Principal Component Analysis Approach to Reduction of Measurement Dimensionality in 3D Impedance Tomography

Magdalena Stasiak, Konrad Nita, Stefan F. Filipowicz, and Jan Sikora

Abstract—The main idea of this paper is to present the Principal Component Analysis (PCA) approach in order to reduce dimensionality of measured data for three-dimensional impedance tomography.

The inverse problem solution provides the identification of the radius and the position of the object placed inside the cylindrical tank.

Index Terms—Electrical impedance tomography, principal component analysis, PCA.

I. INTRODUCTION

The idea of the Electrical Impedance Tomography (EIT) is to evaluate conductivity or permittivity distribution inside the examined object by measuring the voltages between electrodes placed on its surface.

EIT is widely applied to visualization of inaccessible objects or features e.g. to multiphase flows, nondestructive evaluation of structures, determination of underground pollution, and medical imaging [1]-[3].

EIT is nondestructive, noninvasive, radiation-free and relatively inexpensive technique compared with commonly applied methods, such as magnetic resonance or X-ray imaging.

The process of reconstruction of the conductivity and permittivity distribution requires the inverse problem solution. The problem is nonlinear and ill-posed, which makes the task difficult [4].

The most often approach to the inverse problem in EIT is to come it down to the problem of optimization and solve it by deterministic or stochastic methods. Such an approach is prohibitively time expensive, especially in 3D space. It is assumed that artificial neural network may be employed, because it bases on the parallel data processing and has short computation time [3],[5]-[8]. These advantages make neural networks very attractive for EIT applications.

The Principal Component Analysis is used for selection of neural network size. PCA decomposes high-dimensional data into a low-dimensional subspace component. In this way the size of input vector to train the neural networks is limited.

The number of neurons in the hidden layer is determined experimentally. To reduce the complexity of the task and the required computational time for the training process the number of neurons in the hidden layer and consequently the number of weights should be minimal.

With too many hidden neurons and with a large number of weights the neural network will have the problem of over-fitting and would start to memorise the training set rather than generate a robust system. A neural network model with appropriate number of hidden neurons and with enough weights to control the problem has to be designed [5],[6].

II. BASIC PROPERTIES OF PCA

PCA is perhaps one of the oldest and the best-known techniques in multivariate analysis and data mining [9]-[11], but in fact the application of this method to EIT is novel.

What makes this method useful in our research is that we can reduce the dimensionality of a data set, in which there are a large number of interrelated variables, while retaining as much as possible of the variation present in the data set. This reduction is achieved by transforming the input data set to a new set of uncorrelated variables, called the principal components [10],[11].

Generally, PCA is related and motivated by the following two problems [9]:

1. Given random vectors \( x(k) \in \mathbb{R}^m \) with finite second order moments and zero mean, find the reduced \( n \)-dimensional \( (n < m) \) linear subspace that minimizes the expected distance of \( x \) from subspace.

2. Given random vectors \( x(k) \in \mathbb{R}^m \), find the \( n \)-dimensional linear subspace that captures most of the variance of the data \( x \). This problem is related to feature extraction, where the objective is to reduce the dimension of the data while retaining most of its information content.

Both problems have the same optimal solution, which is based on the data covariance matrix. PCA can be converted to the eigenvalue problem of the covariance matrix of \( x \) [9].

First of all, calculation of the covariance matrix is necessary:
III. THE FORWARD AND INVERSE PROBLEM IN EIT

A relationship between the conductivity distribution inside the domain and the measured voltages is described by the Laplace equation with the Neumann and Dirichlet boundary conditions [4].

Mathematical formulation of the forward problem in 3D space can be presented as follows:

\[
\nabla \cdot \gamma(x,y,z) \nabla \varphi(x,y,z) = 0 \quad \text{in } \Omega
\]

where:
- \( \nabla \) - the electric field intensity,
- \( \gamma(x,y,z) \) - the conductivity of the structure,
- \( \Omega \) - region bounded by the close surface \( S \).

With \( n \) being the unit outward normal vector to the boundary surface, \( \varphi \) is subjected to the following boundary conditions:

1. \( \varphi \) is known at the selected pair of the electrodes (4)
2. \( \frac{\partial \varphi}{\partial n} = 0 \) at the rest of the boundary (5)

For three-dimensional space let us consider a set of four layers with 16 electrodes on each layer. Both symmetry of the model and the fact, that supplying electrodes are omitted, because of unknown voltage drop between these electrodes and examined object, were taken into account [4]. For a single layer (16 electrodes - two source and fourteen measurement electrodes) we can get 13 independent electrode-to-electrode voltages and 8 independent potential distributions, therefore 104 linearly independent values of electrode-to-electrode voltages have been collected [4].

