Modeling of Generalized Unified Power Flow Controller for Suitable Location and Power Flow Control

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Abstract—Modeling of generalized unified power flow controller (GUPFC), based on the static consideration, and has been presented in this paper used for power flow control. A Newton-Raphson load flow program has been developed which includes comprehensive control facilities and yet exhibits very strong convergence characteristics. The injection model, which is used to locate GUPFC suitably in the power system, is incorporated into an existing Newton-Raphson load flow algorithm in polar coordinate. The modified Jacobian matrix and power mismatch equations are deduced based on the injection model of GUPFC to control active and reactive powers and voltage magnitude in any combination or to control none of them. Test results are presented on IEEE 30-bus system, which demonstrate the effectiveness of the proposed method.

Index Terms—GUPFC, load flow analysis, optimal location, sensitivity analysis.

I. INTRODUCTION

The flexible AC transmission systems (FACTS) initiative was originally launched to solve the emerging problems in the late 1980s due to restrictions on the transmission line constructions, and to facilitate the growing power export/import and wheeling transactions among the utilities. FACTS devices can enhance transmission system control and increase line loading in some cases all the way up to thermal limits thereby without compromising reliability. These devices can be an alternative to reduce the flows in heavily loaded lines, resulting in an increased loadability, low system loss, improved stability of the network, reduced cost of production and fulfilled contractual requirement by controlling the power flows in the network. These capabilities allow transmission system owners and operators to maximize asset utilization and execute additional bulk transfer with immediate bottom-line benefits. FACTS devices provide new control facilities, both in steady state power control and dynamic stability control [1]-[3].

The unified power flow controller (UPFC) is arguably the most comprehensive device to have emanated so far from the FACTS initiative [4], [5]. In principle at least, the UPFC [6]-[10] offers new horizons in terms of power system control, with the potential to independently control three power system parameters such as bus voltage, line active and reactive power. Provided no operating limits are violated, the UPFC regulates all three variables simultaneously or any combination of them. Using controllable components of UPFC, the line flows can be changed in such a way that thermal limits are not violated, losses minimized, stability margin increased, contractual requirement fulfilled etc. without violating specified power dispatch. With these features, UPFC is probably the most powerful and versatile FACTS device which combines the properties of Thyristor Controlled Phase Angle Regulator (TCPAR) and Static Compensator (STATCOM).

Combining three or more converters working together, called Generalized Unified Power Flow Controller can extend the voltage and power flow control beyond what is achievable with the known two converter UPFC FACTS controllers. The simplest GUPFC consist of three converters one connected in shunt and two connected in series with two transmission lines in a substation. It can control five quantities such as a bus voltage and independent active and reactive power flows of two lines. The real power is exchanged among shunt and series converters via a common dc link.

Power flow (or load flow) analysis [2], [11]-[13] involves the calculation of power flows in lines/transformers and voltages of a power system for a given set of bus bar loads, active power generation schedule and specified bus voltage magnitude at generating buses. Such calculations are widely used in the analysis and design of steady state operation as well as dynamic performance of the system. The power flow problem is formulated as a set of nonlinear equations. Many calculation methods have been proposed to solve this problem. Among them, Newton-Raphson (NR) method and fast-decoupled load flow method are two very successful methods. In general, the decoupled power flow methods are only valid for weakly loaded network with large X/R ratio network. For system conditions with large angles across lines (heavily loaded network) and with special control devices (FACTS devices such as UPFC) that strongly influence active and reactive power flows, NR method may be required. Therefore, when the AC power flow calculation is needed in systems with FACTS devices, NR method can be used with modification of Jacobian matrix.

These FACTS controllers are very powerful devices for enhancing the power system performance but they are expensive too. Therefore, suitable locations of these devices are very important. The objective of placing these
The transformers are connected to three forced and two series (boosting) transformers as shown in Fig. 1.

at the fundamental frequency with variable amplitude ac bus.

compensator providing reactive power compensation and the function of an independent advanced static VAR terminal. Therefore with proper control, it can also fulfill in independent of the active power transfer to (or from) the dc absorb reactive power at its ac terminal, which is from the ac power system. Converter 1 can also generate or converter 2 and converter 3 via a common dc link terminal primarily used to provide the real power demand of configuration, sharing a common dc link. Converter 1 is commuted converters (VSC type) in back-to-back πφ

A. Basic Principles of GUPFC

Three converters GUPFC consist of a shunt (exciting) and two series (boosting) transformers as shown in Fig. 1. All the transformers are connected to three forced commuted converters (VSC type) in back-to-back configuration, sharing a common dc link. Converter 1 is primarily used to provide the real power demand of converter 2 and converter 3 via a common dc link terminal from the ac power system. Converter 1 can also generate or absorb reactive power at its ac terminal, which is independent of the active power transfer to (or from) the dc terminal. Therefore with proper control, it can also fulfill the function of an independent advanced static VAR compensator providing reactive power compensation and thus executing indirect voltage regulation at the controlled ac bus.

