Abstract—Orthogonal frequency division multiplexing (OFDM) is an effective technique for high bit rate applications in multipath channels. Many researches have been carried out to develop and enhance OFDM efficiency. Because of the special structure of OFDM, the channel coding is very important. In this paper, channel coding in OFDM, the efficiency of various codes such as convolutional code, turbo code and their concatenated components with Reed-Solomon code are studied. The techniques of improving the effectiveness of channel coding with soft decoding and the use of channel state information are presented with simulation results. It can be seen that in a high signal to noise ratio, the coding gain of concatenated codes is more than the pure codes. The coding scheme can be chosen according to the complexity, suitable delay, and desired coding gain for the system.

Index Terms—Channel state information, channel delay spread, concatenated codes, OFDM.

I. INTRODUCTION

The recent development of multimedia communication including voice, image, and data in computer networks such as internet and the increased request for usage of mobile services, require the design of such systems that are effective enough to send high bit rates on channels with fading. A suitable choice for full capacity wireless networks is multicarrier modulation, especially orthogonal frequency division multiplexing (OFDM). Many standards like the digital audio broadcasting (DAB) and IEEE 802.11a for wireless local area network (WLAN) networks and other ones, use OFDM as a suitable solution for their tasks [1].

OFDM is very efficient in transmitting high bit rate in multipath channels. It has also other advantages such as encountering impulse noise and narrow band interference. Although large maximum to average power ratio, sensitivity to frequency offset, and hardware complexity are its major drawbacks, some methods are available to alleviate these problems. For example, the use of fast Fourier transform (FFT) and its inverse (IFFT) in the receiver and transmitter considerably reduces its complexity. As a special characteristic, the applied channel coding in OFDM has a higher gain than that of single carrier signals. In this paper, channel coding in the system is explained and its performance is evaluated.

II. THE CODED OFDM SIGNAL

Although different methods have been applied to enhance the efficiency of OFDM, this technique might still have undesirable performance in multipath channel environments [2]. The reason is that the channel frequency response of various subcarriers is not the same and therefore different subcarriers are received by different signal to noise ratios. In other words, some subcarriers are received by low signal to noise ratio and their information is reconstructed with error. On the other hand the signal to noise ratio related to many of the subcarriers is high and their information is not detected with error. Since the damaged subcarriers are usually along the bandwidth, the error probability of the system, defined by damaged subcarriers, is significant. The solution for this problem is the use of channel coding by which the error of weak subcarriers is corrected by the information of the strong subcarriers. Thereby the total error rate is decreased considerably. Using a strong channel coding limits the system efficiency to the received average power. If the channel coding is not used, the system efficiency is limited to the power of the weakest subcarrier. Because of the special structure of OFDM, the application of channel coding in OFDM, which is called coded OFDM (COFDM) [3], is more efficient compared to single carrier signals. The properties of COFDM are explained in the following two sections.

A. Applying Frequency Diversity by Channel Coding

Due to the different channel frequency response in different frequencies, each subcarrier experiments a different frequency response. Therefore the received signal power of each subcarrier is different [2]. If the transmitted power is defined by $P_{st}$ and the channel frequency response in central frequency $f_i$ related to the $i$-th subcarrier by $H(f_i)$, the received power can be expressed as

$$P_r = P_{st} |H(f_i)|^2.$$  \hspace{1cm} (1)

Therefore, the bit error probability of each subcarrier, which is a function of its received power, is not the same. The error correction is achieved by coding nonadjacent subcarriers which are non-correlated in their frequency response regarding their frequency and time. Indeed, the channel coding, using the accuracy of the information of strong subcarriers, corrects the information of the weak subcarriers, if received with errors. This is the same as using frequency diversity to encounter frequency selective fading [3].
B. Using Channel State Information in Soft Decoding

In OFDM, besides soft information, there is another information about reliability of bits to be used in soft decoding which is channel state information (CSI) [4].

As it was noted, frequency response is different for various subcarriers and therefore the received power of each subcarrier is different. Because bit error probability in each subcarrier is proportional to its received power, the bit error probability of each subcarrier is not the same. If the channel frequency response is known at the receiver, a reliability metric can be estimated for each subcarrier, which is applied to the soft decoding bits corresponding to that subcarrier. Subsequently, the decoder has more information about the reliability of input bits and, as a result, the output bit error probability of the decoder will decrease. For example in Viterbi decoder, the metric for each branch is given by

$$\Delta M(r_i / v_j) = \log(p(r_i / v_j)),$$

where $r_i$ is the received sequence of bits at $i$-th time interval and $v_j$ is the transmit sequence of bits at $i$-th time interval of the $j$-th path of the decoder and $p(.)$ is the joint probability of the sequence in each time interval. For example, the number of bits in each time interval for a coding rate of 1/3 is equal to 3.

