A Novel Pareto-Optimal Solution for Multi-Objective Economic Dispatch Problem

S. Muralidharan, K. Srikrishna, and S. Subramanian

Abstract—The modern power system has become very complex in nature with conflicting requirements. Heavy load demands and distributed generation compromise the cost of generation. Also the pollution hazards and inherent losses contribute towards an inefficient system. A scenario of coordination between cost, emission and loss will echo a pareto-optimal generation. The oft handled iterative approaches and optimization techniques have been replaced by a Dynamic Programming technique involving novel recursive approach for realizing production cost minimization, with an emission constrained and loss reduced condition. A study of test system exhibits the computational efficiency and accuracy of the solution.

Index Terms—Economic Dispatch (ED), emission constrained economic dispatch (ECED), emission dispatch (EmD), self adaptive dynamic programming (SADP).

I. INTRODUCTION

Economic dispatch (ED) is defined as the process of allocating generation levels to the generating units so that the system load is supplied entirely and most economically [1]. Many economic dispatch approaches [1], [2]-[17] have been proposed by various researchers to formulate and solve this problem. Reference [1] provided a review of the advances in this field. Equal incremental cost condition has been the criterion for economic scheduling of power generation among units in a plant [18]-[20]. Within a plant, in the absence of transmission loss, the incremental production cost of a plant exactly represents the incremental cost condition among units. In general, optimum plant cost equation can be represented by a second order polynomial using curve-fitting techniques. This formulation could also be done analytically. The solutions are of mathematical interest when applied for cost minimum condition.

Increased power generation and rise in pollution levels go hand in hand. Minimum emission condition provides only theoretical insight to the solution techniques. Hence, the plant cost equation must take into account economic power generation with emission restrictions. This leads to a small deviation in optimum production cost as obtained from economic generation condition. When the emission becomes controlled, the dispatch becomes Emission Constrained Economic Dispatch (ECED).

Various penalty costs of emission have been attempted here for chastising the emission released. Any dispatch neglecting transmission losses is a study by itself for isolated plants alone. In a multi-area, multi-plant system, the transmission loss in between plants is considerable. This forces a coordination to be adopted between cost, emission, and loss. Papers over the past two decades have highlighted very extensive iterative solution procedures involving

1. Formulation of total system equation
2. Differentiation of this equation
3. Incremental cost of received power
4. Curve fitting formulae
5. Iterative techniques without approximations

The best generation pursues a simultaneous realization of various objectives like cost, emission, and loss each at their minimum. Achieving an absolute minimum of all these three objectives concurrently in a real time system is a near impossibility. A best all-round solution attainable in such a case becomes the optimal solution for this multi-objective problem.

A summary of several algorithms of ECED is discussed in [2]. A combined handling of economic and minimum emission dispatch by introducing a price penalty factor is given in [3]. In this paper, the price factor coordinates the emission with actual fuel costs and this is approximated by a curve analysis. This price penalty factor and Back Propagation Neural Network solution is presented in [4]. To avoid local minimum and to achieve global minimum, the authors employ momentum, and adaptive learning rate. The Hopfield neural network approach [5] for optimal economic/environmental dispatching suffers from the number of iterations involved and the difficulty with the Hopfield coefficients.

A systematic procedure for determining the appropriate coefficients for the objective function has been solved using Fuzzy Logic [6]. Reference [7] presented an analysis of the performance of the Hopfield Neural Network methods for economic-emission dispatch problem. Sequential quadratic programming as a tool to solve the two-objective problem, after transforming it into a single objective using weighing factor is adopted in [8]. The choice of weighing factors reflecting the operator’s preference for one of the objectives influences the overall optimization procedure. The implementation of bi-objective generation dispatch by Genetic algorithm [9] is realized within the function by blending power balance equation into it. Multi-objective decision making problem has been attempted in [10], [11] using weighted min-max

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method. To get the optimal efficient dispatch from the non-
inferior set, fuzzy set theory has been applied. A new
 evolutionary algorithm for environmental/economic power
dispatch was proposed in [12], targeting a true multi-
objective problem with competing and non-commensurable
objectives.

