Modelling and Simulation of a Class of Duty-Cycle Modulators for Industrial Instrumentation

J. Mbihi, B. Ndjali, and M. Mbouenda

***Abstract*—In this paper, a class of new Duty-Cycle Modulators (DCM) is investigated. Although the duty-cycle involved in each case is a nonlinear function of the control signal, a numerical analysis indicates that one could develop an excellent linear approximation for an appropriate choice of design parameters. The class of DCM proposed combines duty-cycle modulation and analog signal processing techniques in a single operational amplifier circuit. Simulations are conducted under constant and variable controls using Electronic Work Bench in order to test and validate the proposed class of modulation circuits. As an implication, the class of signal conditioning systems investigated in this study will have a number of applications in industrial instrumentation.**

***Index Terms*—Duty-cycle Modulation (DCM), nonlinear instrumentation, simulation.**

I. Introduction

The operational amplifier is widely used in Electronics for signal processing, signal generation, active filtering as well as signal conditioning. Despite an impressive literature available on the operational amplifier circuits and systems (see [1]-[3]), it turns out that the use of operational amplifiers for duty-cycle modulation has not been investigated in depth.

The duty-cycle modulation is used in instrumentation for signal conditioning from a sensor to a processing station even under a stochastic environment. It is a special voltage control oscillator relying on the fact that the duty-cycle $R_m(u)$ is a function of the input control voltage u. Although the function $R_m(u)$ is generally nonlinear, it is equivalent to its linear approximation around a given operating point, in which case a linear filter could be used for the demodulation process.

The well known duty-cycle modulator (DCM) encountered in Electronics literature is the inverting DCM [1], [2]. When it is used in applications for which the inverted signal is not needed, an additional inverter circuit is needed before or after demodulation. In such cases, a non-inverting DCM could be a suitable choice if any. Both basic modulators could be combined to obtain new modulation issues using a single operational amplifier. These ideas have not been investigated in the literature.

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In this paper, an important class of DCM circuits is considered. It consists of a standard inverting modulator and the following new modulator circuits: a non-inverting modulator, a summing modulator and a difference modulator. In Section II, the proposed class of DCM is described and analysed for constant controls u. In Section III, the results of our analysis are extended to the case of variable controls using approximations. In section IV, the proposed DCM are simulated and tested under constant and variable controls. Finally, the paper is concluded in Section V.

II. Description And Analysis of the Proposed Class of DCM

A. Description of the Proposed Class of DCM

Since a DCM circuit behaves structurally as a voltage control oscillator, it consists conceptually of a modulator involving a high frequency relaxation oscillator and a low frequency control signal u to be modulated. Hence, the modulation technique involved is founded on an appropriate control of switching times associated with the resulting modulated voltage. Consequently, the resulting duty-cycle $R_m(u)$ is a function of u.

Following the previous description and using the operational amplifier technology, we propose the class of DCM shown in Fig. 1. Fig. 1(a) is a standard inverting DCM available in the literature [1], [2]. As it will be shown latter, the non-inverting circuit proposed in Fig. 1(b) is a new alternative solution for modulation problems. The new circuit proposed in Fig. 1(c) is a summing DCM. It is a new DCM structure providing averaging and modulation. Finally, the difference DCM proposed in Fig. 1(d) is also a new issue resulting from an intuitive association of both inverting and non inverting modulators. It is useful for practical problems requiring differencing and modulation.

For the sake of clarity, we shall consider here and elsewhere the following notations

$$\alpha_1 = \frac{R_1}{R_1+R_2},\ \alpha_2 = \frac{R_2}{R_1+R_2},\ \alpha_a = \frac{R_a}{R_a+R},$$
$$\alpha = \frac{R}{R_a+R},\ E_0 = \pm E, \tau_a = R_a C,\ \tau_b = R_b C,\ \tau = R\,C \quad (1)$$

where E and $-E$ are saturated positive and negative voltages, respectively.

B. Analysis of the Proposed Class of DCM for Constant Controls

In Fig. 1, the operational amplifier is used typically in all cases as a switching device since it operates under a

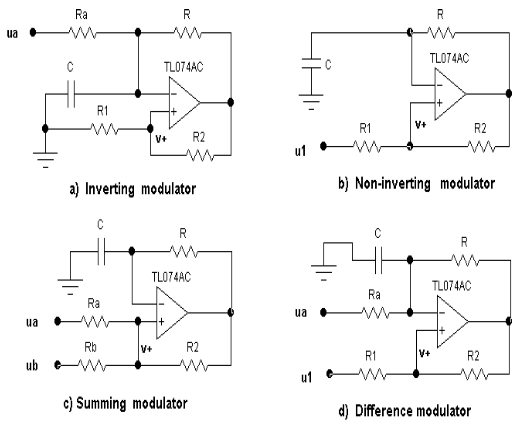


Fig. 1. A class of duty-cycle modulators (DCM).

positive feedback. As an implication, the output voltage switches each time the capacitor voltage tries to cross each of the thresholds associated with v+.

