Abstract—This paper investigates resonance phenomenon in active filters. The paper studies the effect of system/filter parameters on resonance conditions. It also proposes an active technique for damping such resonances. The study is based on modal analysis in which unstable modes or modes with slow damping are first extracted. Then, using a new technique, such modes are considerably damped. PSCAD/EMTDC software is also employed to verify the performance of the proposed technique.

Index Terms—Resonance phenomenon, active filters, damping resonance, modal analysis.

I. INTRODUCTION

Resonance is stated as harmonic current magnification phenomenon. This phenomenon occurs where a load includes resonance components such as capacitor banks for power factor correction or passive filters [1]. Resonance conditions may also happen during operation of active filters due to the interaction between filter dc link capacitor and system/load inductances. Such undesired conditions deteriorate the filter performance and may also damage its components. To use active filters with line current detection, a control method to suppress the harmonic current magnification phenomenon is needed. To study this, a system composed of a load, source and an active filter is selected. Then, time domain equations governing the system behavior are transformed to dq frame and modal analysis is performed. Three main parts of the system are:

- Active filter power circuit
- Active filter control circuit
- Load and source power circuit

The structure of this paper is as follows. In the next section, block diagrams of the three aforementioned parts are derived, and the modes with slow/no damping are extracted using modal analysis. Then, a new method for damping such modes is introduced. Since the damping action is achieved by using the active filter; the method is called active damping [2]-[4]. The main contribution of this work with respect to previous ones is that in this paper, the system modes with low damping are found first. Then, the contribution of state variables on these modes is determined. Therefore, the control action is taken based on the state variables with maximum contribution.

II. SYSTEM BLOCK DIAGRAM

In this section, the modeling of the system under study is explained.

A. Voltage Source Inverter Model

Fig. 1 depicts a voltage source inverter (VSI) which is connected to an ac source through \( R_f \) and \( L_f \). A three-phase mathematical model for the voltage source converter (VSC) of Fig. 1 is derived in [5]-[6]. In [7], by using this mathematical model, it is shown that a VSC in balanced three phase operation without neutral wire can be modeled as shown in Fig. 2.

In this figure, \( v_{qf} \) and \( v_{df} \) are the transformed voltages of the inverter output, i.e., \( v_{df}, v_{bf} \), and \( v_{of} \) using Park Transformation and \( f_a, f_b, \) and \( f_c \) are defined as: \( f_k = \frac{v_{kf}}{V_e} \), where \( k = a, b, \) and \( c \), \( f_a, \) and \( f_c \) also represent \( f_a, f_b, \) and \( f_c \) in the transform dq axes. If the system is a four-wire system and the filter can pass neutral current through the dc link center point, then power balance equations for zero sequence voltage/current can be written as

\[
3v_{of}i_{of} = -\frac{V_e}{2}i_{0}\kappa
\]

(1)

where \( v_{of} \) and \( i_{of} \) are the zero sequence of the inverter output voltage and current, and \( i_{0}\kappa \) is the dc link current due to the zero sequence current at the ac side and \( V_e \) is the dc link voltage.

If \( f^0 = \frac{V_{of}}{V_e} \), from (1)
Fig. 3. 3-phase 4 wire VSI model.

Fig. 4. Load and voltage source.

Fig. 5. Load and voltage source model, (a) three phase three wire system, and (b) neutral wire model.

Fig. 6. Control system block diagram.

Table I

<table>
<thead>
<tr>
<th>SYSTEM DATA FOR SIMULATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_f = 1.5\text{mH}$</td>
</tr>
</tbody>
</table>

III. RESONANCE CONDITION ANALYSIS

To study the resonance behavior of the system, modal analysis is employed. Different operating cases are considered. First, a constant nonlinear load connecting to a stiff source is studied. Then, the effect of source impedance is also taken into consideration. The final study is carried out on a combination of a nonlinear load and a capacitor bank.

A. Case 1: Constant Nonlinear Load

Fig. 7 depicts the block diagram of a complete system, i.e. filter, load, and source. The effect of source impedance is neglected. System data is also given in Table I.