The 60’s saw the introduction of Dynamic programming
as a tool for solving unit commitment. In the earlier
methods [13], [14], the commitment of generating units
was determined independently for every time period. A
heuristic truncated window DP [15], employs a variable
window size according to the load demand increments.
Fuzzy dynamic programming (FDP) approach with a fuzzy
objective function characterized by the fuzzy set related to
total cost is used in [16]. An iterative dynamic
programming method for solving economic dispatch
including transmission line losses [17] used a modified cost
function to obtain the optimal dispatch.

A conventional Dynamic programming procedure
consists of two parts:

1. An evaluation of all possible configurations from the
   beginning to the end of the problem, and
2. Back-track operation from the end to the beginning
   over the optimal path.

In contrast to this, Self Adaptive Dynamic Programming
(SADP) technique provides at once the generation schedule
of individual generators in an analytical form for a large
number of variables without the traditional Lagrange form.

Unlike linear programming, there is a dearth of standard
mathematical formulation for the Dynamic Programming.
Therefore, problem solving involves - developing the
functional equations for the problem and solving functional
equations for determining the optimal solution.

Increase in the number of states at each stage is the curse
of dimensionality in the literature of Dynamic Programming.
The result is spectacular in computational savings, if the state variables are three or less. Against this
background, it has been established that the format
developed in this paper can even be extended to higher
number of state variables with well-defined mathematical
approach. Hence, it has been aptly called Self Adaptive
Dynamic Programming (SADP) approach.

This paper presents SADP technique eliminating
common Lambda approach for loss included emission
constrained economic dispatch problem. The regular
quadratic cost equations, a penalty factor for emission and
a price factor for charging the transmission losses are
incorporated. A way of recording the impact of fossil fuel
based electrical energy production on human health and the
environment is achieved by estimating a monetary value
for the generated emissions. The blending of emission with
actual fuel costs is facilitated by the use of a price penalty
factor (F/E ratio). The price factor (g), which monetizes
the transmission loss, has been defined as the ratio of the cost
of generation to the power of generation (F/P ratio). This
analytic concept gives encouraging results very closely
following the conventional methods. In this work, a plant
having six generators has been considered for study.
Proposed algorithm can also be extended for a
larger system.

II. DYNAMIC PROGRAMMING

Programming (DP) is a mathematical technique [21]-
[22] dealing with the optimization of multistage decision
process. The word ‘programming’ is used in the
mathematical sense of selecting an optimum allocation of
resources and it is ‘dynamic’ as it is particularly useful for
problems where decisions are taken at several distinct
stages. Discrete, continuous, deterministic, as well as
probabilistic models can be solved by this method.

A system in its initial state, described by a vector $s_N$, 
finally reaches the state $s_0$ as a result of certain decisions
denoted by the vector ‘d’. The transformation $T_N$ can
be functionally explained as $s_0 = T_N(s_N,d)$. Let a real valued
function $\psi(s_N,d)$ called the objective or the return
function be associated with the transformation ($T_N$) which
measures the effectiveness of the decisions made and the
output that results from these decisions. The objective is
to determine a given input $s_N$ to optimize (minimize or
maximize) $\psi(s_N,d)$ subject to the constraint $s_0 = T_N(s_N,d)$.

This multistage problem is decomposed into ‘j’ stages,
where $1 \leq j \leq N$, and $s_j$ represents the input at the $j^{th}$
stage. Starting from the initial state $s_N$, the system is
classified to pass through successive states
$s_N,s_{N-1},s_{N-2},s_{N-3}, ..., s_2,s_1$ before reaching the final state
$s_0$. Thus each state $s_{j-1}$ is the function of the input state
$s_j$ and the decision vector $d_j$, i.e. $s_{j-1} = T_j(s_j,d_j)$. There
results a stage return function $f_j(s_j,d_j)$. In
addition, the return function $\psi_N$ is a function of stage
returns, i.e., $\psi_N = \psi_0(f_N,f_{N-1},...,f_2,f_1)$. From the
above discussion, it would seem to suggest that if $\psi_N$ is of the
form $\psi_N = f_N \circ f_N \circ f_{N-1} \circ f_{N-2} \circ f_1$ where “$\circ$” represents
a composition operator indicating either addition or
multiplication, then $\psi_N = f_N \circ \psi_{N-1}$, where
$\psi_{N-1} = f_{N-1} \circ f_{N-2} \circ f_2 \circ f_1$. It is possible to separate all
$\psi_N, \psi_{N-1},...\psi_2$ successively in this order, and thus the
recursive equation may now be proposed as

$$F_i(s_i) = \min [f_i(s_i) \circ F_{i+1}(s_{i+1})], 2 \leq i \leq N$$

With $F_i(s_i) = \min_d d_i$, subject to $s_{i+1} = T_j(s_i,d_j)$, $2 \leq i \leq N$.