Converters 2 and 3 are used to generate voltage sources at the fundamental frequency with variable amplitude (0 ≤ Vs1 ≤ Vumn ) and phase angle (0 ≤ \( \phi_s \) ≤ 2\( \pi \) ), and amplitude (0 ≤ V\( \phi_s \) ≤ V\( \phi_s \) ) and phase angle (0 ≤ \( \phi_s \) ≤ 2\( \pi \) ), respectively, which are added to the ac transmission lines by the series connected boosting transformers, in order to control the reactive and active powers through the lines. GUPFC [3], [14], also known as multi-line UPFC, can control bus voltage and power flows of more than one lines or even of sub-networks. The simple GUPFC consisting of three converters is capable of simultaneously controlling five power system quantities, e.g. the bus voltage at substation, real and reactive power flows on two lines. For control of GUPFC, proportional-integral (PI) loops are utilized. In this scheme the gains of controller parameters are being selected to provide stable operation of GUPFC under steady state and faulty conditions.

The controller measures the regulated bus voltage at the ac bus (\( V_1 \)), dc link capacitor voltage (\( V_{dc} \)), real and reactive power flows in the lines (\( P_{line-1}, P_{line-2}, Q_{line-1}, \) and \( Q_{line-2} \)) and compared with the respective reference settings. The GUPFC, as proposed in [3], can also be used in modeling other members of the CSC family in power flow and OPF analysis. The strong control capability of the GUPFC with controlling bus voltage and multi-line power flows offers a great potential in solving many of problems facing the electric utilities in a competitive environment.

B. Static Representation of GUPFC

The equivalent circuit of GUPFC placed in line-\( i \) having impedance \( r_{i} + jx_{i} (=1/(g_{i} + jb_{i})) \) connected between bus-\( i \) and bus-\( j \) and in line-\( m \) having impedance \( r_{m} + jx_{m} (=1/(g_{m} + jb_{m})) \) connected between bus-\( i \) and bus-\( k \) is shown in Fig. 2. Let there be \( p \) (> 2) numbers of lines connected at bus-\( i \). GUPFC has five controllable parameters, namely the magnitude and the angle of inserted voltage (\( V_{s1}, \phi_{s1} \)) in line-\( i \), the magnitude and the angle of inserted voltage (\( V_{s2}, \phi_{s2} \)) in line-\( m \) and the magnitude of the current (\( I_{q} \)). The current in shunt converter can be delineated into two components viz. the current (\( I_{p} \)) in phase with the voltage at bus-\( i \) and current (\( I_{q} \)) in quadrature with the voltage at exciting substation.

Based on the principle of GUPFC operation and the circuit diagram, the basic mathematical relations can be written as

\[
\bar{l}_{ij} = (\mathbf{V}_{i} + \mathbf{V}_{s1} - \mathbf{V}_{j}) \mathbf{\bar{V}}_{ij}
\]

\[
\bar{l}_{ik} = (\mathbf{V}_{i} + \mathbf{V}_{s2} - \mathbf{V}_{k}) \mathbf{\bar{V}}_{ik}
\]

\[
\text{Arg}(\bar{l}_{q}) = \text{Arg}(\mathbf{V}_{i}) \pm \pi / 2 \ , \ \text{Arg}(\bar{l}_{r}) = \text{Arg}(\mathbf{V}_{j})
\]

\[
\bar{l}_{l} = \text{Re}[\mathbf{V}_{s1} \mathbf{I}_{q} + \mathbf{V}_{s2} \mathbf{I}_{l}]
\]

The power injection at bus-\( i \) can be written as

\[
\bar{S}_{i} = \mathbf{P}_{i} + j\mathbf{Q}_{i} = \mathbf{V}_{i} \mathbf{I}_{q} + \mathbf{V}_{i} \mathbf{I}_{l} + \mathbf{V}_{i} (\mathbf{I}_{p} + j\mathbf{I}_{q}) + \sum_{j=1}^{p} \mathbf{V}_{ij} \mathbf{I}_{ij} + \mathbf{V}_{ij} \mathbf{I}_{ij}
\]

where \( \mathbf{I}_{sh} \) is the shunt current due to line charging. All the bold quantities represent the complex variables.
The effect of GUPFC can be represented as injected power with the network without GUPFC as shown in Fig. 3. The injected complex powers \( S_{ij}, S_{jk}, S_{kg} \) at bus-\( i \), bus-\( j \) and bus-\( k \) can be derived as