The system has 10 state variables as

$$x_1: \Delta v, x_2: i_{df}^f, x_3: v_{df}^f, x_4: \Delta i_{loss}, x_5: i_{df}^f, x_6: i_{dh}^f, x_7: i_{df}^f, x_8: v_{df}^f, x_9: i_{df}^f, x_{10}: v_{df}^f$$

The differential equations (state equations) of the system are extracted from Fig. 7. They are listed as below

$$\frac{d}{dt} \Delta v_c = -\frac{\Delta i}{c} = -\frac{3}{2} i_{df}^f - \frac{3}{2} i_{df}^f - 6 f i_{df}^f \quad (4a)$$

$$\frac{d}{dt} i_{df}^f = \frac{R_f}{L_f} i_{df}^f + \frac{v_{df}^f - v_{df}^f}{L_f} \quad (4b)$$

$$V_{df}^f(s) = (k_c + k_i) (i_{df}^f - i_{df}^f) \Rightarrow \quad (4c)$$

$$\frac{d}{dt} V_{df}^f = k_p \left( \frac{d}{dt} i_{df}^f - \frac{d}{dt} i_{df}^f \right) + k_i \left( i_{df}^f - i_{df}^f \right)$$

$$\Delta V_c(s) = \frac{k_i}{1 + s T_{dc}} (\Delta v_c) \Rightarrow \quad (4d)$$

$$\frac{d}{dt} \Delta v_c = -\frac{1}{T_{dc}} \Delta v_c + \frac{1}{T_{dc}} \Delta v_c$$
As it can be seen, (4a) is the product of state variables, so the differential equations of the system are nonlinear and they are linearized about nominal operating point. 

For calculating the system modes, the following steps are performed:

1. The state equations of system are extracted.
2. The nonlinear state equations are linearized about the nominal operating point.
3. The modes of system are the eigenvalues of the linear system extracted in step 2.

By writing the governing differential equations and linearizing the equations about nominal operating point, the following modes are obtained:

\[ \lambda_d = ( -1/67 \pm j \times 0.30 ) \times 10^4 \], \[ \lambda_4 = -1.8\times10^4 \], \[ \lambda_5 = -1.55\times10^4 \], \[ \lambda_6 = -114 \pm j \times 181 \], \[ \lambda_7 = -1.9\times10^3 \], \[ \lambda_8 = -1.67 \pm j \times 0.19 \] \times 10^4 \], \[ \lambda_9 = -251 \].

Since the source is stiff and the load is considered as an input block in Fig. 7, the resulted modes illustrate the interaction between the ac and dc side of the filter. It can be seen that all the modes have desirable damping. Using participation factors (participation factors indicate the effect of different state variables on different modes of the system) described in [8], the degree of contribution of each state variable in system modes can be detected. In Table II, the state variables with maximum contribution to the system modes are given.
Increasing the filter inductance has little effect on the damping and no mode will get close to the unstable region. However, decreasing the capacitor value will result in oscillation of capacitor modes. Therefore, the capacitor value must be greater than a critical value.

For $C_1 = C_2 = 1600 \ \mu F$, the modes corresponding to capacitors are $\lambda_{5,6} = -111 + j 400$. If source impedance is considered in the block diagram of Fig. 7, $L_f$ and $R_f$ will change to $L_f - L_s$ and $R_f - R_s$, respectively. However, the effect of source impedance is negligible and even with $R_f = R_s = 0$, no mode will become unstable.

For $R_f = R_s = 0$, $L_s = 0.5 \ \text{mH}$ and $L_f = 1.5 \ \text{mH}$ system modes are:

- $\lambda_5 = -2.92 \times 10^9$, $\lambda_6 = -3.88 \times 10^9$, $\lambda_{1,2} = -114 \pm j 184$
- $\lambda_9 = -1.11 \times 10^9$, $\lambda_{10,11} = -1.07 \times 10^9$
- $\lambda_{12,13} = -3.9 \times 10^9$, $\lambda_{14,15} = -1.1 \times 10^9$, $\lambda_{16} = -251$

Fig. 8 shows dc-link voltage variation in case 1, $c_1 = c_2 = 1500 \ \mu F$, (a) voltage variation across dc-link capacitor $c_1 (\psi_1^0)$, (b) voltage variation across total dc-link capacitor $c_1 + c_2 (\psi_{1,2}^0)$, (c) voltage variation across dc-link capacitor $c_2 (\psi_2^0)$. As it can be seen, since all modes have sufficient damping, the dc link voltage variation is small and normal.