The survivor path will be the one that satisfies

$$\max_j \sum_i \log(p(r_i / v_j))) = \max_j \left[ \log \left( \prod_i \sum_k \frac{1}{2 \pi \sigma_n^2} \exp \left( \frac{-d_n^2}{2 \sigma_n^2} \right) \right) \right]_{i,j} = \min_j \left[ \sum_i \frac{\sigma_n^2 (\sigma_n^2 - \sigma_{ij}^2)}{\sigma_{ij}^2} \right]$$

(3)

where $\sigma_n^2$ is the variance of noise in $i$-th bit and $j$-th time interval. Although the variance of actual noise in subcarriers is constant and the power of received signal is different, it can be assumed that the power of received signal is constant and the power of noise is variable

$$\frac{S}{N} \frac{E(s_j^2)}{E(n_j^2)} = \frac{P_s}{\sigma^2} = P_s \left| H(f_j) \right|^2,$$

(4)

where $|H(f_j)|$ is the frequency response of channel at central frequency of subcarrier which sends $ij$-bit. $P_s$ is the power of received signal according to $ij$-bit and $P_s$ is the power of transmitted signal which is the same for every bit and does not depend on frequency response of channel. The noise of the bits will be different and the power of received signal in bits is the same if the variance of noise defined as

$$\frac{1}{\sigma_{ij}^2} = \frac{P_s \left| H(f_j) \right|^2}{\sigma^2}.$$

By substituting (5) in (3) the following result will achieve

$$\max_j \left[ \sum_i \frac{\sigma_n^2 (\sigma_n^2 - \sigma_{ij}^2)}{\sigma_{ij}^2} \right] = \max_j \left[ \sum_i \frac{\sigma_n^2 (\sigma_n^2 - \sigma_{ij}^2)}{\sigma_{ij}^2} \right]_{i,j} = \min_j \left[ \sum_i \frac{\sigma_n^2 (\sigma_n^2 - \sigma_{ij}^2)}{\sigma_{ij}^2} \right]$$

(6)

So before decoding, received vectors are multiplied by squared frequency response and then applied to the decoder as a received vector. In this case the importance of the channel state information or $|H(f_j)|^2$ is expressed in soft decoder. This information defines survivor path more carefully and reduces the output error probability.

III. EFFECT OF CHANNEL DELAY SPREAD ON THE ERROR PROBABILITY OF CODED OFDM SIGNAL

As it was mentioned before, in an uncoded OFDM signal, an increase in delay spread, greater than the guard time interval, causes an increment in error probability. The reason is the increase ofICI and ISI errors. But in coded OFDM, a parameter called code diversity gain is affected by channel delay spread which is important in error probability of the system. Code diversity gain is a function of channel delay spread. Increasing the delay spread improves the code diversity gain, because shorter delay spread corresponds to greater convergence bandwidth [1]. Short delay spread can create deep fading for several adjacent subcarriers, and so there is no frequency diversity because of the same channel frequency response. Therefore the channel coding gain decreases. By increment of the channel delay spread, channel frequency will be more selective, frequency diversity increases and also bit error rate is reduced until ICI and ISI errors, resulting from deviation of delay spread and guard time interval, increase the system’s error rate. The third factor of the limitation of error probability is received signal to noise power ratio, such that with more increment in delay spread, the ratio will be more limited. By simulation, the effect of channel delay spread on bit error rate will be explained.

Fig. 1 shows the bit error rate of the OFDM signal with 64 carriers and with a convolutional code of rate 1/2 and soft decoding without interleaving. The modulation used is DPSK. Signal to noise ratio is equal to 3dB and guard time interval is 50% of symbol duration and in form of cyclic prefix.

![Fig. 1. BER of COFDM.](http://www.SID.ir)
In order to show the real effect of channel delay spread on the error probability of COFDM, there is no interleaving in this simulation. In part (a) the main limitation of error is the channel diversity and more delay spread causes more diversity gain and then the bit error rate is reduced. In part (b) the main parameter of limitation is the channel signal to noise power ratio. The error rate is constant and approximately independent of delay spread. In part (c) the system error rate is limited by ICI and ISI errors and more delay spread increases ICI and ISI errors, so the system error rate increases. The increment of power signal in comparison to noise reduces the limitation of signal to noise power ratio and therefore part (b) is shorter and parts (a) and (c) are longer, respectively. As mentioned before, for showing the real effect of channel delay spread on the error probability of coded OFDM, there is no interleaving in our simulations.