This type of approach constitutes the backward
recursion. This backward recursion can be conveniently
used only when optimization with respect to a specific
input $s_N$ is needed, and the output $s_0$ is not considered.

To optimize the system with respect to a prescribed
output $s_0$, it would therefore be natural to reverse the
direction, i.e. treat $s_j$ as the function of $s_{j+1}$ and $d_j$, and
substitute $s_j = T_j(s_{j+1},d_j)$, $1 \leq j \leq N$. Also express stage
returns as functions of stage output and then proceed from
stage N to stage 1. Such a procedure, known as the forward
recursive approach is adopted in this paper.

III. FORMULATION OF ECED PROBLEM WITH LIMITED
LOSSES

A final operating condition with minimum production
cost at reduced emission rate while achieving an acceptable
loss value leads to a multi-objective problem which is dealt
successfully in this work.

In general, an emission constrained economic dispatch
problem starts with a mathematical cost equation (1) [$/hr]$, which
is modeled to represent each individual generator in
terms of generation and cost coefficients and a
mathematical emission equation (2) [kg/hr], which is
formulated to relate the emission coefficients with the individual generation.

\[ F_i(P_i) = a_i P_i^2 + b_i P_i + c_i \]  
(1)

\[ E_i(P_i) = d_i P_i^3 + e_i P_i + f_i \]  
(2)

where, \( P_i \) is the individual generation from unit ‘i’; \( a_i, b_i \) and \( c_i \) are its cost coefficients and \( d_i, e_i \) and \( f_i \) are its emission coefficients. Transmission losses are given by \( P_i \) where \( P_i = P_i B_i P_i \). Here P and B are in the form of matrices representing power generation and transmission loss coefficients respectively. And P is the transpose of P. A load balance equation will impose constraint upon generation as

\[ \sum_{i=1}^{n} P_i - R_i - P_a = 0 \]  
(3)

where \( P_a \) is the total system load demand and \( P_i \) is the transmission loss.

A generation limit will also act as a constraint over the operating range of individual generators i.e.

\[ P_{min} \leq P_i \leq P_{max} \]  
(4)

An appreciable increase in the volume or weight of emission is governed by the magnitude of generation, which in turn governs the cost and hence the economical operation of the system. These costs are coordinated with a penalty cost of emission (h), given by

\[ h = F_i/E_i \]  
(5)

where \( F_i, E_i \) are the cost and emission respectively corresponding to specific conditions of generation including the limits of generation (minimum and maximum) and average costs of generation as given below and discussed fully in [23]

\[ h_{max} = F_{max}/E_{max}, \quad h_{min} = F_{min}/E_{min} \]

\[ h_{ave} = (h_{max} + h_{min})/2, \quad h_{com} = \left(\sum_{i=1}^{n} h_{ave} \right)/n \]

For a sample three generator system, the emission constrained cost equation [S/hr] is

\[ f_i = \sum_{i=1}^{3} \left( (a_i P_i^2 + b_i P_i + c_i) + h_i (d_i P_i^3 + e_i P_i + f_i) \right) \]  
(6)

The cost of transmission losses in between the plants is accounted with the actual fuel costs by a price factor \( g \) which is taken as

\[ g_i = F_i/P_i \]  
(7)

where, \( F_i \) & \( P_i \) are in turn the cost and generation corresponding to \( i \) th generator for specific conditions of generation including the limits of generation and the average costs of generation as given in (8).

\[ g_{max} = F_{max}/P_{max}, \quad g_{min} = F_{min}/P_{min} \]

\[ g_{ave} = (g_{max} + g_{min})/2, \quad g_{com} = \left(\sum_{i=1}^{n} g_{ave} \right)/n \]  
(8)