B. Case 2: Nonlinear Load in Parallel with a Capacitor Load

This case presents the study results when the load is a combination of a nonlinear load in parallel with a capacitive load. The source is also connected to the filter through an impedance of $R_s$ and $L_s$. Fig. 9 depicts the system block diagram. System data is also given in Table III.

<table>
<thead>
<tr>
<th>$L_f$</th>
<th>$R_f$</th>
<th>$V_c$</th>
<th>$R_s$</th>
<th>$L_s$</th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 mH</td>
<td>0.3 Ω</td>
<td>340 V</td>
<td>1 mΩ</td>
<td>0.1 mH</td>
<td>313 µF</td>
<td>7000 µF</td>
</tr>
</tbody>
</table>

System modes are:

- $\lambda_2 = (-1.67 \pm 0.32) \times 10^4$, $\lambda_3,4 = (-1.67 \pm 0.33) \times 10^4$
- $\lambda_{5,6} = -115 \pm j 177$, $\lambda_7 = -19$
- $\lambda_{8,9} = (-0.51 \pm 0.27) \times 10^4$, $\lambda_{10,11} = (-0.51 \pm 0.21) \times 10^4$
- $\lambda_{12,13} = (-1.67 \pm 0.23) \times 10^4$, $\lambda_{14,15} = (-0.51 \pm 0.25) \times 10^4$
- $\lambda_{16} = -251$

Using the participation factors, the degree dependence of system modes and state variables can be found. In this case, all the modes have acceptable damping. Modes $\lambda_{5,6}$ correspond to the dc link capacitor. Now, $R_s$ is set to zero and the new modes are extracted. The results are

- $\lambda_{2} = (-1.67 \pm 0.33) \times 10^4$, $\lambda_{4} = (-1.67 \pm 0.0604) \times 10^4$
- $\lambda_{5,6} = -114 \pm j 181$, $\lambda_{7} = -19$, $\lambda_{8,9} = (-12 \pm j 5951)$
- $\lambda_{10,11} = (-11 \pm j 5324)$, $\lambda_{12,13} = (-1.67 \pm 0.24) \times 10^4$
- $\lambda_{14,15} = (-11 \pm j 5637)$, $\lambda_{16} = -251$

It can be seen that six modes of the system, i.e., $\lambda_{10,11}, \lambda_{8,9}, \lambda_{14,15}$, have low damping. In Table IV, the state variables with maximum contribution to the system modes are given.

It can be seen that the source current, capacitor current and source voltage, have maximum contribution in modes with low damping. These modes result in an increase in capacitor and source current. This current will also result in a considerable change in capacitor/source voltage and current as shown in Figs. 10 and 11.

Fig. 10 shows simulation without active damping and $R_s = 0$ (a) source voltage ($v_s$), (b) filter current ($i_f$), (c) source current ($i_s$). Fig. 11 shows dc-link voltage variation in case 2, $c_1 = c_2 = 7000 \ \mu F$, (a) voltage across total dc-link capacitor $c_1 + c_2 (\psi_{1,2}^0)$, (b) voltage variation across dc-link capacitor $c_1 (\psi_1^0)$, (c) voltage variation across total dc-link capacitor $c_1 + c_2 (\psi_{1,2}^0)$.
System damping can be increased by adding extra resistance in the system, e.g., increasing. But this would result in higher losses and lower efficiency. To increase system damping, active damping is employed. In this method, the modes with little damping are found first. Then, the state variables corresponding to such modes are determined. At the end, by adding suitable controllers, system damping is improved.

IV. RESONANCE ACTIVE DAMPING

Consider the filter current controller of Fig. 12. The task of active damping is to decrease voltage/current oscillation by a modulating signal, i.e., \( v_m \) or filter output voltage \( v_f \) by the use of a suitable signal.