Fig. 1 presents the 3D model for computer simulation of detecting the dimensions and position of the cylinder placed inside the measuring space. In our experiment we change the radius and position of the object.

Simulations were conducted for cylinder-shaped inside object with radius \( R = 0.5, 0.75, 0.8, 1.0, 1.2, 1.5, 1.75, 2.0, 2.25, 2.5 \) cm. Moreover, simulations for different locations of the inside cylinder, which was being moved along the radius to the center of the outer cylinder and along \( z \)-axis, were conducted.

In order to simulate the electrical potential on the electrodes, the finite element method was used [12]. The model was discretized by tetrahedron elements with 45000 nodes. The generated mesh is shown in Fig. 2.

IV. DATA REDUCTION USING PRINCIPAL COMPONENT ANALYSIS

One of the most important problems in PCA is the determination of the output signal dimension. PCA transforms the input data from the original high-dimensional space into a low-dimensional output subspace performing dimensionality reduction and retaining most of the intrinsic information of the input data.

Fig. 3 presents the distribution of the first 100 eigenvalues of measured data. It can be observed that the majority of the eigenvalues is very close to zero. It is possible to remove some redundant information by performing the PCA transformation. In this case, the dimension was reduced from the initial value of 416 to the range from 4 to 20. The noise part of the signal, corresponding to small eigenvalues, is removed. Thanks to this, the neural network is taught only with the most significant part of information that is contained in the signal of reduced dimensionality.

The number of connectors in the input layer is adapted to the chosen dimensionality of measured data after the PCA transformation.
TABLE I
PERCENT OF THE RETAINED INFORMATION AFTER PCA TRANSFORMATION

<table>
<thead>
<tr>
<th>PCA transformation</th>
<th>416-6</th>
<th>416-8</th>
<th>416-12</th>
<th>416-20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retained information</td>
<td>95.2%</td>
<td>97.1%</td>
<td>99.15%</td>
<td>99.95%</td>
</tr>
</tbody>
</table>

After extensive computer simulations with the PCA data dimensionality reduction procedure, we have chosen two structures of PCA data reduction system: 416-8, which retains 97.1% of information (measured as sum of the retained eigenvalues), and 416-6 – retaining 95.2% of information (see Table I).

From many tested network structures three best structures of the neural networks: 6-4-3, 6-6-3, 8-4-3 have been selected.

In this experiment we used the Multi Layer Perceptron (MLP) neural network [13] to solve the inverse problem. MLP teaching algorithm is based on gradient methods (Conjugate Gradients or Variable Metric Method), which are known to be the most efficient in the optimization theory [6]. However, the errors of local minimum of an objective function are a major difficulty in solving of real live technical problems. The number of local minimums increases contemporarily with the number of net weights, therefore the likelihood of a prematurely stopping of calculation rises for the nets of bigger sizes [5], [6], [8].

In our case the neural network size was significantly limited by unattended selection of dimension (PCA approach), what allowed providing proper robust generalization. Moreover, using PCA is necessary in EIT cases, when finding of independent measurements is very complicated. The number of measurements includes usually reciprocal pairs and the actual number depends on the geometry of model. One of the main advantages of PCA is its usage in EIT for different shapes of both the inside and outer object.

The neural networks allow identifying some of the vital parameters, for example like the position and radius of cylindrically shaped object.

Selected neural networks (see Fig. 4) consist of:

- 6 or 8 input connectors,
- one hidden layer with 4 or 6 neurons with sigmoidal function,
- \[ f(x) = \frac{1}{1 + e^{-x}} \] (6)

V. RESULTS

The neural networks were tested for the samples, which we got for a 2.5 cm high cylinder-shaped object with radius \( R = 0.8, 1.2, 2.0 \) cm, located in different positions inside the tank.

For each neural network we carried out a significant number of training procedures for different, initial weight values randomly generated within the range of \((0,1)\).

With the aid of the three-output neurons network structure, we have determined not only the radius \( R \) [13], but also the \( d \) and \( z \) coordinates of the internal object position (where \( d \) and \( z \) are the distance from the center of the cylinder to the center of the object, and the distance from the bottom of the tank to the center of the object, respectively).

The results for different neural networks are presented in Tables II and III.

Mean absolute error and maximum absolute error are calculated as:

\[ MEAN = \frac{1}{N} \sum_{i=1}^{N} |R^{(i)} - R_r^{(i)}| \] (7)

\[ MAX = \max_i |R^{(i)} - R_r^{(i)}| \] (8)

where

- \( R^{(i)} \) - exact radius (distance) of the object in the \( i \)-th test,
- \( R_r^{(i)} \) - reconstructed radius in the same test,
- \( N \) - number of testing data records.