1) Case of a Standard Inverting DCM

For the standard inverting DCM shown in Fig. 1(a), while the associated switching thresholds associated with v+ are fixed and equal to

$$\begin{cases} u_M^+ = \alpha_1 \; E \\ u_m = -\alpha_1 \; E \end{cases} \quad (2)$$

the capacitor voltage evolves according to the linear differential equation

$$RC\frac{du_c}{dt} + \frac{R+R_a}{R_a}u_c = \frac{R}{R_a}u_a + E_0 \quad (3)$$

The general solution of (3) in terms of the set of parameters defined in (1) is given by

$$u_c(t) = \left(u_c(0) - \alpha\, u_a - \alpha_a E_0\right) e^{-\frac{1}{\alpha_a \tau}t} + \alpha\, u_a + \alpha_a E_0 \quad (4)$$

along with the boundary conditions

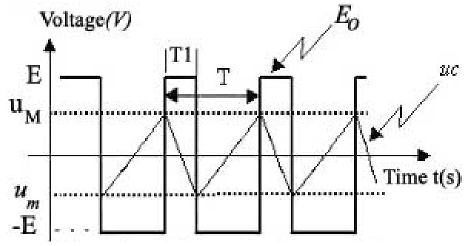
$$\begin{aligned} &u_c(0) = u_m{}^+ \rightarrow u_c(T_1) = u_M{}^+ \;\; for\; E_0 = E, \\ &u_c(0) = u_m{}^+ \rightarrow u_c(T_2) = u_M{}^+ \;\; for\; E_0 = -E \end{aligned} \quad (5)$$

Notice from (4) that under fixed switching thresholds, the control voltage u_a will dictate the whole dynamics of the capacitor voltage as well as the resulting switching times of the operational amplifier. From the preceding analysis, one obtains the steady-state behaviour of signals shown in Fig. 2. As an implication, $T_1(u_a)$ and $T(u_a)$ can be computed as follows

$$\begin{aligned} T_1(u_a) &= \tau\, \alpha_a \ln\left(\frac{-\alpha\; u_a - (\alpha_a + \alpha_1)E}{-\alpha\; u_a + (\alpha_1 - \alpha_a)E}\right), \\ T(u_a) &= \tau\, \alpha_a \ln\left(\frac{(-\alpha\; u_a)^2 - ((\alpha_a + \alpha_1)E)^2}{(-\alpha\; u_a)^2 - ((\alpha_a - \alpha_1)E)^2}\right) \end{aligned} \quad (6)$$

Thus, the resulting duty cycle is given by the following non linear expression

$$R_m(u_a) = \frac{T_1(u_a)}{T(u_a)} = \frac{\ln\left(\dfrac{-\alpha\; u_a - (\alpha_a + \alpha_1)E}{-\alpha\; u_a + (\alpha_1 - \alpha_a)E}\right)}{\ln\left(\dfrac{(-\alpha\; u_a)^2 - ((\alpha_a + \alpha_1)E)^2}{(-\alpha\; u_a)^2 - ((\alpha_1 - \alpha_a)E)^2}\right)}. \quad (7)$$



Note: T_1 and T depend on the control u_a.

Fig. 2. Steady state of an inverting DCM with a constant control.

In Fig. 4, numerical analysis using Matlab [4] indicates that, the duty-cycle remains approximately proportional to the control voltage around $(R_m = 1/2, u = 0)$ for a suitable choice of design parameters.

2) Case of the Non-inverting DCM

The idea of a non-inverting DCM shown in Fig. 1(b) is suggested in [5] with little development and none application. In this case, while the associated switching thresholds are equal to

$$\begin{cases} u_M^+ = \alpha_1 \mathrm{E} + \alpha_2 u_1 \\ u_m^+ = -\alpha_1 \mathrm{E} + \alpha_2 u_1 \end{cases} \quad (8)$$

the capacitor voltage evolves according to the linear differential equation

$$RC\frac{du_c}{dt} + u_c = E_0 \quad (9)$$

The general solution of (9) in terms of some parameters defined in (1) is given by

$$u_c(t) = \left(u_c(0) - E_0\right) e^{-\frac{1}{\tau}t} + E_0 \quad (10)$$

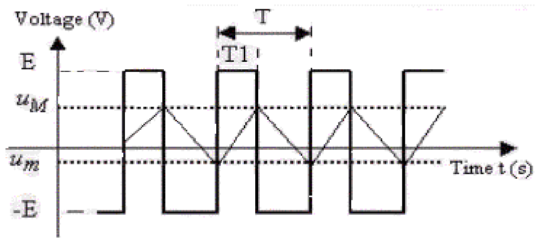
Notice from (10) that the behaviour of the capacitor voltage is independent to the control voltage u_1, while the switching thresholds defined by (8) of the operational amplifier as well as the resulting switching times depend on u_1. Although it is structurally simple, the proposed non inverting duty-cycle modulation circuit depicted on Fig. 1(b) is an alternative candidate for DCM problems. The long run behaviour of signals obtained in this case is shown in Fig. 3. As an implication, one can compute $T_1(u_1)$ and $T(u_1)$ as follows

$$\begin{aligned} T_1(u_1) &= \tau \ln\left(\frac{\alpha_2 u_1 - (1+\alpha_1)E}{\alpha_2 u_1 + (\alpha_1 - 1)E}\right), \\ T(u_1) &= \tau \ln\left(\frac{(\alpha_2 u_1)^2 - ((1+\alpha_1)E)^2}{(\alpha_2 u_1)^2 - ((\alpha_1 - 1)E)^2}\right) \end{aligned} \quad (11)$$