For a sample three-generator system, modified form of cost equation [S/hr] now becomes

\[ f_i = \sum_{i=1}^{3} \left( (a_i P_i^2 + b_i P_i + c_i) + h_i (d_i P_i^3 + e_i P_i + f_i) \right) \]

\[ = \sum_{i=1}^{3} \left( a_i P_i^2 + b_i P_i + c_i + g_i (P_i B_i P_i) \right) \]  
(9)

where, \( a_i' = (a_i + h_i d_i) \), \( b_i' = (b_i + h_i e_i) \) and \( c_i' = (c_i + h_i f_i) \). Now the loss formula for the first generator can be modified as

\[ P_i = P_i B_i P_i = P_i B_1 P_i + P_i B_2 B_3 (P_i / P_i) + P_i B_3 B_2 (P_i / P_i) \]

\[ = P_i B_1 (a_1' + a_2' + a_3') \]

\[ = P_i B_1 (a_1' + a_2' + a_3') \]  
(10)

where, \( B_1' \) is the modified form of self-coefficient.

Using this, cost equation [S/hr] for the first generator can be rewritten as

\[ f_i = (a_1' P_i^2 + b_1' P_i + c_1' + g_i (P_i B_1' P_i)) \]

\[ = (a_1' + g_i B_1') P_i^2 + b_1' P_i + c_1' \]

Substituting \( a_i = a_i + g_i B_i', \) we arrive at the cost equation for the first generator as

\[ f_i = (a_i' P_i^2 + b_i' P_i + c_i') \]  
(11)

Similar cost equations can be derived for the second and the third generators respectively.

The whole formulation as observed is purely analytic in nature with high possibility for accurate solutions. A best choice is chosen for the penalty factor of emission and the price factor of loss. An optimum penalty factor for emission arrived at a previous paper [24] is made use of in this work.

IV. IMPLEMENTATION OF RECURSIVE APPROACH TO ECED PROBLEM

Let \( s_i \) be the output from the \( i \) th stage of this multistage problem. Considering the three-stage system, \( s_3 \) is the output from the third stage and it equals \( P_1 + P_2 + P_3 \). Outputs from earlier stages are respectively, \( s_2 = P_1 + P_2 = s_3 - P_3 \) (for the second stage) and \( s_1 = P_1 = s_2 - P_2 \) (for the first stage). Now

\[ f_i(s_i) = \min_{0 \leq P_1 \leq P_{min}} (a_i' P_1^2 + b_i' P_1 + c_i') \]  
(12)

Since \( c_1', c_2' \) and \( c_3' \) are constants, they are removed from the respective equations and their sum can be added to the cost equation [S/hr] at the end.

\[ f_2(s_2) = \min_{0 \leq P_2 \leq P_{min}} (a_2' P_2^2 + b_2' P_2 + f_i(s_2)) \]

\[ = \min_{0 \leq P_2 \leq P_{min}} (a_2' P_2^2 + b_2' P_2 + a_i (s_2 - P_2)^2 + b_i (s_2 - P_2)) \]  
(13)

For the second generator, minimum is attained when the above equation (13) is differentiated with respect to \( P_2 \) and equated to 0. This gives the value of \( P_2 \) in terms of \( s_2 \) and constant, i.e.

\[ P_2 = A_2 s_2 + B_2 \]  
(14)

where \( A_2 = 2a_i'/(2a_i' + 2a_i') \) and \( B_2 = (b_i - b_i')/(2a_i' + 2a_i') \)

Similarly for the third generator,
As Bith generator under optimum condition. This procedure can also be suitable for loss included matrix, which has made the discussed dynamic triangularization has been adopted for the loss coefficient technique to achieve this simple analytical form. A transmission loss have helped the suggested recursive flowchart in Fig. 1. High accuracy and the same is aptly demonstrated by a point using the above technique is found for the emission constraint economic dispatch condition, which neglects all the three objectives mentioned in this paper, to be at their best possible values. Since constrained economic dispatch condition with loss, using modified form of B-coefficient matrix [18]. In the subsequent approach, a few iterations are required so that the same format suits the total generation with loss inclusion. Thus, an all round satisfactory performance forms the basis of system planning. The best performance addresses all the three objectives mentioned in this paper, to be at their best possible values. Since simultaneous realization of their minima is impossible, the comparison chart helps to find a near optimal solution satisfying multi-objective criterion, with a small deviation from their individual minima. As a subsequent process, various values are considered for price factor of loss. This is done on a systematic basis wherein the cost comparison, emission comparison and loss comparison are performed. The comparison chart has been proposed for an overall multi-objective performance monitoring.

