), \( A_t(h) \) defines mean time of \( h(t) \) and \( A_m(h) \) deals with the dispersion of \( h(t) \) around its mean time [9]. (4) is rewritten as

\[ H(s) = \sum_{n=0}^{\infty} (-1)^n s^n A_n(h) \] (6)

Let \( H(s) = \sum_{n=0}^{\infty} \frac{S^n}{n!} \frac{d^nH(s)}{ds^n} \). Then, time moments can be expressed as

\[ A_n(h) = \frac{(-1)^n}{n!} \left[ \frac{d^nH(s)}{ds^n} \right]_{s=0} \] (7)

#### B. Moments and Parameters of a Transfer Function

Let \( y(t) \) the step response of the studied system. We propose to identify the system by the model

\[ H(s) = \frac{Y(s)}{E(s)} = K, \frac{1+b_1s+b_2s^2+....+b_ms^n}{1+a_1s+a_2s^2+....+a_ms^n} \] (8)

from the final value theorem, as time approaches infinity for a stable linear system, the system response approaches a steady state value \( K_y \) given by

\[ K_y = \lim_{t \to \infty} y(t) = y(\infty) \] (9)

if a step input is applied to the system described in equation (8), by taking the Laplace transform of the normalized response gives

\[ H(s) = s \cdot y(s) \] (10)

let us consider \( \varepsilon(t) \) an error function with

\[ \varepsilon(t) = K_y - y(t) \] (11)

by introducing the Laplace transform in equation(11), (8) can be written as

\[ \varepsilon(s) = K_y \left[ \frac{1}{s} - \frac{1}{s-\omega^2} \right] \] (12)

the development of (12) gives

\[ \varepsilon(s) = K_y \left[ (a_1-b_1) + ... + (a_n-b_n) s^{n-1} + ...(a_m-b_m) s^{n-1} \right] \] (13)

which is equivalent to

\[ (1+a_1 s+a_2 s^2+....+a_m s^n) \varepsilon(s) = K_y \left[ (a_1-b_1) + ... + (a_n-b_n) s^{n-1} + ...(a_m-b_m) s^{n-1} \right] \] (14)

then, using (6) we can write

\[ \varepsilon(s) = \sum_{n=0}^{\infty} (-1)^n s^n A_n(\varepsilon) \] (15)

the development of (15) gives

\[ \varepsilon(s) = A_s(\varepsilon) - sA_1(\varepsilon) + s^2A_2(\varepsilon) + ... + (-1)^n s^n A_n(\varepsilon) + ... + (-1)^n s^n A_n(\varepsilon) \] (16)

according to (14) and (16) we can write

\[ K_y (a_1-b_1) = A_s(\varepsilon) \]

\[ K_y (a_2-b_2) = a_1 A_s(\varepsilon) - A_1(\varepsilon) \]

\[ K_y (a_3-b_3) = a_2 A_s(\varepsilon) - a_1 A_1(\varepsilon) + A_2(\varepsilon) \]

\[ \ldots \]

\[ K_y (a_n-b_n) = a_{n-1} A_s(\varepsilon) - a_1 A_{n-1}(\varepsilon) + ... + A_n(\varepsilon) \] (17)
finally we can deduce the coefficients of the transfer function \( H(s) \) by solving the following matrix system

\[
\begin{bmatrix}
K_1(a_1 - b_1) \\
K_2(a_2 - b_2) \\
K_3(a_3 - b_3)
\end{bmatrix}
\begin{bmatrix}
A_0(\varepsilon) \\
-A_1(\varepsilon)A_0(\varepsilon) \\
-A_2(\varepsilon)A_1(\varepsilon)
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
a_3
\end{bmatrix}
= \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

(18)

where \( A_n(\varepsilon) \) is the \( n \)th order temporal moment.

C. DC Motor Transfer Function and its Moments

For our cases, when \( n = 2 \) and \( m = 1 \), the transfer function (28) becomes

\[
H(s) = K_1 \frac{1+b_1s}{1+a_1s+a_2s^2}
\]

(19)

system (18) is reduced to the following matrix system

\[
\begin{bmatrix}
K_1(a_1 - b_1) \\
K_2(a_2 - b_2) \\
K_3(a_3 - b_3)
\end{bmatrix}
\begin{bmatrix}
A_0 \\
-A_1A_0 \\
-A_2A_1
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
a_3
\end{bmatrix}
= \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

(20)

the resolution of this matrix system (20), gives the following coefficients

\[
a_1 = \frac{A_0A_1 - K_1A_0}{A_0 - K_1A_1}, a_2 = \frac{A_1 + A_2}{K_1}, b_3 = a_1 - A_0
\]

(21)

D. Parametric Identification

After having deduced the mathematical forms which are used for calculation of the transfer function coefficients and which enable us at the same time to calculate the electric and mechanical motor parameters, we present here, the stages to be followed at the time of the determination of these parameters.