Thus, the resulting duty-cycle for a non-inverting modulator is given by

$$R_m(u_1) = \frac{T_1(u_1)}{T(u_1)} = \frac{\ln\left(\dfrac{\alpha_2 u_1 - (1+\alpha_1)E}{\alpha_2 u_1 + (\alpha_1 - 1)E}\right)}{\ln\left(\dfrac{(\alpha_2 u_1)^2 - ((1+\alpha_1)E)^2}{(\alpha_2 u_1)^2 - ((\alpha_1 - 1)E)^2}\right)} \quad (12)$$

Fig. 4 indicates also that, the range for which the duty-cycle remains approximately proportional to the control voltage, is wider compare to the previous case.



Note: u_m, u_M, T_1 and T depend on the control u_I .

Fig. 3. Steady state of a non- inverting DCM with a constant control.

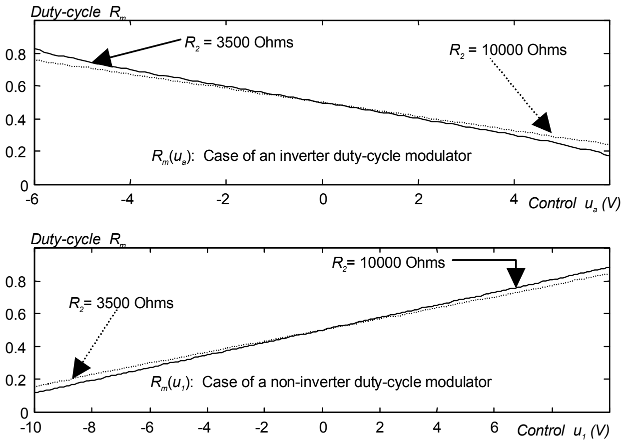


Fig. 4. Graphs associated wits duty-cycles within the control space.

3) *Case of the Summing DCM*

For a summing duty-cycle modulator proposed in Fig. 1(c), the analysis could be conducted in the same way developed in Section II.B.2. In fact, using the following parameters

$$\begin{cases} R_1 = \dfrac{R_a R_b}{R_a + R_b} \\ u_1 = \dfrac{R_a}{R_a + R_b} u_b + \dfrac{R_b}{R_a + R_b} u_a \end{cases} \tag{13}$$

relying on Thevenin equivalent input circuit, one could write similarly the duty-cycle obtained as follows

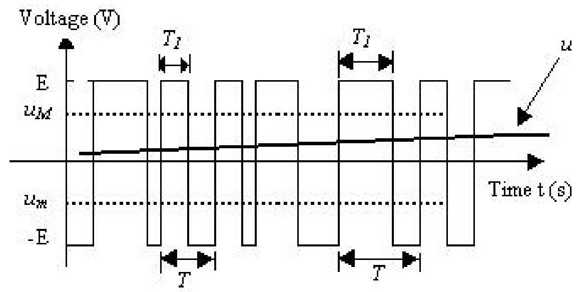
$$T_1(u_1) = \tau \ln\left(\frac{\alpha_2 u_1 - (1+\alpha_1)E}{\alpha_2 u_1 + (\alpha_1 - 1)E}\right),$$
$$R_m(u_1) = \frac{T_1(u_1)}{T(u_1)} = \frac{\ln\left(\dfrac{\alpha_2 u_1 - (1+\alpha_1)E}{\alpha_2 u_1 + (\alpha_1 - 1)E}\right)}{\ln\left(\dfrac{(\alpha_2 u_1)^2 - ((1+\alpha_1)E)^2}{(\alpha_2 u_1)^2 - ((\alpha_1 - 1)E)^2}\right)} \tag{14}$$

Note from (13) that if $R_a = R_b$, then $R_1 = R_a / 2$ and $(u_a + u_b)/2$ in (14). This leads to he averaging DCM.

4) *Case of a Difference DCM*

For a difference duty-cycle modulator, a straightforward analysis gives rise the following expressions

$$T_1(u_a) = \tau\, \alpha_a \ln\left(\frac{\alpha_2 u_1 - \alpha\ u_a - (\alpha_a + \alpha_1)E}{\alpha_2 u_1 - \alpha\ u_a + (\alpha_{\ 1} - \alpha_a)E}\right),$$
$$R_m(u_a) = \frac{T_1(u_a)}{T(u_a)} = \frac{\ln\left(\dfrac{\alpha_2 u_1 - \alpha\ u_a - (\alpha_a + \alpha_1)E}{\alpha_2 u_1 - \alpha\ u_a + (\alpha_1 - \alpha_a)E}\right)}{\ln\left(\dfrac{(\alpha_2 u_1 - \alpha\ u_a)^2 - ((\alpha_a + \alpha_1)E)^2}{(\alpha_2 u_1 - \alpha\ u_a)^2 - ((\alpha_1 - \alpha_a)E)^2}\right)} \tag{15}$$



Note: Control u is approximately constant for each cycle while T_1 and T depend on the control u_I .