This signal is the one with the most contribution to the corresponding mode(s), i.e., source current, capacitor current, and source voltage. Fig. 13 shows active damper and current control loop. Fig. 14 explains principles of operation of the active damper.

![Active damper and current control loop](image)

### TABLE IV

<table>
<thead>
<tr>
<th>Modes ( \lambda )</th>
<th>States with the largest participation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{14} )</td>
<td>( x_4, x_3, x_2, x_1 )</td>
</tr>
<tr>
<td>( \lambda_{13} )</td>
<td>( x_4, x_3, x_2, x_1 )</td>
</tr>
<tr>
<td>( \lambda_{12} )</td>
<td>( x_4, x_3, x_2, x_1 )</td>
</tr>
</tbody>
</table>

From Fig. 14, one can write

\[
\begin{align*}
    i_s + i_f &= i_c + i_i \\
    v_f(t) &= L_f \frac{di_f(t)}{dt} + v_s(t) \\
    v_c &= L_s \frac{di_i(t)}{dt} + v_s(t)
\end{align*}
\]

From (4)-(6) and knowing that \( \frac{dv_f}{dt} = c' \frac{dv_s}{dt} \), then

\[
\frac{d^2v_s(t)}{dt^2} + \frac{1}{c'} \left( \frac{1}{L_f} + \frac{1}{L_s} \right) v_s(t) = \frac{1}{L_f c'} v_f(t) + \frac{1}{L_s c'} v_f(t) - L_s \frac{di_i(t)}{dt}
\]

Characteristic equation for (8) is

\[

s^2 + \frac{1}{c'} \left( \frac{1}{L_f} + \frac{1}{L_s} \right) = 0
\]

It can be seen that for \( R_s = 0 \), \( L_s = 0.1 \), and \( L_f = 1.5 \) mH the system damping is zero and the natural frequency is

\[

\omega_n = \sqrt{\frac{1}{c'(L_f + L_s)}} = 5838 \text{ rad/s}
\]

If \( v_f \) has a term in opposite phase with \( \frac{dv_s}{dt} \) (c' current), then a positive damping is achieved.
Therefore, if a term of \(-K_{\text{damp}}'i^c\) is added to \(v_f\), the characteristic equation will change to

\[
s^2 + \frac{k_{\text{damp}}'}{L_f} + \frac{1}{c} \left( \frac{1}{L_f} + \frac{1}{L_c} \right) = 0.
\]

Considering a damping factor of \(\xi = 0.5\), \(k_{\text{damp}}'\) can be found as \(k_{\text{damp}}' = 8.76\).

Once again system modes are extracted, but this time a term of \(-k_{\text{damp}}'i^c\) is added to the output current regulator. The modes are:

\[
\begin{align*}
\lambda_{2,3} &= (-1.35 \pm j 0.90) \times 10^4, \\
\lambda_{5,6} &= -114 \pm j 181, \\
\lambda_7 &= -19, \\
\lambda_{8,9} &= (-845 \pm j 6099), \\
\lambda_{10,11} &= (-815 \pm j 5442), \\
\lambda_{12,13} &= (-1.34 \pm j 0.93) \times 10^4, \\
\lambda_{14,15,16} &= (-831 \pm j 5770), \quad \lambda_{16} = -251.
\end{align*}
\]

As it can be seen, adding the term of \(-k_{\text{damp}}'i^c\) has increased the damping of the source and capacitor current. Simulation results also indicate that increasing \(k_{\text{damp}}'\) will deteriorate the situation. We expected to see an increase of 0.5 in the damping, but this did not happen. This is due to the fact that in choosing \(k_{\text{damp}}'\), all system dynamics were not considered but the dominant modes of filter, capacitor and source, 2nd order system, were considered. This time, we try to design another damper by getting a feedback from the source current. Substituting (7) into (8) yields

\[
\frac{d^2i_s(t)}{dt^2} + \frac{1}{c} \left( \frac{1}{L_s} + \frac{1}{L_c} \right) \frac{di_s(t)}{dt} - \frac{1}{L_f L_c} v_f'(t) + \frac{1}{L_f L_c} v_f'(t) + \frac{1}{L_c} \cdot v_f'(t) = \frac{1}{L_s} \cdot \frac{di_s(t)}{dt} + \frac{1}{L_s} \cdot \frac{di_s(t)}{dt}.
\]

