Probabilistic OPF Approach for Transmission Expansion Planning in Restructured Power Systems

M. Parsa Moghaddam, H. Abdi, and M. H. Javidi

Abstract—Transmission expansion planning (TEP) is one of the most important parts of expansion planning in power systems. Competition in these systems has resulted in essential changes in TEP models and criteria. While methods introduced so far are essentially based on dc load flow, the proposed method utilizes the ac optimal power flow (OPF) in order to model the real world condition and obtaining optimal plans. Furthermore, in order to model the operation and investment costs, transmission tariffs are used. Investigation on 8-bus system confirms the advantages of the proposed method as compared with previously presented approaches.

Index Terms—Transmission expansion planning, competition, electricity market, probabilistic locational marginal price.

NOMENCLATURE

- $\ell$: Line number between buses $q$ and $r$
- $G$: Set of generators
- $L$: Set of lines
- $N$: Set of network buses
- $C$: Set of PQ buses
- $M$, $m$: Indices for maximum and minimum limits
- $c_{\ell}$: Duration for construction of line $\ell$ [year]
- $g$: General inflation
- $b$: Profit factor, equal to 1.2
- $x_{\ell}$: Impedance of line $\ell$ [p.u.]
- $s$: Sample number
- $K$, $t$: Positive numbers [K is app. 100]
- $\alpha$: Variable coefficient
- $T_{\ell}$: Transmission tariff of line $\ell$ [S/MWh]
- $LMP_k$: LMP at $k$th bus [S/MWh]
- $LMP_{ave}$: Total mean value of LMPs [S/MWh]
- $\sigma_k$: Standard deviation of pdf of LMP of line $k$ [S/MWh]
- $\sigma^2$: Variance of a defined pdf [S/MWh]
- $C_i(P_{gi}, Q_{gi})$: Cost of generated power of bus $i$ [S/h]
- $P_{gi}, Q_{gi}$: Active and reactive generated powers at bus $i$ [MW, MVar]
- $P_{loss \ell}$: Loss of line $\ell$ at base case [MW]
- $P_{loss \ell}$: Active and reactive loads at bus $k$ [MW, MVar]
- $P_{flow \ell}$: Power flow of line $\ell$ [MW]
- $v_k$: Voltage magnitude of bus $k$ [p.u.]
- $\delta_k$: Voltage angle of bus $k$ [rd]
- $AC_{\ell}$: Annual cost of line $\ell$ [S]
- $PVC_{\ell}$: Present value of construction for line $\ell$ [S]
- $PVL_{\ell}$: Present value of land for line $\ell$ [S]
- $PVO_{\ell}$: Present value of operation for line $\ell$ [S]
- $FCR_{\ell}$: Fixed charge rate of line $\ell$
- $FCRL_{\ell}$: Fixed charge rate of land for line $\ell$
- $CRF$: Capital return factor for line $\ell$
- $TE_{\ell}$: Total energy transmitted by line $\ell$ [MWh]
- $LTEC_{\ell}$: Levelized transmitted energy cost for line $\ell$ [S/MWh]
- $Y_1, Y_2, Y$: Random variables
- $U$: Random number
- $Z$: Normal random variable

I. INTRODUCTION

World-wide focus on competition and open access transmission networks has resulted in restructuring of power systems. Different goals such as providing resources, increasing efficiency and customer choice have been considered in numerous countries and therefore, various strategies have been adopted [1], [2]. Power industry and its related aspects have experienced dominant changes, consequently. However, transmission networks, as the interfaces between generation and load sections, have preserved their monopolistic characteristics in most power systems. They play a vital role in order to facilitate competition in the generation and electricity retailing. While reliability and stability of a system as the most important features should be considered in TEP, the plan should also provide fair and no discriminatory access to the system for all consumers.

Furthermore, there are many differences between TEP in vertically integrated utilities (VIUs) and competitive environments. Some of these important differences are as follows:

- In vertically integrated systems, TEP is considered only as a part of general expansion planning for the whole integrated systems, while in competitive environments it is normally an isolated expansion planning for the transmission systems [3].
- Comparing deregulated power systems with vertically integrated ones, there are a lot of uncertainties in input data of the new systems. Therefore, TEP in these
systems should be robust against those uncertainties [4]-[5].

- In the new environments, transmission service pricing has more impact on TEP [6].

Different models for solving TEP have been proposed. Generally, planning models are classified into three different categories including mathematical, heuristic and meta-heuristic optimization models [4].