Fig. 5. Principle of DCM with a variable control u .

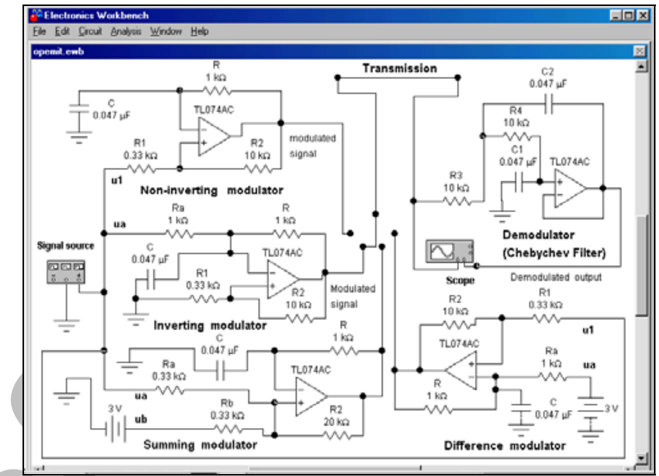


Fig. 6. Schematic diagram of modulators/demodulator in EWB.

Equation (15) is similar to (7) for the same α given an equivalent control $u_a - (\alpha_2/\alpha)\,u_1$. As a result, the difference DCM operates as an inverting DCM for the input signal $u_a - (\alpha_2/\alpha)\,u_1$.

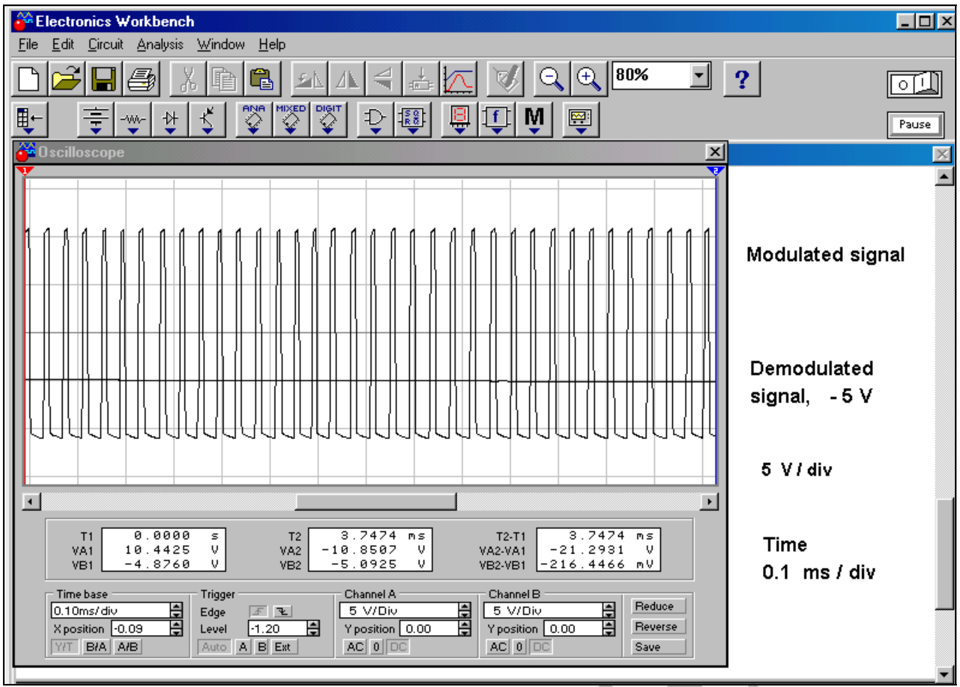
III. Extension of the Analysis of DCM for Variable Controls

Under variable controls, the duty-cycle of any DCM varies from cycle to cycle. Hence, an exact analysis of events involved is not obvious. However, an approximate analysis could be conducted under a reasonable assumption in order to extend previous results from cycle to cycle as it is illustrated in Fig. 5.