If \(v_f(t)\) has a component in phase with the source current, the damping will increase. Therefore, by adding a term of \(k_{\text{damp}}'i_s'\) in the control loop the effects are studied. The value of \(k_{\text{damp}}'\) is chosen such that it has little effect on the source current and capacitor voltage. A value of 2 is selected. The new system modes in the case of adding \(k_{\text{damp}}'i_s'\) and \(-k_{\text{damp}}'i_s'\) are:

\[
\begin{align*}
\lambda_{1,2} &= (-1.29 \pm j 0.99) \times 10^4, \\
\lambda_{3,4} &= (-1.29 \pm j 0.93) \times 10^4, \\
\lambda_{5,6} &= -114 \pm j 181, \\
\lambda_7 &= -19, \\
\lambda_{8,9} &= (-1529 \pm j 6062), \\
\lambda_{10,11} &= (-1482 \pm j 5388), \\
\lambda_{12,13} &= (-1.29 \pm j 0.96) \times 10^4, \\
\lambda_{14,15,16} &= (-1505 \pm j 5724) \times 10^4, \quad \lambda_{16} = -251.
\end{align*}
\]

Corresponding simulations results are shown in Figs. 15 and 16. Fig. 15 shows simulation after active damping (a)source voltage (\(v_s\)), (b) filter current (\(i_f\)), (c) source current (\(i_s\)). Fig. 16 shows dc-link capacitor voltage after active damping (a) voltage variation across \(c_1+c_2\) (\(v_{cc,\text{in}}\)), (b) voltage across \(c_1+c_2\) (\(v_{cc,\text{out}}\)), (c) voltage variation across \(c_1\), and (d) voltage across \(c_1\).

V. CONCLUSIONS

In this paper, the resonance phenomenon in an active filter operation is studied. It is shown that in some cases, the system may become unstable due to the interaction of filter components and load/source inductances. The equivalent block diagram of a complete system is extracted, and different operating cases are presented. It is shown that the modes of the system may have little damping in some cases. To alleviate the problem, a resonance active damping method is proposed. It is shown analytically and using PSCAD/EMTDC software that the new method can considerably increase system damping and prevent the system from becoming unstable.

REFERENCES

M. Rahimi obtained his B.Sc. degree in Electrical Engineering from Esfahan University of Technology, Esfahan, Iran, in 2001, and the M.Sc. degree from Sharif University of Technology, Tehran, Iran, in 2003. He is presently a Ph.D. student in the Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran.

H. Mokhtari was born in Tehran, Iran on Aug. 19, 1966. He obtained his B.Sc. degree in Electrical Engineering in 1989 from Tehran University in Tehran, Iran. He joined Electric Power Research Center Institute in 1989 and worked for three years in the consulting division of power dispatching projects. In 1992 he was granted a scholarship from Iran Ministry of Culture and Higher Education for his M.Sc. degree. He finished his masters program in the field of power electronics in University of Newbrunswick, Canada in 1994. In 1994, he entered Toronto University in Toronto, Canada. In 1999, he finished his Ph.D. degree. Since 2000, he has been with the School of Electrical Engineering at Sharif University in Tehran, Iran. He is currently an associate professor in this department. His research interests include power quality, power electronics and application of power electronics in power distribution systems. He is also a senior consultant to several utilities and industries.

G. Zafarabadi was born in Shiraz, Iran, in 1978. He received his B.Sc. degree in Electrical Power engineering from Mazandaran University in 2001. In 2004 he received his M.Sc. in Power Engineering from Sharif University of Technology, Tehran, Iran. From 2004 to 2007, he was employed as a researcher at Iranian Academic Center for Education, culture and Research (ACECR) - Sharif Branch. He is now researcher at the Electric Power Systems Research Center of Niroo Research Institute (NRI). His research interests include dynamic of powerplants (identification, tuning of powerplant controllers such as AVR, PSS and Governor and …) and distributed generations.