\[
S_{ij} = S_{ij}^0 - S_{ij} = -[\overline{V}_{ij}^* y_{ij} + \overline{V}_{ij}^* y_{ij}^*] - (I_T + jI_q)^* \quad (6)
\]

\[
S_{jk} = S_{jk}^0 - S_{jk} = -[\overline{V}_{jk}^* y_{jk} + \overline{V}_{jk}^* y_{jk}^*] \quad (7)
\]

\[
S_{kg} = S_{kg}^0 - S_{kg} = -[\overline{V}_{kg}^* y_{kg} + \overline{V}_{kg}^* y_{kg}^*] \quad (8)
\]

where \( S^0 \) is the complex power injection when there was no GUPFC.

From (6), the real and reactive power injections at bus-\( i \) can be derived as

\[
P_{ig} = -\text{Re}[\overline{V}_{ij}^* y_{ij} + \overline{V}_{ij}^* y_{ij}^*] - V_{i}I_T^* \quad (9)
\]

\[
Q_{ig} = -\text{Im}[\overline{V}_{ij}^* y_{ij} + \overline{V}_{ij}^* y_{ij}^*] + V_{i}I_q \quad (10)
\]

From (4), we have

\[
V_{ij}^* = \text{Re}\left[\left(V_{ij}^0 y_{ij} + V_{ij}^0 y_{ij}^* V_{ij}^0 + V_{ij}^0 y_{ij}^* V_{ij}^0 y_{ij}^* \right) y_{ij}^* \right] +\]

\[
\text{Im}\left[\left(V_{ij}^0 y_{ij} + V_{ij}^0 y_{ij}^* V_{ij}^0 + V_{ij}^0 y_{ij}^* V_{ij}^0 y_{ij}^* \right) y_{ij}^* \right] \]

Thus,

\[
V_{ij}^* = V_{ij}^0 g_{ij} + V_{ij}^0 g_{ij}^* + V_{ij}^0 g_{ij}^* \quad (11)
\]

The real and imaginary values of \( V_{ij}^* \) can be written as

\[
\text{Re}(\overline{V}_{ij}^* y_{ij}) = V_{ij}^0 g_{ij} \cos(\phi_{ij} - \delta_{ij}) + b_{ij} \sin(\phi_{ij} - \delta_{ij}) \quad (12)
\]

\[
\text{Im}(\overline{V}_{ij}^* y_{ij}) = V_{ij}^0 g_{ij} \sin(\phi_{ij} - \delta_{ij}) - b_{ij} \cos(\phi_{ij} - \delta_{ij}) \quad (13)
\]

The injected active and reactive powers at bus-\( i \) will be

\[
P_{ig} = -V_{ij}^0 g_{ij} - V_{ij}^0 g_{ij}^* - 2V_{ij}^0 g_{ij} \cos(\phi_{ij} - \delta_{ij}) - 2V_{ij}^0 g_{ij}^* \cos(\phi_{ij} - \delta_{ij}) +\]

\[
V_{ij}^0 g_{ij} \cos(\phi_{ij} - \delta_{ij}) + b_{ij} \sin(\phi_{ij} - \delta_{ij}) + V_{ij}^0 g_{ij}^* \cos(\phi_{ij} - \delta_{ij}) + b_{ij} \sin(\phi_{ij} - \delta_{ij}) \quad (14)
\]

\[
Q_{ig} = V_{ij}^0 I_q + V_{ij}^0 I_q \quad (15)
\]

Similarly the real and reactive powers injections at bus-\( j \) and bus-\( k \) can be derived as

\[
P_{jg} = -V_{ij}^0 g_{ij} - V_{ij}^0 g_{ij}^* - 2V_{ij}^0 g_{ij} \cos(\phi_{ij} - \delta_{ij}) - 2V_{ij}^0 g_{ij}^* \cos(\phi_{ij} - \delta_{ij}) +\]

\[
V_{ij}^0 g_{ij} \cos(\phi_{ij} - \delta_{ij}) + b_{ij} \sin(\phi_{ij} - \delta_{ij}) + V_{ij}^0 g_{ij}^* \cos(\phi_{ij} - \delta_{ij}) + b_{ij} \sin(\phi_{ij} - \delta_{ij}) \quad (16)
\]

\[
P_{kg} = -V_{ij}^0 g_{ij} - V_{ij}^0 g_{ij}^* - 2V_{ij}^0 g_{ij} \cos(\phi_{ij} - \delta_{ij}) - 2V_{ij}^0 g_{ij}^* \cos(\phi_{ij} - \delta_{ij}) +\]

\[
V_{ij}^0 g_{ij} \cos(\phi_{ij} - \delta_{ij}) + b_{ij} \sin(\phi_{ij} - \delta_{ij}) + V_{ij}^0 g_{ij}^* \cos(\phi_{ij} - \delta_{ij}) + b_{ij} \sin(\phi_{ij} - \delta_{ij}) \quad (17)
\]

\[
Q_{jg} = -V_{ij}^0 I_q + V_{ij}^0 I_q \quad (18)
\]

\[
Q_{kg} = -V_{ij}^0 I_q + V_{ij}^0 I_q \quad (19)
\]