As the TEP problem is stated as a large-scale, non-linear and non-convex optimization problem, heuristic or meta-heuristic optimization algorithms should be applied. This is due to the fact that such algorithms can reach to better solutions as compared with those obtained through classical techniques [7]. Based on the above mentioned description, various methods such as multi-objective planning [8], fuzzy algorithm [9], cooperative game theory [10], multi-agent coalition formation [11], non-linear mixed integer programming [12], genetic algorithm (GA) [7], [13], and locational marginal price (LMP) [5], [14]-[17] have been proposed so far for TEP in competitive environments. It should be noted that due to uncertainties, most publications emphasize on probabilistic approaches for TEP in competitive environments [13], [18].

Since, the pricing system for purchasing and selling the electric energy in competitive markets is based on nodal pricing or LMP, therefore methods based on LMP, which model the real condition of these environments are of more importance. LMP is the price of supplying an additional MW of load at each bus in the system, considering generator and load bidding prices, the transmission system components experiencing congestion, losses and the electrical characteristics of the system.

Generally, the previous methods proposed for TEP in competitive markets have two drawbacks as follows [4]:

- The network modelling does not consider the ohmic losses. Although the dc load flow decreases the calculation time, but the accuracy of optimal plans decreases, seriously. Furthermore, the importance of finding optimal plans in the planning stage, justifies the long time of ac OPF calculations.
- The operation costs are not taken into account by the planning algorithm. Usually these costs are calculated a posteriori considering a set of alternative plans that were selected.

In this paper the above mentioned weak points are eliminated in the TEP problem of competitive environments. The case study results show that the location of optimal plans will change in this case.

II. FUNDAMENTALS OF THE PROPOSED METHOD

Load, generation costs, bid of generators, power and bids of independent power producers (IPPs) are the main sources of random uncertainties in deregulated power systems. Probabilistic load flow (PLF) is used for modeling these uncertainties in regulated power systems. It models only the technical criteria. However, to achieve the objectives of TEP in deregulated power systems, market based criteria are needed in addition to the technical criteria. Also, to define and compute probabilistic market based criteria, it is need to calculate the pdfs of variables that market performance can be assessed by their analysis.

To facilitate a perfect and fair competition, electricity suppliers and consumers should have no constraint for bidding and offering the energy. Therefore, TEP in competitive electricity markets must encourage the competition and provide fair access to cheap generation.

A good approach to facilitate a fair competition is to expand the network in a way which flats the LMP profile as much as possible. Thus, flating the LMP profile is selected as planning criterion.

In the proposed method, pdfs of LMPs in all network buses are calculated using OPF based on PLF. As a result, the pdfs of output variables are computed instead of their accurate values.

Then, the buses are divided into source and sink sets based on their LMP mean values. Those buses at which mean of LMPs are smaller than the $LMP_{ave}$ are grouped in the source set and the rest are considered as the set of sink buses.

To reduce the number of lines nominated for expansion planning, only few buses among each of the above mentioned sets are selected to be connected through new lines. The criteria for choosing the nominated buses are as follows

- Buses in the source set

\[ LMP_{ave} - LMP_k > \alpha \sigma_k \]  

- Buses in the sink set

\[ LMP_k - LMP_{ave} > \sigma_k \]  

Finally, the option plan for expansion is characterized by new lines that should be constructed between any of the nominated buses from sink and source sets for flating the LMP profile, as much as possible.

To specify the flatness of a price profile, some indices are defined. In a network with $n$ buses, the pdf of LMPs have been computed for a given pdf for each input. Consider MLMP be a $1 \times n$ vector such that its $k$th element is the mean of LMP at bus $k$, and VLMP be a $1 \times n$ vector such that its $k$th element is the variance of LMP at bus $k$. The following parameters can be defined for determining the flatness of price profile [5].

- Mean of MLMP or $LMP_{ave}$: The less mean of MLMP indicates that cheaper generators are dispatched. This means a better condition for competition.
- Variance of MLMP: the smaller variance of MLMP indicates the flatter price profile and consequently better competition.
- Variance of VLMP: the smaller variance of VLMP indicates the more similar volatility of LMP at different buses and consequently the more similar risk in purchasing the power from different buses.