It is possible from a terminal voltage step \( \Delta U_o \) to determine the majority of the dc motor parameters, which is also possible from the abrupt terminal voltage variation. We measure the initial and final currents and speeds values shown on table I. At the steady state we can write

\[
U_{\text{at}} = R_1i_{\text{at}} + K_1\omega_1
\]

(22)

\[
U_{\text{at}} = R_1i_{\text{at}} + K_1\omega_1
\]

(23)

where \( U_{\text{at}}, i_{\text{at}} \) and \( \omega_1 \) respectively denote the terminal voltage armature current and the rotor speed at initial regime subscripted "0". \( U_{\text{at}}, i_{\text{at}} \) and \( \omega_1 \) respectively denote the terminal voltage, armature current and the rotor speed at final regime subscripted "1".

According to (22) the torque and back-EMF constant \( K \) can be written as

\[
K = \frac{U_{\text{at}} - i_{\text{at}}U_{\text{o0}}}{i_{\text{at}} - i_{\text{o0}}}
\]

(24)

the steady state check

\[
\Delta U_o = R_1\Delta i_o + K\Delta \omega
\]

(25)

where \( \Delta U_o, \Delta i_o \) and \( \Delta \omega \) respectively denote the terminal voltage variation, armature current variation and rotor speed variation.

The armature resistance is given from (25) as

\[
R_1 = \frac{\Delta U_o - K\Delta \omega}{\Delta i_o}
\]

(26)

according to (27) and (28) we can obtain the two transfer functions of the armature current and rotor speed

\[
\Delta U_o = R_2\Delta i_o + L_2 \frac{d\Delta i_o}{dt} + K\Delta \omega
\]

(27)

\[
K\Delta \omega = \frac{d\Delta \omega}{dt} + f\Delta \omega
\]

(28)

the armature current transfer function is given as

\[
H_i(s) = \frac{\Delta i_o(s)}{\Delta U_o(s)} = \frac{f}{K^2 + R_2f} + \frac{1}{1 + \tau_n \tau^2 + (\tau_n + \mu \tau) s}
\]

(29)

the rotor speed transfer function is given as

\[
H_\omega(s) = \frac{\Delta \omega(s)}{\Delta U_o(s)} = \frac{K}{K^2 + R_2f} + \frac{1}{1 + \tau_n \tau^2 + (\tau_n + \mu \tau) s}
\]

(30)

where \( \tau_e = L_e / R_e \) is Electrical time constant, \( \tau_m = R_m f / (K^2 + R_f) \) is Mechanical time constant, \( \mu = R_f f / (K^2 + R_f) \) is Usually small coefficient and the calculation of \( K_i \) and \( K_\omega \) gains of the tow outputs \( i_o(t) \) and \( \omega(t) \) respectively, by taking into account equations (29) and (30) gives

\[
K_i = \frac{\Delta i_o(\infty)}{\Delta U_o(\infty)} = \frac{f}{K^2 + R_2f}
\]

(31)

\[
K_\omega = \frac{\Delta \omega(\infty)}{K^2 + R_2f}
\]

(32)

according to (31) and (32) we deduce

\[
f = \frac{K_\Delta \omega(\infty)}{\Delta \omega(\infty)} = \frac{K}{K^2 + R_2f}
\]

(33)

by identification of \( H_i(s) \) and \( H_\omega(s) \) denominators with \( H(s) \) denominator we obtain

\[
a_i = \tau_m + \mu \tau_e
\]

(34)

\[
a_i = \tau_m \tau_e
\]

(35)

according to (34) and (35) we can obtain a second order equation

\[
\mu \tau_e^2 - a_i \tau_e + a_2 = 0
\]

(36)
The resolution of the (36) gives two roots one is positive, the other is negative (rejected).

- According to (34) and (35) we deduce \( \tau_w \)
- The deduction of \( \tau_m \) and \( \tau_e \) gives \( L_a \) and \( J \).