Equations (14) to (19) are derived for three converters (one shunt and two series) and can be generalized for multi-converter UPFC, where one shunt converter is connected at bus-\( i \) and \( n \) series converters are connected between the lines connected at bus-\( i \), as

\[
P_{ig} = -\sum_{n} \left\{ V_{ij}^0 g_{ij} \cos(\phi_{ij} - \delta_{ij}) + b_{ij} \sin(\phi_{ij} - \delta_{ij}) \right\} - (20)
\]

\[
Q_{ig} = \sum_{n} \left\{ V_{ij}^0 I_q \cos(\phi_{ij} - \delta_{ij}) + b_{ij} \sin(\phi_{ij} - \delta_{ij}) \right\} \quad (21)
\]

\[
P_{jg} = -\sum_{n} \left\{ V_{ij}^0 g_{ij} \cos(\phi_{ij} - \delta_{ij}) + b_{ij} \sin(\phi_{ij} - \delta_{ij}) \right\} \quad (22)
\]

\[
Q_{jg} = \sum_{n} \left\{ V_{ij}^0 I_q \cos(\phi_{ij} - \delta_{ij}) + b_{ij} \sin(\phi_{ij} - \delta_{ij}) \right\} \quad (23)
\]

where \( n = f, k,... \)

### III. Modified Nonlinear Power Flow Equation

The effect of GUPFC on power system can be modeled by injecting real and reactive power flows at three related buses as shown in Fig. 3, thus, it has no effect on the bus admittance matrix \( Y_{bus} = (G + jB) \). The load flow equations at bus-\( i \) for having \( n \) buses in the system and without GUPFC, can be expressed as below

\[
P_{ia} = P_{Gi} - P_{Li} = \sum_{j=1}^{n} V_{ij} G_{ij} \cos(\delta_{ij} + \theta_{ij}) \quad (24)
\]

\[
Q_{ia} = Q_{Gi} - Q_{Li} = \sum_{j=1}^{n} V_{ij} G_{ij} \sin(\delta_{ij} + \theta_{ij}) \quad (25)
\]

For each PQ and PV bus, there is an active power mismatch equation and for each PQ node, there is reactive power mismatch equation. These equations can be formulated as follows:

\[
\Delta P_i = P_{Gi} - \sum_{j=1}^{n} V_{ij} G_{ij} \cos(\delta_{ij} + \theta_{ij}) \quad (26)
\]

\[
\Delta Q_i = Q_{Gi} - \sum_{j=1}^{n} V_{ij} G_{ij} \sin(\delta_{ij} + \theta_{ij}) \quad (27)
\]

where \( P_i \) and \( Q_i \) are injected bus-generated power. \( P_{Gi} \) and \( Q_{Gi} \) are injected bus power at bus-\( i \) caused by the installation of GUPFC. \( m \) is the number of network PQ buses. Here, the \( n \)-th bus is supposed to be the slack bus. \( G_{ij} \) and \( B_{ij} \) are the \( ij \)-th elements of \( Y_{bus} \) matrix.

If two series transformers of GUPFC are being installed in branch \( k1 \) (connected between bus-\( l \) to bus-\( m \)) and in branch \( k2 \) (connected between bus-\( l \) and bus-\( o \)) respectively, the power mismatch equations at bus-\( l \) bus-\( m \) and bus-\( o \) will be modifies as
\[ \Delta P_l = P_{ls} - P_{lg} - \sum_{j=1}^{n} V_j (G_{lj} \cos \delta_j + B_{lj} \sin \delta_j) \] (28)

\[ \Delta Q_l = Q_{ls} - Q_{lg} - \sum_{j=1}^{n} V_j (G_{lj} \sin \delta_j - B_{lj} \cos \delta_j) \] (29)

\[ \Delta P_m = P_{ms} - P_{mg} - \sum_{j=1}^{n} V_j (G_{mj} \cos \delta_{mj} + B_{mj} \sin \delta_{mj}) \] (30)

\[ \Delta Q_m = Q_{ms} - Q_{mg} - \sum_{j=1}^{n} V_j (G_{mj} \sin \delta_{mj} - B_{mj} \cos \delta_{mj}) \] (31)

\[ \Delta P_o = \Delta P_{og} - V_o \sum_{j=1}^{n} V_j (G_{oj} \cos \delta_{oj} + B_{oj} \sin \delta_{oj}) \] (32)

\[ \Delta Q_o = \Delta Q_{og} - V_o \sum_{j=1}^{n} V_j (G_{oj} \sin \delta_{oj} - B_{oj} \cos \delta_{oj}) \] (33)