III. TEP PROBLEM FORMULATION

To formulate the TEP problem in a competitive environment, important features should be simulated and considered in their mathematical forms. The most important parameters are as follows

A. Uncertainties

Generation bid prices and load quantities are the major input uncertainties which have been modelled as normal pdfs.
B. Probabilistic LMP

As explained before, probabilistic LMPs are estimated using PLF. In fact, LMPs are calculated in their probabilistic forms using OPF. To do this, the objective function of costs including generation bids and transmission costs based on transmission tariffs is minimized subject to load, voltage and generation constraints. The objective function and AC OPF are modeled as follow.

\[
\text{Min } J_k = \sum_{i=1}^{G} C_i (P_{gi}) + \sum_{l=1}^{L} T_l P_{flow, l}
\]

S.T:
\[
\begin{align*}
\sum_{i=1}^{G} P_{gi} - \sum_{l=1}^{L} P_{loss, l} - \sum_{k=1}^{N} P_{di} &= 0 \\
Q_{gi} - Q_{di} - Q_i (v, \delta) &= 0 & i \in C \\
P_{gi}^m &\leq P_{gi} & \leq P_{gi}^M \\
Q_{gi}^m &\leq Q_{gi} & \leq Q_{gi}^M \\
|P_{flow, l}| &< P_{flow}^M \\
v_{k}^m &\leq v_{k} & \leq v_{k}^M \\
\delta_{k}^m &\leq \delta_{k} & \leq \delta_{k}^M
\end{align*}
\]

It should be noted that (4) as the loss equation can be written in terms of voltage and angle. However, presentation of this equation as an individual relation is only for emphasizing on its importance in the proposed model.

General structure of the proposed OPF for calculating LMPs is presented in Fig. 1. Its main features are as follows:

- Modeling line resistance and transmission tariffs in LMP calculations.
- \( kLMP \) is calculated by subtracting base case cost from increased cost, \((J_{increased, k} - J_{base, k})\), instead of using differentiation function, which is very complex, when the AC OPF is used.
- Using the well known loss coefficient method developed by Kron and adopted by Kirchmayer for loss calculation [19].
- Using the linear programming based on utilizing generation shift factors for adjusting flows in overloaded lines [20].

C. Transmission Tariffs

Justification of costs is very important in competitive environments. Therefore, transmission costs must be taken into account. In this paper for modelling the construction, operation, and maintenance costs of transmission lines, transmission tariffs are used. Transmission tariff is calculated according to the levelized transmitted energy cost (LTEC) as follows.

\[
AC_l = (1 + g)^{\ell} (PV_{C, i} FCR_l + PV_{L, i} FCRL_l + PV_{O, i} CRF_l)
\]

\[
TE_l = 8760 P_{flow, l}^M
\]

\[
LTEC_l = \frac{AC_l}{TE_l}
\]

\[
T_{l} = b LTEC_l
\]

At first, the annual cost of line \( \ell \) is calculated using (12) based on economic parameters. Then, \( LTEC_l \) is calculated dividing the annual cost by total energy transmitted for line \( \ell \). Finally, the transmission tariff for line \( \ell \) is obtained using (15).

IV. THE PROPOSED ALGORITHM

The proposed algorithm for TEP in competitive electricity markets is depicted in Fig. 2.
It should be noted that the algorithm is continued till the mean value of LMP of each bus is converged to a limit value. Therefore, the number of samples is variable with a range from 150 to 500.

Also, simulating normal random variables and selecting the magnitudes for input pdfs are performed applying the method as described in [21].

V. CASE STUDY

The proposed algorithm has been applied to a typical 8-bus and standard IEEE 30 bus networks [5], [22]. The 8-bus network structure and its related parameters are presented in Appendix. However, $R$ and $X$ parameters of the lines have been modified to be more realistic. Also, the related parameters of IEEE 30 bus network are presented in [22].

A. 8- Bus Network

Using the random generated samples from the pdfs of loads and generation bid prices, MLMP and VLMP vectors, and $LMP_{ave}$ for the base case are found in $$/MWh as follows:


$$LMP_{ave} = 21.0261$$

By comparing MLMP values at any bus in the network with $LMP_{ave}$, the sink and source buses and therefore, candidate lines are specified.

- Set of source buses $\{1, 3, 5, 6\}$.
- Set of sink buses $\{2, 4, 7, 8\}$.
- Transmission line candidates $\{1-2, 1-4, 1-7, 1-8, 3-2, 3-4, 3-7, 3-8, 5-2, 5-4, 5-7, 5-8, 6-2, 6-4, 6-7, 6-8\}$

Finally, based on the proposed algorithm, the optimal candidate lines for expansion are specified as $\{1-8, 3, 4, 5-4\}$ and $6-8\}$ (Table I).