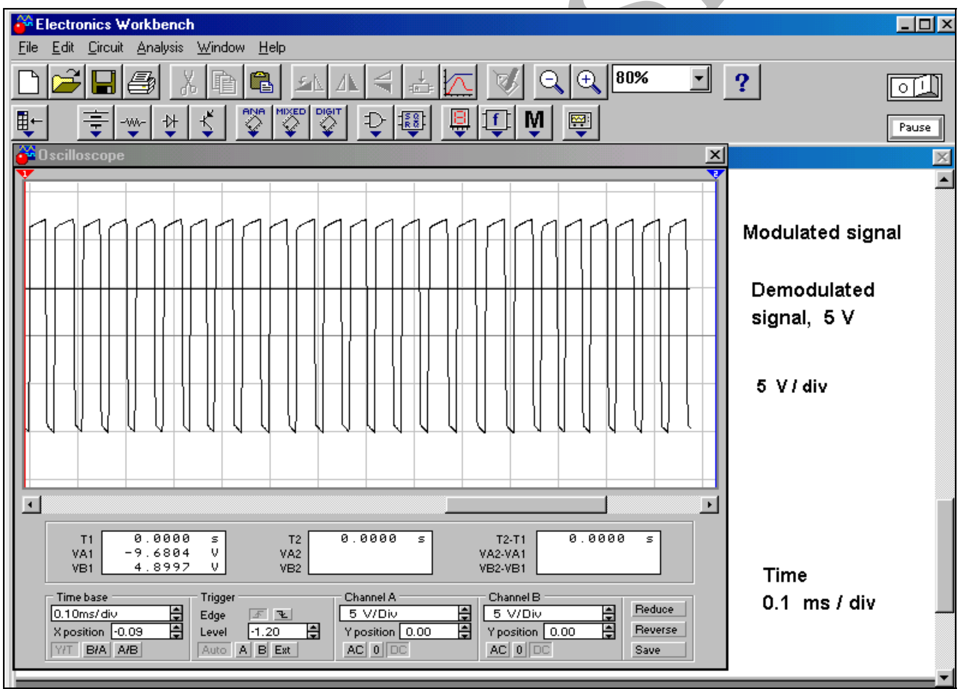
We conjuncture that if the frequency of the basic oscillator is sufficiently large compared to the maximum frequency contained in the spectrum of the control signal, then the graph of the control signal could be approximated in each cycle by a constant value. Thus, the previous analytical results for constant controls could be used and updated from cycle to cycle under variable controls. An important question arising at this point is how to choose a good oscillator frequency for a given constant or variable control signal? The search of a good value done using simulation. Then, from (6), (11), (14) and (15), the basic oscillator frequency (given the parameters found in simulation), could be computed by setting the control signals to zero.

IV. Simulation and Test of the Proposed Class of Duty-Cycle Modulators

Fig. 6 shows a schematic diagram of modulators and a demodulator circuit used in Electronic Work Bench® [6] to



(a)



(b)

Fig. 7. (a) Inverting modulator with a constant control $u_1 = 5\,v$, and (b) Non-inverting modulator with a constant control $u_a = 5\,v$.

test the proposed class of DCM. Constant and triangle-type controls are used to test the class of DCM under study.

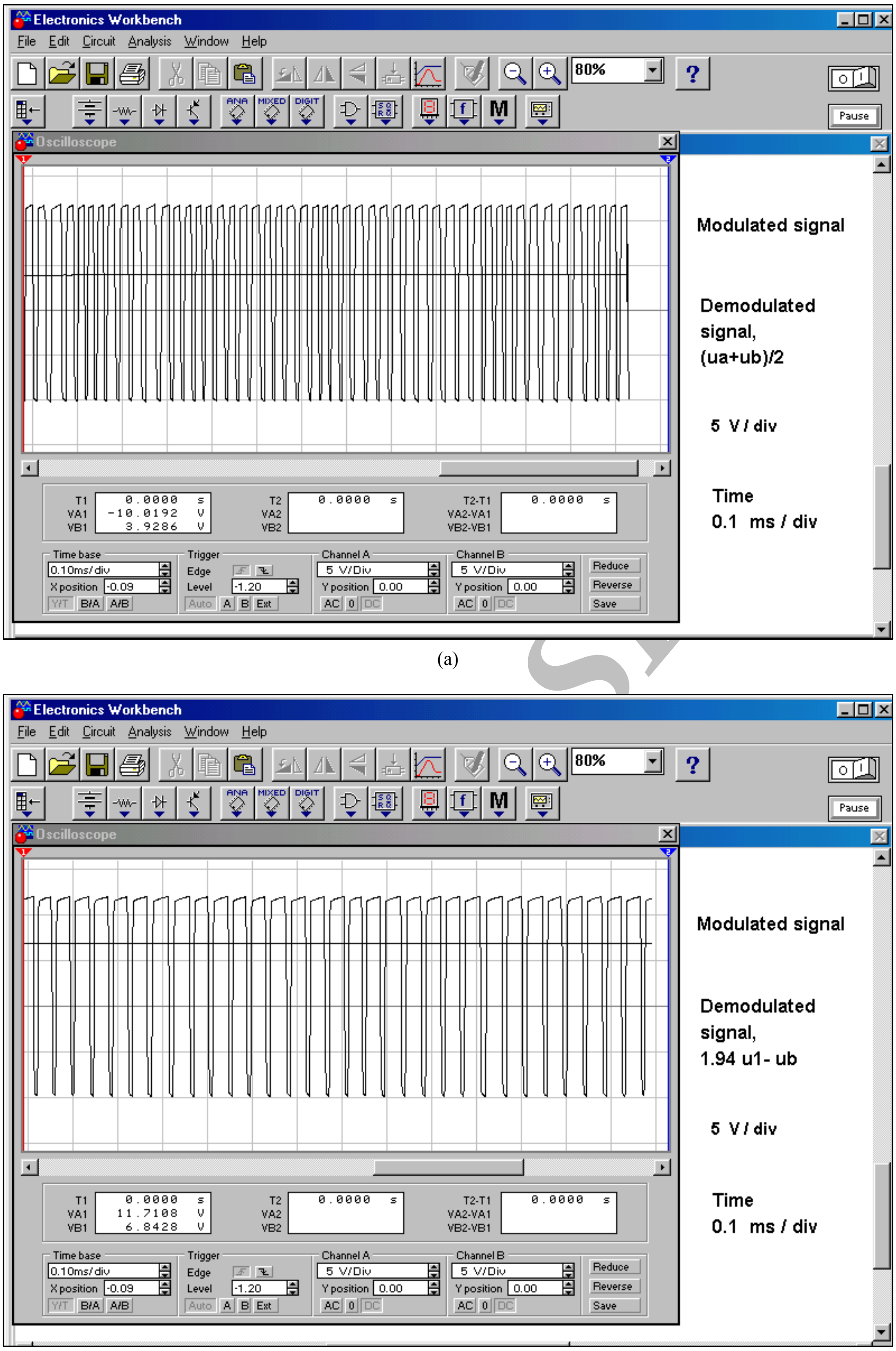
Good parameter values found using both analytical results and simulation are specified for each circuit shown in Fig. 6. In all cases, the linear demodulator considered is a second order Chebychev filter with a transfer function