The injected active power at buses (\( P_{lg}, P_{mg}, P_{og} \)), and reactive powers (\( Q_{lg}, Q_{mg}, Q_{og} \)) having a GUPFC are calculated using (8) to (19). Thus, the relationship are obtained for small variations in \( V \) and \( \delta \), by forming the total differentials,

\[
\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta V \\ \Delta \delta \end{bmatrix} + J_2 \begin{bmatrix} \Delta \delta \end{bmatrix} \] (34)

\[
J = J_1 + J_2
\] (35)

where \( J_1 \) is the normal N-R power flow Jacobian matrix and \( J_2 \) is the partial derivative matrix of injected power with respect to the variables. When bus-\( l \) and bus-\( m \) are PQ buses, \( J_2 \) may have 16 nonzero elements (36) to (61) as if bus-\( l \) is a PV bus corresponding elements of row and column will not exist. When more than one UPFC are installed in the network their effects are added to \( J_2 \). In this situation the non-zero elements may be more than 16. Now we can see that the power flow can be solved by NR method in the normal way except the small differences in \( J \) and power mismatch equations. The computation formulas of \( J_2 \) are given below

\[
\frac{\partial P_{lg}}{\partial \delta_1} = -2V_{s1}^2 g_{lm} \sin (\phi_{s1} - \delta_1) - 2V_{s2}^2 g_{lo} \sin (\phi_{s2} - \delta_1) \] (36)

\[
\frac{\partial P_{lg}}{\partial \delta_m} = V_{s1} V_m [g_{lm} \sin (\phi_{s1} - \delta_m) - b_{lm} \cos (\phi_{s1} - \delta_m)] \] (37)

\[
\frac{\partial P_{lg}}{\partial \delta_o} = V_{s2} V_o [g_{lo} \sin (\phi_{s2} - \delta_o) - b_{lo} \cos (\phi_{s2} - \delta_o)] \] (38)

\[
\frac{\partial P_{mg}}{\partial \delta_m} = V_{s1} V_m [g_{lm} \sin (\phi_{s1} - \delta_m) + b_{lm} \cos (\phi_{s1} - \delta_m)] \] (39)

\[
\frac{\partial P_{mg}}{\partial \delta_o} = 0 = \frac{\partial P_{mg}}{\partial \delta_o} \] (40)

\[
\frac{\partial P_{og}}{\partial \delta_o} = V_{s2} V_o [g_{lo} \sin (\phi_{s2} - \delta_o) + b_{lo} \cos (\phi_{s2} - \delta_o)] \] (41)
\[ \frac{\partial Q_{eg}}{\partial V_j} = 0 = \frac{\partial Q_{eg}}{\partial V_m}. \]  

(61)

With help of (36) to (61) the power flow Jacobian matrix can be modified and power flow equations can be solved by conventional N-R method.

IV. OPTIMAL LOCATION OF GUPFC

The costs of some FACTS devices are quite high especially those devices which use self-commutated converters. Generalized unified power flow controllers (GUPFC) use 3 or more converters and thus very expensive compared to the other FACTS controllers. Therefore, it is very important to locate few devices optimally in the system for specific objectives. By controlling the congestion which occurs for limited period of time, system’s security and stability [14] are improved whereas reducing the losses for the remaining period, cost of operation is reduced.

The severity of the system loading under normal and contingency cases can be described by a real power line flow performance index [2], as given below.

\[ P_I = \sum_{m=1}^{N} \frac{W_m \left( \frac{P_{lm}}{P_{lm}^{\text{max}}} \right)^{2a}}{2a} \]  

(62)

where \( P_{lm} \) is the real power flow and \( P_{lm}^{\text{max}} \) is the rated capacity of line- \( m \). \( a \) is the exponent and \( w_m \) a real non-negative weighting coefficient which may be used to reflect the importance of the lines. \( N \) is the total number of lines in the network.

\( P_I \) will be small when all the lines are within their limits and reach a high value when there are overloads. Thus, it provides a good measure of severity of the line overloads for a given state of the power system. Most of the works on contingency selection algorithms utilize the second order performance index which, in general, suffers from masking effects. The lack of discrimination, in which the performance index for a case with many small violations may be comparable in value to the index for a case with one huge violation, is known as Masking effect. By most of the operational standards, the system with one huge violation is much more severe than that with many small violations. Masking effect to some extent can be avoided by using higher order performance indices (i.e. \( a > 1 \)). However, in this study, the value of exponent has been taken as 2 and \( w_j = 1.0 \). It was found that masking effect was removed with this value for the considered examples.