### IV. EXPERIMENTAL RESULTS

The separately excited dc motor used for experimental tests has the nominal characteristics shown on table II. The first experiment is the determination of electric dc motor parameters according to direct tests, like armature resistance \( R_a \) and armature inductance \( L_a \), as well as the back-EMF constant \( K \). Mechanical parameters are also determined by direct tests (static torque \( T_{st} \) and viscous friction coefficient \( f \)). The deceleration test enables us to determine the moment of inertia \( J \). The second experiment to be carried out is to identify the dc motor parameters from the dynamic test. According to a step amplitude \( \Delta U \) of terminal voltage applied to the armature circuit of the dc motor. The initial and final values of the armature current and the angular speed obtained from this test are shown in table I. The back-EMF constant \( K \), armature resistance \( R_a \), and the viscous friction coefficient \( f \), can be determined by using respectively equations (24), (26) and (33). Table III shows the first, second and third order moments values, as well as the transfer function coefficients values successively calculated from the trapezoids method.

Let us know \( a_1 \), \( a_2 \) and \( \mu \) then, we have the second order equation \( 0.00874\tau_e^2 - 0.0556\tau_e + 0.00144 = 0 \). The resolution of this equation gives \( \tau_{e1} = 0.026s \) and \( \tau_{e2} = 6.34s \) (\( \tau_{e2} \) is a rather large time constant, is thus rejected). The deduction of \( \tau_e \) and \( \tau_m \) enables to calculate \( J \) and \( L_a \). Table IV summarizes the values of the parameters calculated from the two identification methods (direct tests and moments method). Finally to check the

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**TABLE II**

**SPECIFICATION OF EXPERIMENTAL DC MOTOR**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>180 W</td>
</tr>
<tr>
<td>Rated speed</td>
<td>1500 rpm</td>
</tr>
<tr>
<td>Armature voltage</td>
<td>270 V</td>
</tr>
<tr>
<td>Field voltage</td>
<td>220 V</td>
</tr>
<tr>
<td>Armature current</td>
<td>1.1 A</td>
</tr>
<tr>
<td>Field current</td>
<td>0.4 A</td>
</tr>
</tbody>
</table>

**TABLE III**

1, 2, AND 3 ORDER MOMENTS AND TRANSFER FUNCTION COEFFICIENTS VALUES

<table>
<thead>
<tr>
<th>( a(t) )</th>
<th>( i_e(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_0 )</td>
<td>-0.29657</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>-0.152533</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>-0.000504</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>0.470565</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>0.240323</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>-5.962602</td>
</tr>
</tbody>
</table>

- The static torque can be calculated from steady state as
  \[
  T_{st} = K\nu_0 - f\omega_0
  \]
  (37)
TABLE IV

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Direct tests</th>
<th>Moments method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$ [Ω]</td>
<td>28</td>
<td>30.9</td>
</tr>
<tr>
<td>$L_s$ [H]</td>
<td>0.820</td>
<td>0.803</td>
</tr>
<tr>
<td>$K$ [N.m.A⁻¹]</td>
<td>1.34</td>
<td>1.323</td>
</tr>
<tr>
<td>$J$ [kg.m²]</td>
<td>0.0028</td>
<td>0.0031</td>
</tr>
<tr>
<td>$f$ [N.m.s/rad]</td>
<td>0.00054</td>
<td>0.0005</td>
</tr>
<tr>
<td>$T_a$ [N.m]</td>
<td>0.127</td>
<td>0.128</td>
</tr>
</tbody>
</table>

precision of each method, we have simulated the dynamic test applying a step amplitude of terminal voltage $\Delta U_s$=188 V to the armature circuit of dc motor, as well as, deceleration test and mechanical characteristic. According to Figs. 2-4 the curves simulated from the dynamic test parameters with the moment method are close to the real curve (experimental curve) that those simulated from the direct tests parameters. With regard to the steady state (Fig. 5) the curves simulated from the dynamic test and direct tests parameters are almost identical and close to real measurement (experimental curve).

V. CONCLUSION

In this work, we tried to contribute our share in the discipline of the dc motors modeling. This contribution, which can be classified in a very wide field of identification methods, can be summarized in the following results: we have developed a dynamic model based on the moments method, this method especially makes it possible to have a model closer to reality in transitory mode. We have proposed a comparison between various models based on the identification methods (direct tests and moments method), this comparison is made on the basis of real measurements taken in laboratory on a separately excited dc motor, with 180 W of rated power. It shows the advantage of the only dynamic test for identification, coupled to the moments method.

REFERENCES


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