$$F(s) = \frac{1}{R_3 R_4 C_1 C_2\ s^2 + (R_3 C_1 + R_4 C_1)\ s + 1} \tag{16}$$

It is designed as in [7] for the class of control signals considered. It provides 0.3 dB of maximum attenuation in the pass band and 40 dB of minimum attenuation in the stop band.

A. Simulation with Constant Controls

Fig. 7 illustrates the behaviour of inverting and non-inverting DCM with constant controls $u_a = 5$ V, $u_1 = 5$ V respectively. In the first case, the demodulated signal obtained in Fig. 7(a) is –5V, while in the second case shown in Fig. 7(b) it is equal to 5V. Fig. 8 illustrates the behavior of summing and difference DCMs with constant controls. In Fig. 8(a), $u_a = 5$ V, $u_b = 3$ V and the demodulated value is exactly $(u_a + u_b)/2 = 4$ V. Thus the summing DCM is an averaging modulator. In Figure 8(b), $u_a = 3$ V, $u_1 = 4$ V, and this yields a demodulated voltage $(\alpha_2/\alpha)\ u_1 - u_a = 1.94\ u_1 - u_2 = 4.76$ V. Note that, in all cases, the period $T(u)$ and the associated duty-cycle of the modulated signal are constant for constant controls.



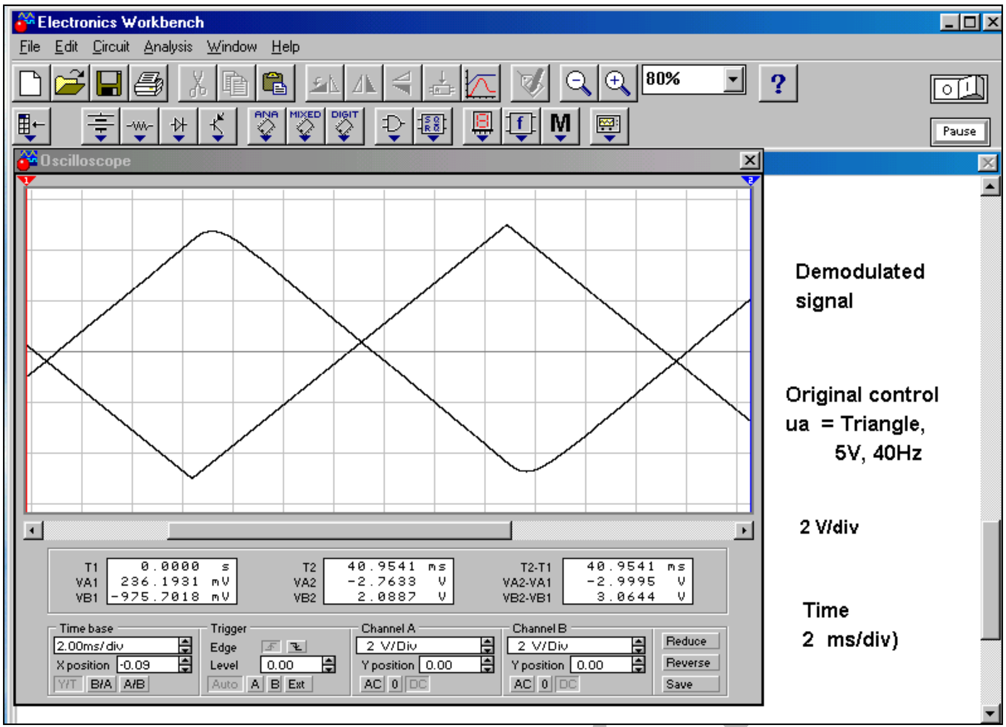
(a)