The real power flow PI sensitivity factors with respect to the control parameters of GUPFC can be defined as

\[ c_1^k = \left. \frac{\partial P_I}{\partial V_{sl}} \right|_{V_{sl}=0} = P_I \text{ sensitivity with respect to } V_{sl} \]

\[ c_2^k = \left. \frac{\partial P_I}{V_{sl} \partial \phi_{sl}} \right|_{\phi_{sl}=0} = P_I \text{ sensitivity with respect to } \phi_{sl} \]

\[ c_3^k = \left. \frac{\partial P_I}{\partial I_q} \right|_{I_q=0} = P_I \text{ sensitivity with respect to } I_q \]

\( c_1^k \) and \( c_2^k \) with respect to \( V_{sl} \) and \( \phi_{sl} \) respectively as will be the same as \( c_1^k \) and \( c_2^k \) with respect to \( V_{sl} \) and \( \phi_{sl} \). Using (62), the sensitivity of PI with respect to GUPFC parameter \( X_k(V_{sl}, \phi_{sl}, \text{ and } I_q) \) connected between bus- \( i \) and bus- \( j \), can be written as

\[ \left. \frac{\partial P_I}{\partial X_k} \right|_{X_k} = \sum_{m=1}^{N} w_m \left( \frac{P_{lm}}{P_{lm}^{\text{max}}} \right)^{2a} \left( \frac{1}{P_{lm}^{\text{max}}} \right)^{2a} \left( \frac{\partial P_{lm}}{\partial X_k} \right) \]  

(63)

The real power flow in a line- \( m \) (\( P_{lm} \)) can be represented in terms of real power injections using power flow equations [2] where \( s \) is slack bus, as

\[ P_{lm} = \left\{ \begin{array}{ll}
S_{mt} P_t & \text{for } m \neq k \\
S_{mt} P_t + P_{js} & \text{for } m = k
\end{array} \right. \]  

(64)

where \( S_{mt} \) is the \( mt \)-th element of \( S \) matrix which relates line flow with power injections at the buses without GUPFC and \( n \) is the number of buses in the system. Observe that line- \( k \), from bus- \( i \) to bus- \( j \), is the line containing the series converter of GUPFC, \( P_{lg}^k \), therefore, is the addition flow, at bus- \( j \), in the line containing the GUPFC, due to the presence of the device.

Using (63) and (64), the following relationship can be derived,

\[ \frac{\partial P_{lm}}{\partial x_k} = \left\{ \begin{array}{ll}
S_{mt} \left[ \frac{\partial P_t}{\partial x_k} + S_{mj} \frac{\partial P_{lg}}{\partial x_k} \right] & \text{for } m \neq k \\
S_{mt} \left[ \frac{\partial P_t}{\partial x_k} + S_{mj} \frac{\partial P_{lg}}{\partial x_k} + S_{ij} \frac{\partial P_{lg}}{\partial x_k} \right] & \text{for } m = k
\end{array} \right. \]  

(65)

The derivatives of real and reactive powers with respect to control parameters of GUPFC are given below.

\[ \frac{\partial P_{lg}}{\partial V_{sl}} \bigg|_{V_{sl}=0} = -2V_{gj} \cos \phi_{sl} \delta_j + V_j (g_{j} \cos \phi_{sl} - \delta_j) \]  

(66)

\[ \frac{\partial P_{lg}}{\partial \phi_{sl}} \bigg|_{\phi_{sl}=0} = -2V_{gj} \sin \delta_j + V_j (g_{j} \sin \delta_j + b_{j} \cos \delta_j) \]  

(67)

\[ \frac{\partial P_{lg}}{\partial I_q} \bigg|_{I_q=0} = 0 \]  

(68)

\[ \frac{\partial P_{lg}}{\partial I_q} \bigg|_{I_q=0} = V_j (g_{j} \cos \delta_j + b_{j} \sin \delta_j) \]  

(69)

\[ \frac{\partial P_{lg}}{\partial V_{sl}} \bigg|_{V_{sl}=0} = V_j (g_{j} \sin \delta_j - b_{j} \cos \delta_j) \]  

(70)

\[ \frac{\partial P_{lg}}{\partial I_q} \bigg|_{I_q=0} = 0 \]  

(71)

The derivatives of real power injection with respect to phase angle of GUPFC are considered around zero although the phase angle in GUPFC can be controlled from
0° to 360°. For any practical system, the angle difference for both ends of a line is generally very small and it is limited to 30° due to stability and security reasons [2]. In a practical power system control of an angle of UPFC or GUPFC are generally not high because small change in angle with increases the power flow in lines to its maximum loadability limit. Therefore, the derivatives with respect to phase angle around zero are correct. However, it can be calculated around any angle, as derivation is very simple.