It should be considered that as much as $\alpha$ is smaller, the candidate set of expansion buses will be bigger (Table II).

B. IEEE 30 Bus Network

Using the described method, MLMP and VLMP vectors, and $LMP_{ave}$ for the base case are found in $$/MWhr as:


$$VLMP = [12.02 10.43 17.98 13.45 23.53 15.92 13.45 21.11 20.63 30.66 19.38 23.34 23.17 27.94]$$

$$LMP_{ave} = 19.7233$$

- Set of source buses $\{1, 2, 3, 5, 7, 8, 10, 12, 13, 14, 15, 16, 20, 21, 27\}$.

- Set of sink buses

Error! Objects cannot be created from editing field codes.

- Transmission line candidates $\{1-4, 1-6, ..., 27-30\}$. This set has 225 members.

Finally, based on the proposed method, the optimal candidate lines for TEP are found as:

$$\{1-24, 2-19, 3-18, 5-26, 7-25, 8-11, 10-23, 12-6, 13-19, 14-28, 15-17, 16-30, 20-23, 21-17, 27-11\}$$

To investigate the validity of the proposed algorithm, all possible candidates in the 8-bus network are selected in another approach without applying any screening procedure (Table I). The results confirm that none of the optimal candidates are among those filtered candidates. The major conclusions of the proposed method are as follows:
TABLE I
FINAL RESULTS

<table>
<thead>
<tr>
<th>Line</th>
<th>Mean of MLMP ($/MWh)</th>
<th>Variance of MLMP ($/MWh)</th>
<th>Variance of VLMP ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>20.9538</td>
<td>3.9800</td>
<td>49.5549</td>
</tr>
<tr>
<td>1-4</td>
<td>20.9925</td>
<td>4.9366</td>
<td>66.343</td>
</tr>
<tr>
<td>1-7</td>
<td>20.9328</td>
<td>3.8316</td>
<td>19.9108</td>
</tr>
<tr>
<td>2-3</td>
<td>21.1863</td>
<td>3.8722</td>
<td>7.6887</td>
</tr>
<tr>
<td>3-4</td>
<td>20.6038</td>
<td>3.9237</td>
<td>4.4382</td>
</tr>
<tr>
<td>3-7</td>
<td>21.0808</td>
<td>4.8186</td>
<td>42.9999</td>
</tr>
<tr>
<td>3-8</td>
<td>20.6038</td>
<td>3.9237</td>
<td>4.4382</td>
</tr>
<tr>
<td>5-2</td>
<td>20.3250</td>
<td>2.7620</td>
<td>3.6992</td>
</tr>
<tr>
<td>5-4</td>
<td>20.2713</td>
<td>2.8295</td>
<td>2.0665</td>
</tr>
<tr>
<td>5-7</td>
<td>20.8450</td>
<td>4.9366</td>
<td>6.6343</td>
</tr>
<tr>
<td>5-8</td>
<td>20.4113</td>
<td>4.1499</td>
<td>2.2607</td>
</tr>
<tr>
<td>6-2</td>
<td>20.9863</td>
<td>3.1817</td>
<td>3.4456</td>
</tr>
<tr>
<td>6-4</td>
<td>21.3688</td>
<td>4.2670</td>
<td>4.1954</td>
</tr>
<tr>
<td>6-7</td>
<td>21.0525</td>
<td>4.1499</td>
<td>2.2607</td>
</tr>
<tr>
<td>6-8</td>
<td>20.5338</td>
<td>2.7085</td>
<td>0.9166</td>
</tr>
</tbody>
</table>

Other feasible candidate lines

1-3  | 21.8143              | 5.2022                   | 1.0007                   |
1-5  | 21.9547              | 5.0124                   | 8.0076                   |
1-6  | 21.0207              | 4.5565                   | 13.8966                  |
2-4  | 21.6741              | 5.4219                   | 0.4531                   |
2-7  | 21.6109              | 5.3909                   | 0.2233                   |
2-8  | 21.6573              | 5.1677                   | 0.3987                   |
3-5  | 21.4084              | 4.8034                   | 1.2227                   |
3-6  | 21.6183              | 5.0499                   | 2.1217                   |
4-7  | 21.0039              | 3.2660                   | 44.5071                  |
4-8  | 21.5231              | 6.5671                   | 13.9812                  |
5-6  | 21.3273              | 4.1896                   | 0.4850                   |
5-7  | 21.9871              | 6.9801                   | 10.2214                  |