(b)

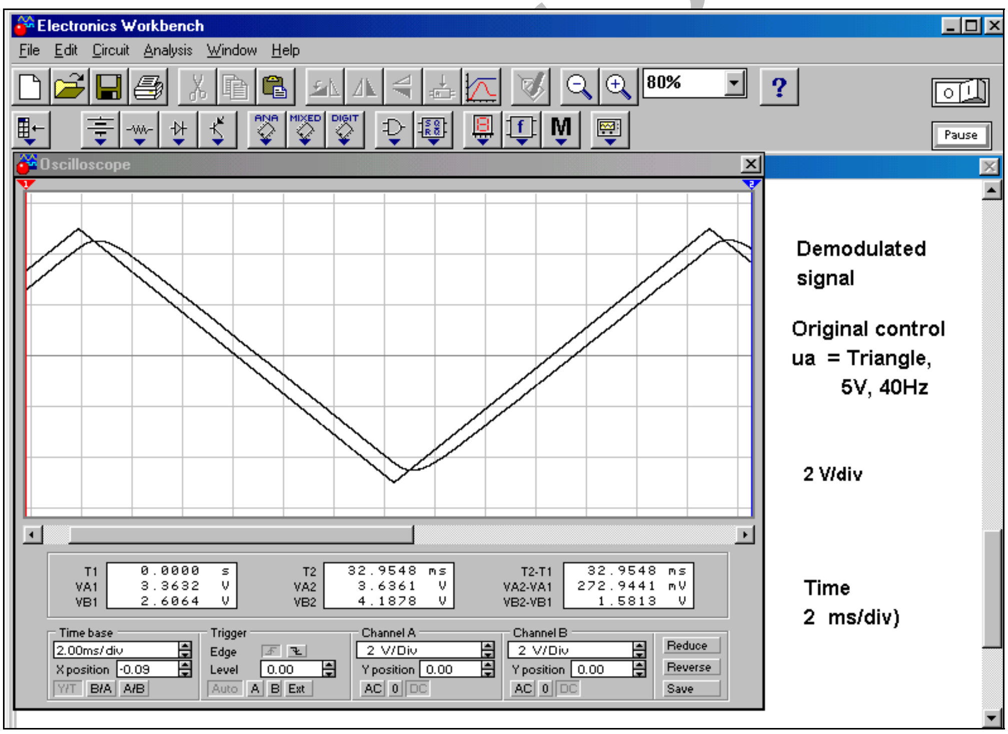
Fig. 8. (a) Summing modulator with a constant control $(u_a + u_b)/2$, and (b) Difference modulator with a constant control $(\alpha_2/\alpha)\,u_1 - u_a$.

B. Simulation with Variable Controls

Fig. 9 illustrates the behavior of inverting and non-inverting DCMs under a triangle-type control (40 Hz, 5 V). Fig. 9(a) indicates that the demodulated signal is close to $(-u_a)$ while it is close to u_1 in Fig. 9(b) for the non-inverting modulator. In addition, Fig. 10 illustrates the behavior of summing and difference DCMs under a triangle-type control (40 Hz, 5 V). In Fig. 10(a), $u_a = 3$ V, u_b being the triangle-type control indicated above. As a result, the demodulated voltage obtained is close to the original equivalent control $(u_a + u_b)/2$ (Its graph has been produced from u_a and u_b using an auxiliary circuit). Note that, the demodulated voltage varies approximately from $(5+3)/2 = 4$ V to $(-5+3)/2 = -1$ V. In Fig. 10(b), the demodulated output is also close to the original equivalent input $1.94\,u_1 - u_a$, where the coefficient 1.94 corresponds to α_2/α. In all cases related to variable controls, there is a constant delay between each control and the associated demodulated signal obtained. In addition, there is a steady-state error in the demodulated signal. However, these dynamic problems could be negligible depending of the range of low frequency signals considered in instrumentation.



(a)



(b)

Fig. 9. (a) Inverting modulator with a triangle-type control , and (b) non-inverting modulator with a triangle-type control.

V. Conclusion

An important class of duty-cycle modulators has been analysed and simulated in this work. The standard inverting duty-cycle modulator and three knew circuits have been investigated in depth. Specific properties associated with each circuit will dictate its use in an instrumentation context. While our analysis is exact under constant controls, the extension of the analytical results obtained under variable controls relies on approximations. However, simulations using Electronic Workbench allowed us to validate the good quality of the proposed circuits for signal conditioning even under variable controls. Although the delay and the steady-state error observed on modulated signals have received our attention, they have not been studied in depth. These dynamic problems will be investigated in a future research.