V. CASE STUDIES

The effectiveness of proposed method has been tested on IEEE 30 bus test systems [12] with and without GUPFC by modifying an existing NR load flow method to allow inclusion of proposed GUPFC model. Before solving the power flow equations, the suitable location of GUPFC has been obtained using method suggested in Section IV.

A. Location of GUPFC

Sensitivity factors have been calculated for GUPFC placed in every line one at the time for the same operating conditions. The sensitivities of real power flow performance index with respect to the control parameters of GUPFC for some lines are given in Table I. Line 1-2 is overloaded as real power flow is 1.62 pu, whereas line limit is 1.5 pu, and other line flows are under limit. Line 1-27 has the most negative \( c^4_1 \) value and so it is more sensitive to overall system loading. When increasing series injection \( (V_{s1}) \) line overloading decreases. Positive values of \( c^4_1 \) show that it cannot change line overloading because series injected voltage is always positive value. Increasing series injection voltage magnitude will increases \( P_l \), thus congestion of the system.

Table I (column 4, \( c^4_1 \)) shows the placement of GUPFC in line 1-27 has highest absolute value and is negative which indicates that phase angle shift of the GUPFC should be positive. Placing of GUPFC in line 1-27 will reduce the line loading of line 1-2 and increases the loading of line 1-27. Since other line connected at bus-1 is line 1-2 which is also having high \( c^4_2 \) value, therefore, it is other line in which GUPFC series transformer is placed. The sensitivity factor \( c^4_2 \) is always zero because the reactive power component of the shunt current \( (I_q) \) cannot control the real power flow of the line as it is in 90° phase with input voltage. The real power component of shunt current controls real power flow in the line as it is in phase with the input voltage.

B. Case -A: Without GUPFC

From the NR load flow results, it is found that with the system loading \( P_{load} = 274.8 \) MW; \( Q_{load} = 126.0 \) MVAR, power flows in line 1-2 is 162.42–j22.14 MVA and in line 1-27 it is 76.06+j3.68 MVA. The total system loss was found to be 15.28–j10.78 MVA. To verify the results, the proposed approach is also used taking all the control parameters of GUPFC to zero. The result obtained is the same as compared to conventional NR load flow method.

C. Case -B: With GUPFC

To improve the system performance (to mitigate the congestion of system), GUPFC was placed in lines 1-2 and 1-27 near to bus 1. The overall system losses will also be reduced along with control of line power flows to their limiting values. Moreover, the control parameters of GUPFC can be simultaneously or selectively controlled to enhance the power system performance. Therefore, in this case study, the impact of control parameters of GUPFC is investigated. For three-converter GUPFC, five control parameters can be varied and there are several possible combinations of control strategies. However, following case studies are performed and the results are presented below.

(i) \( V_{s1} \) and \( V_{s2} \) varied and rest are kept constant
(ii) \( \phi_{s1} \) and \( \phi_{s2} \) varied and rest are kept constant
(iii) \( V_{s1} \) and \( \phi_{s2} \) varied and rest are kept constant
(iv) \( V_{s2} \) and \( \phi_{s1} \) varied and rest are kept constant
(v) \( I_q \) varied and rest are kept constant
(vi) When all parameters of GUPFC varied

Among all cases, the optimum range of parameter can be found such that the improvements of system performance are the best.

1) \( V_{s1} \) and \( V_{s2} \) Are Varied and Rest Are Kept Constant

As line 1-2 is highly loaded and line 1-27 is less loaded, the flow in line 1-2 is reduced by series injected voltages in lines 1-2 and 1-27. The GUPFC parameters \( \phi_{s1} \), \( \phi_{s2} \) and \( I_q \) are set to zero values and, while, \( V_{s1} \) and \( V_{s2} \) are varied from 0 to 0.3 pu in this case. At \( V_{s1} = 0.02 \) pu and \( V_{s2} = 0.08 \) pu the total system loss is 14.53–j16.73 MVA and line flow reduced to 145.64–j18.04 MVA from 162.42–j22.14 MVA in line 1-2 and increased to 94.33+j68.10 MVA from 76.06+j3.68 MVA in line 1-27. It is seen that only series voltage injection control \( V_{s1} \) and \( V_{s2} \) variation on the both real and reactive power are very small.