V. RESULTS AND DISCUSSION

A study is made on IEEE six-generator, 30-bus test system [2], [6], [9] with cost and emission coefficients and power limits as given in Table I and loss coefficients taken from Table II.

Cost, emission and loss results, for a sample load of 800 MW obtained by recursive method for various price factors are presented in Table III. A single price factor (g) does not provide a solution for the best configuration, where all the three objectives are at their minimum. It necessitates a comprehensive study about various price factors (8). This table also helps to identify the price factor with which compromise between the objectives are satisfied. Comparison of total cost, total emission and total loss (for 900 MW) under ECED condition with loss restriction for various price factor combinations are graphically presented in Fig. 2. This pictorial representation simplifies the task of identifying an all round solution. Results obtained for 700 MW using recursive approach have been compared with that arrived through conventional Lambda iterative method and quick method [4], in Table IV. Accuracy of the proposed algorithm has been

### Table I

<table>
<thead>
<tr>
<th>Unit</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
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<td>756.79886</td>
<td>0.00420</td>
</tr>
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<td>0.10587</td>
<td>46.15916</td>
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### Table II

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<th>Loss Coefficients for Six-Generator System (×10^5)</th>
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<td>140</td>
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<td>17</td>
</tr>
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### Table III

<table>
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<th>Cost, Emission and Loss (for 800 MW) with Varied Price Factors</th>
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<td>Cost ($/hr)</td>
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<tr>
<td>-------------</td>
</tr>
<tr>
<td>gmin</td>
</tr>
<tr>
<td>gmax</td>
</tr>
<tr>
<td>gmed</td>
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### Table IV

<table>
<thead>
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<th>Comparison of Results for Six Generator System (for 700 MW)</th>
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<tr>
<td>---------------------</td>
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<tr>
<td>P1 (MW)</td>
</tr>
<tr>
<td>P2 (MW)</td>
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<tr>
<td>P3 (MW)</td>
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<tr>
<td>P4 (MW)</td>
</tr>
<tr>
<td>P5 (MW)</td>
</tr>
<tr>
<td>P6 (MW)</td>
</tr>
<tr>
<td>Total cost ($/hr)</td>
</tr>
<tr>
<td>Total Emission (kg/hr)</td>
</tr>
<tr>
<td>Total Loss (MW)</td>
</tr>
</tbody>
</table>
Fig. 1. SADP Flowchart.

Fig. 2. Comparison of total cost, total emission and total loss (for 900 MW).

The cost and emission under ECED condition are compared with their matching parts under pure economic dispatch and pure emission conditions in Figs. 3 and 4. These graphs illustrate the fact that the proposed algorithm provides a best compromise in cost and emission, since minimum of both cannot be achieved simultaneously.

Fig. 5 clearly projects the reduction in the number of iterations required for the proposed approach when compared with the conventional iterative method. Fig. 6 visually presents the CPU time requirement for the SADP approach.

VI. CONCLUSION

A novel form of dynamic programming technique is presented in this paper. All the three factors of power dispatch viz. cost, emission and loss are combined and the pareto-optimal economic dispatch for emission constrained and loss-restricted case is the objective of this paper. The twin objectives of cost and emission are conflicting in nature and a compromise has to be reached to obtain an acceptable power dispatch strategy within the various system constraints.

Minimum cost, minimum emission, and minimum loss conditions are individually of theoretical fancy and are not a simultaneously realizable phenomenon. Therefore, any economic power dispatch problem must consider emission properties and loss inclusion. Emission limits, must be
satisfied even though there is a slight deviation with regard to economy. An attempt to reduce loss is mandatory.

While achieving this, the regular long and laborious conventional Lambda approach involving incremental production cost and incremental transmission losses is totally avoided. In spite of this, the realization of total cost production and incremental transmission losses is obtained.

From Table III, the price factor $\varepsilon_{\text{min}}$ is found to be an acceptable solution in reality. It is hoped that the variation in the penalty cost of emission for emission restriction and variation in price factor in charging transmission losses can provide better performance solution. This requires an in-depth study for an overall comparison and thus refinement of economic generation may be a realistic achievement.

REFERENCES


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