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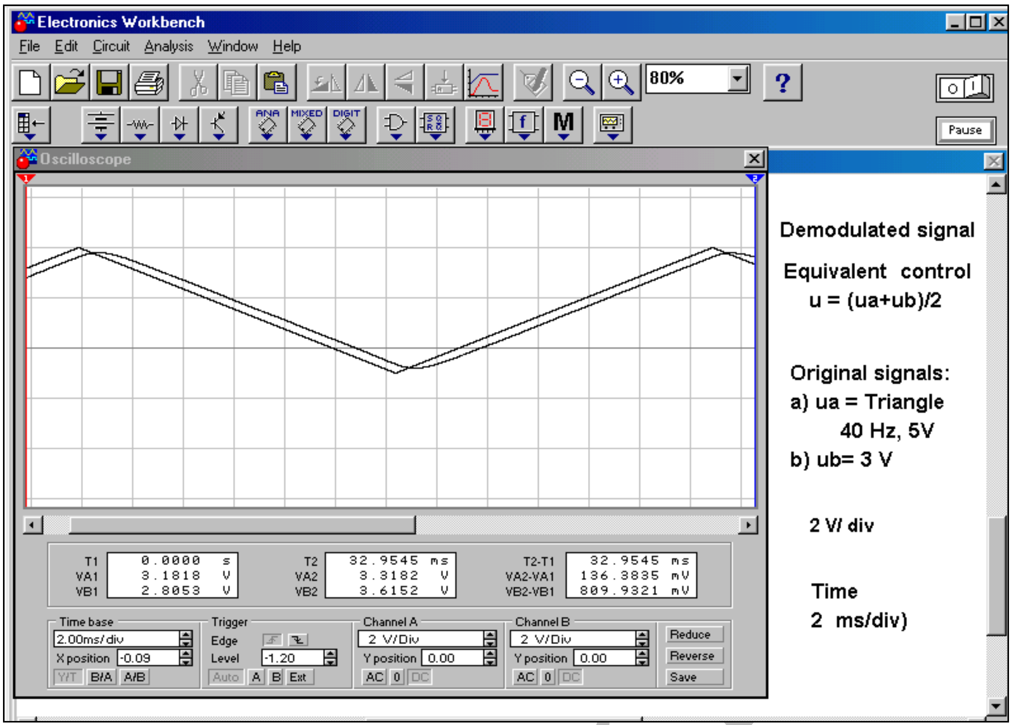
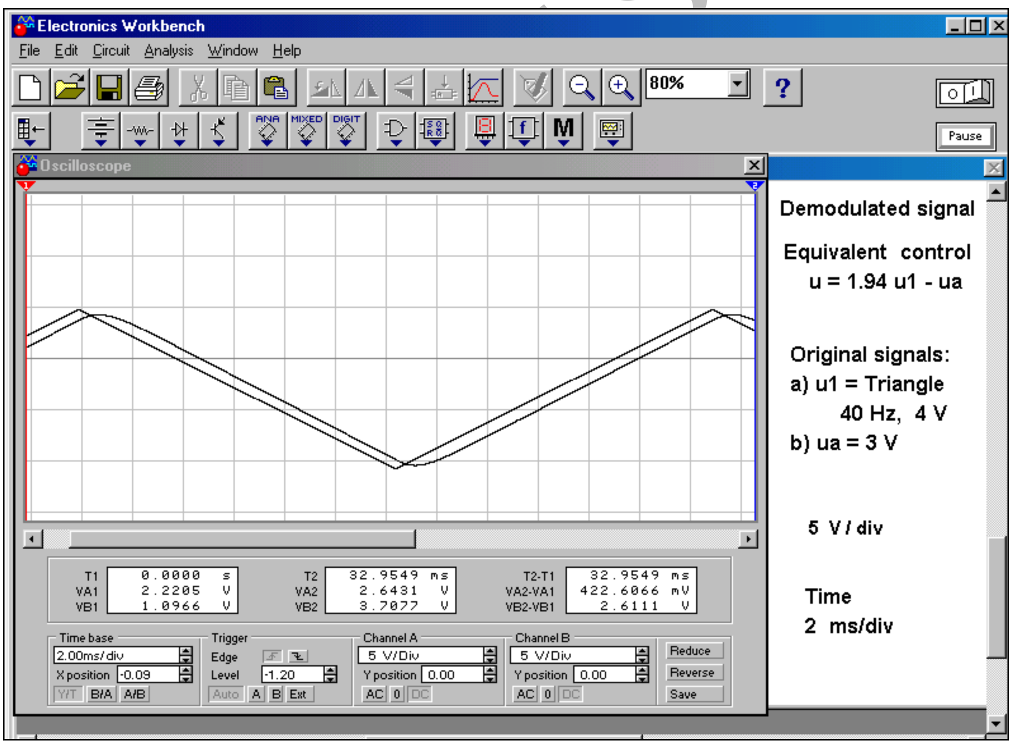


(a)



(b)

Fig. 10. (a) Summing modulator with a variable control $(u_a + u_b)/2$, and (b) difference modulator with a variable control $(\alpha_2/\alpha)\,u_1 - u_a$.

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