2) \( \phi_{s1} \) and \( \phi_{s2} \) Are Varied and Rest Are Kept Constant

From case (1), it was found that the system loss, keeping the power flow in the lines within limits, is small at \( V_{s1} = 0.02 \) pu, \( V_{s2} = 0.08 \) pu. In this case, phase angle control of lines 1-2 and 1-27 was varied keeping rest
GUPFC’s parameters at $V_{s1} = 0.02$ pu, $V_{s2} = 0.08$ pu and $I_q = 0$. In previous case we have seen that with variation of $V_{s1}$ and $V_{s2}$ do not give favorable results. But we know that real power flow can be controlled more effectively by changing the phase angle. Variations of system losses, line flows and bus-27 voltage are shown in the Figs 4 to 8. It was found that minimum system loss is 10.94–j25.94 MVA which is reduced from base case value 15.28–j10.78 MVA. And line flows are 141.97–j2.94 MVA in line 1-2 and 93.18+j1.01 MVA in line 1-27 at $\theta = 85.94^\circ$ and $\phi = 57.30^\circ$ with $V_{s1} = 0.02$ pu and $V_{s2} = 0.08$ pu.

Figs. 4 and 5 show real and reactive power loss with the variation of phase angle of series injected voltages of GUPFC, respectively. The real power loss varies in the range 0.1 to 0.2 pu and reactive power loss in the range of 0.1 to -0.3 pu. The real power flow in lines 1-2 and 1-27 are shown in Fig. 6. It is observed that real power flows in the lines are very sensitive to the phase angles of injected series voltages. The voltage magnitude of bus-2 (PV bus) is almost constant which can be seen from Fig. 7. The voltage is changed from the specified value when the reactive power limit of generator-2 is violated whereas the voltage at bus-27 is very sensitive to the variation of $\phi_{s1}$ and $\phi_{s2}$ (in these figures $\phi_{s1}$ and $\phi_{s2}$ are shown as $\text{angs}1$ and $\text{angs}2$, respectively).

3) $V_{s1}$ and $\phi_{s2}$ Are Varied and Rest Are Kept Constant

$V_{s1}$ and $\phi_{s2}$ are varied keeping $V_{s2} = 0.08$ pu, $\phi_{s1} = 0^\circ$ and $I_q = 0$. It was found that at $V_{s1} = 0.06$ pu and $\phi_{s2} = 68.75^\circ$ system losses are 10.98–j26.03 MVA and line flows are 144.11–j91.12 MVA in line 1-2 and 93.39+j6.98 MVA in line 1-27.

4) $V_{s2}$ and $\phi_{s1}$ Are Varied and Rest Are Kept Constant

In this case overloaded line is controlled by series injected voltage and under loaded line is controlled by phase angle while keeping rest parameters constant at $V_{s1} = 0.02$ pu, $\phi_{s2} = 0^\circ$ and $I_q = 0$. The system power losses are shown in Fig. 8. It is seen that reactive power loss is always negative for all the values of $V_{s2}$ and $\phi_{s1}$. The variation of real power loss is very small as compared to reactive power loss.

5) $I_q$ Is Varied and Rest Are Kept Constant

Since bus -1 is slack bus, the effect of $I_q$ variation will have no effect. To see the effect of $I_q$ variation, GUPFC is placed in lines 8-21 and 8-22. In base case, system loss is 15.28–j10.78 MVA. But at the value of $I_q = 0.27$ pu losses reduces to 15.14–j12.87 MVA and line flows increased to 13.29+j8.22 MVA and 5.98+j3.46 MVA in lines 8-21 and 8-22, respectively. There is little impact on real power loss and real line flows. But bus voltages improved and so reactive power flows changed. In base case bus voltages are $V_{b1} = 1.05578$, $V_{b2} = 1.04552$ and $V_{b22} = 1.04672$ pu.

6) All Parameters Are Varied

In this case, all the control parameters of GUPFC are varied simultaneously. The results of real power less than 0.0878 pu are given in Table II. In Table II, the angles are in degree and rests are in pu. It is found that minimum system real power loss is 8.70 MW at $V_{s1} = 0.1$ pu, $V_{s2} = 0.2$ pu, $\phi_{s1} = 58.97^\circ$, $\phi_{s2} = 70.43^\circ$ and $I_q = 0$ pu.
VI. CONCLUSIONS
NR method is a suitable power flow calculation method in the system with FACTS devices when high accuracy is required. The NR method with proper modification is proposed to incorporate GUPFC. The corresponding formulas are deduced based on the injection model. The additional computational work is small due to the special features of FACTS technology. Case studies show the effectiveness of the proposed methods. The main feature of the approach is that it does not increase any additional buses in the system. Final parameter selection can be done based on the minimization of particular objective such as total cost, total loss etc. of the system.

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REFERENCES

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