TABLE II
DIFFERENT VALUES OF $\alpha$ AND OPTIMAL OPTIONS FOR TEP

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.00</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sink buses feasible candidates</td>
<td>1, 3, 5, 6</td>
<td>1, 3, 5, 6</td>
<td>1, 3, 5, 6, 7</td>
<td>5, 6</td>
<td>5, 6</td>
</tr>
<tr>
<td>Source buses candidates</td>
<td>2, 4, 7, 8</td>
<td>2, 4, 7, 8, 9</td>
<td>2, 4, 7, 8, 9</td>
<td>2, 4, 7, 8, 9</td>
<td>2, 4, 7, 8, 9</td>
</tr>
<tr>
<td>Candidate lines</td>
<td>28</td>
<td>16</td>
<td>16</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>Optimal options for each sink bus</td>
<td>1-8, 3-4, 5-4</td>
<td>1-8, 3-4, 5-4, 6-8</td>
<td>1-8, 3-4, 5-4, 6-8</td>
<td>1-8, 3-4, 5-4, 6-8</td>
<td>1-8, 3-4, 5-4, 6-8</td>
</tr>
</tbody>
</table>

TABLE III
LOADS DATA (\(\mu = 0.95\))

<table>
<thead>
<tr>
<th>Load No.</th>
<th>Bus No.</th>
<th>Load (MW) ((\mu = \sigma^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>(300, 10)</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>(300, 12)</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>(300, 15)</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>(300, 5)</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>(250, 9)</td>
</tr>
</tbody>
</table>

Calculations, the most suitable number of samples is around 250 and the best quantity for $\alpha$ is about 0.25.

Partial changes in some parameters such as power and profit factors may lead to essential changes in LMPs profile and consequently in final optimum plans.

It should be noted that a complete discussion on the numerical results of both case study models are presented in [22].

VI. CONCLUSIONS

Expansion planning for networks in competitive environments relies seriously on parameters which are not deterministic. In such power systems, there may be many uncertainties about load and generation in the network. Therefore, probabilistic and heuristic methods instead of classic approaches may be applied to get better solutions.

In this paper a new algorithm for TEP has been proposed. While the method uses pdfs of the parameters
instead of their accurate values, the priority criteria for the expansion plan is based on LMPs at different buses. This is due to the fact that as much as LMP values are uniform in a network, the competition is more fair and non-discriminatory.

The investigation confirms that modelling ac OPF and transmission tariffs in the TEP problem of competitive environments changes the optimal plans, basically. Also, while the method benefits acceptably calculation time, the results are reliable and optimal.

APPENDIX

Network Data

The data of the case study network which is depicted in Fig. 3 are presented in Table III to Table VII.

REFERENCES


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TABLE VII

<table>
<thead>
<tr>
<th>Line</th>
<th>R(p.u.)</th>
<th>X(p.u.)</th>
<th>Limit (MW)</th>
<th>Length (km)</th>
<th>Tariff ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>0.0375</td>
<td>0.2537</td>
<td>140</td>
<td>330</td>
<td>4.689</td>
</tr>
<tr>
<td>1-7</td>
<td>0.0360</td>
<td>0.2284</td>
<td>185</td>
<td>400</td>
<td>4.275</td>
</tr>
<tr>
<td>1-8</td>
<td>0.0301</td>
<td>0.2520</td>
<td>190</td>
<td>450</td>
<td>4.683</td>
</tr>
<tr>
<td>2-4</td>
<td>0.0127</td>
<td>0.1069</td>
<td>200</td>
<td>190</td>
<td>1.878</td>
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<td>390</td>
<td>250</td>
<td>1.267</td>
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<td>2-6</td>
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<td>0.19985</td>
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<td>350</td>
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<td>4.237</td>
</tr>
<tr>
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<td>190</td>
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<tr>
<td>3-6</td>
<td>0.03240</td>
<td>0.21636</td>
<td>180</td>
<td>360</td>
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<tr>
<td>3-7</td>
<td>0.02950</td>
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<td>140</td>
<td>250</td>
<td>3.351</td>
</tr>
<tr>
<td>4-6</td>
<td>0.02700</td>
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<td>180</td>
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<td>0.22840</td>
<td>185</td>
<td>400</td>
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<tr>
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<td>0.23920</td>
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