IV. CONCATENATED CODES

Instead of using a single block code or convolutional code, it is also possible to combine or concatenate two codes [5]. The main advantage of a concatenated code is its larger coding gain and less hardware complexity compared to a single code. Usually in this system, the inner code is a convolutional code and the outer one is a block code, for example a Reed-Solomon code. The reason is that convolutional coding can better correct random errors, while Reed-Solomon code cleans up the relatively few remaining errors in the decoded output of the convolutional decoder. The task of the outer and inner interleaver is to break up bursts of errors as much as possible. Compared with a single-coded system, concatenated coding has more delay because of extra interleaving, encoding and decoding which is a disadvantage for packet communications. There are several other combinations of codes to construct a useful one, such as parallel-concatenated codes and turbo codes [1].

V. SIMULATIONS

The parameters used in simulations are chosen according to IEEE 802.11a standard for WLAN systems. The channel model is designed as a weighted tapped delay line with Rayleigh distribution for each delay and exponential distribution for the power of line factors, which is a very popular model [6]. The protection time interval is a cyclic prefix and its duration is 25% of symbol duration. Because differential modulations are used, there is no need for channel estimators. Block interleaving with length of 52 bits is done in frequency domain over subcarriers. In order to extract information from the channel, the impulse response of the channel is used. It is assumed that the practical receiver can fully estimate the channel frequency response using the information of pilot symbols.

The high performance of turbo code in AWGN channels suggests the idea of analyzing its performance in OFDM signals and multipath channels using the channel information. In Fig. 2 a comparison between the error probability of turbo code using channel state information and convolutional code in the same condition is made. As it can be seen, both codes represent the same bit error probability and hence the convolutional code is preferred due to less complexity. The improper performance of turbo code in a Rayleigh channel versus AWGN channel can be explained this way that in AWGN channels there are no burst errors, therefore the possibility that both decoder receive a number of serial errors and follow an invalid path is very low. However, in a multipath Rayleigh channel there is another condition and the errors may occur in burst form. If one of the decoders loses the correct path, it can find it again using the other decoder, but if both decoders lose the correct path, then more time is needed to get back to the correct path due to the feedback they have on each other and that is why the burst error will be longer. So, turbo code does not present a proper performance in multipath channels.

As Fig. 2 shows, it seems that although the bit error probability in convolutional and turbo code is the same, but as mentioned above the packet error probability will be less in a turbo code. In packet switching networks, packets with at least one bit error are dropped and are preferred over convolutional code because of their lower packet error probability.

Fig. 3 shows a comparison between error probability diagram of a concatenated RS-CC code and a convolutional code.

The soft decoding of convolutional code in both cases is done using the channel state information. The combine
code rate of RS-CC approximately is equal to the code rate of convolutional code so that a fair comparison is made. As it can be seen, the gain of concatenated code is more than convolutional code, and hence the importance of Reed-Solomon code can be easily noticed.

The combination of turbo coding and Reed-Solomon coding is compared with pure turbo code in Fig. 4. It is clear that the coding gain in concatenated RS-TC code is more than turbo code. The reason is that the Reed-Solomon code corrects the remaining burst errors of turbo code to a great deal and reduces the error probability of the system. As it can be seen in a signal to noise ratio lower than 6dB the performances of both codes are the same, the reason is that the output error of the turbo code decoder is more than the correction ability of the RS code. RS code does not spread error, while other codes like convolutional and turbo code present a negative coding gain if the probability of uncoded error is less than a specific level.

The behavior of RS code with the output errors of turbo and convolutional code is supposed to be different, however the curves of error probability of RS-CC and RS-TC do not show significant difference. The reason is that although the average length of burst error in the two codes is different, RS code can precisely correct burst errors with specific length and if the average length of the output error of turbo or convolutional decoders is outside of this region, the performance of the RS code and the distribution of the length of burst errors in convolutional and turbo code and the size of interleaving block is very useful in the design of concatenated codes. Also the difference between the average error length in turbo and convolutional codes can make more latitude in selecting the RS code and the length of interleaving block. In summary it can be said that the use of any of these two codes in OFDM signal does not have any significant priority on the other one and any of these two codes can be used in specific conditions.

VI. Conclusion

As it is shown in this paper, use of soft decoding, channel state information and concatenated codes increase the channel coding gain. The coding gain of turbo code is the same as convolutional code and the error probability curves of RS-CC and RS-TC codes do not show significant difference for OFDM in multi-pass channel. Also turbo code method has less packed error probability than convolutional code. For networks with packed switching, the packed error probability is one of the most important problems and using turbo-codes is preferred in these networks. So a combination of the above techniques could be used according to the complexity, suitable delay and desired coding gain for the system.

REFERENCES


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