We obtained the best results, i.e. the smallest absolute error (for the radius and the vector of movement in 3D), in case of the network with 6 input nodes and four neurons in the hidden layer (6-4-3), and rather poor results for the 8-4-3 network structure.
TABLE II
THE RESULTS IN THE FORM OF MEAN ABSOLUTE ERROR (IN CENTIMETERS)

<table>
<thead>
<tr>
<th>Distance from the center</th>
<th>6-4-3</th>
<th>6-6-3</th>
<th>8-4-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>1.72</td>
<td>1.67</td>
<td>1.89</td>
</tr>
<tr>
<td>Distance from the bottom</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>z</td>
<td>1.70</td>
<td>1.69</td>
<td>1.82</td>
</tr>
<tr>
<td>Radius of the object</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>0.41</td>
<td>0.42</td>
<td>0.57</td>
</tr>
</tbody>
</table>

TABLE III
THE RESULTS IN THE FORM OF MAXIMUM ABSOLUTE ERROR (IN CENTIMETERS)

<table>
<thead>
<tr>
<th>Distance from the center</th>
<th>6-4-3</th>
<th>6-6-3</th>
<th>8-4-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>2.86</td>
<td>2.95</td>
<td>2.01</td>
</tr>
<tr>
<td>Distance from the bottom</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>z</td>
<td>4.59</td>
<td>4.72</td>
<td>4.94</td>
</tr>
<tr>
<td>Radius of the object</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>1.401</td>
<td>1.39</td>
<td>1.90</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

Our results look promising and indicate that it is possible to reduce the time of the inverse problem solution, in relation to classical approach to EIT, by using neural networks.

However our experiments confirm that in this case, the selection of neural network size is very important. The PCA approach to the reduction of the number of measurements in EIT is novelty and gave the satisfactory results – significant limitation of the number of neurons in the input layer. In order to get acceptable results, it is necessary to conduct many teaching procedures for different number of neurons in the hidden layer of neural network.

REFERENCES


Magdalena Stasiak
Magdalena Stasiak was born in Lodz, Poland, in 1973. She was graduated from the Faculty of Technical Physics, Computer Science and Applied Mathematics of the Technical University in Lodz (Poland). She received the M.Sc. degree in computer science in 1997.

Since 1997 she has worked as an assistant at the Institute of Electrical Apparatus of the Technical University in Lodz. Since 2001 she has been doing Ph.D. in the Institute of Theory of Electrical Engineering, Measurement and Information Systems, Warsaw University of Technology. The subject of her Ph.D. is connected with Electrical Impedance Tomography. She is an author about 15 publications in conference proceedings and journals.

Konrad Nita
Konrad Nita was graduated from the Faculty of Electrical Engineering Warsaw University of Technology and received the M.Sc. degree in 2002.

The subject of his master thesis was connected with Electrical Impedance Tomography. He is a design engineer of several impedance tomographs. Since 2002 he has been a candidate for doctor’s degree and the subject of his Ph.D. thesis is connected with Boundary Element Method (BEM) and Electroencephalography Methods. He is an author about 10 publications in conference proceedings and journals.

Stefan F. Filipowicz
Stefan F. Filipowicz received the M.Sc. degrees in electrical engineering from the Technical University of Warsaw, in 1974 and the Ph.D. degree in electrical engineering in 1985.

Since 1974 he has been employed in the Institute of Theory of Electrical Engineering, Measurement and Information Systems, in the Faculty of Electrical Engineering Warsaw University of Technology.

He was for short periods Visiting Scientist in RIKEN Brain Science Institute in Japan and MEI in Moscow. He was engaged in many research projects for industry and research work, which was supported by Polish Science Research Committee. He is an author over 90 publications in conference proceedings and journals. His research interests include Image Reconstruction in Electrical Impedance Tomography (EIT), Industrial Process Tomography and Electroencephalography Techniques.

Jan Sikora
Jan Sikora received the B.Sc. and M.Sc. degrees in electrical engineering from the Warsaw University of Technology, in 1973, and the Ph.D. degree in electrical engineering from the same university, in 1979.

From 1973 to 1979, he was employed as an assistant and then as Assistant Professor in the Institute of Electrical Engineering and Electrical Measurements working on Numerical Methods of Electromagnetic Field Theory. He was promoted to Associate Professor in 1991. From 1997 till now, he is a Professor in the Department of Electrical Engineering at the Warsaw University of Technology. His research interests include Numerical analysis with the aid of Finite Element Method (FEM) and Boundary Element Method (BEM), Inverse Problems of Electromagnetic Fields Theory, Optimal Shape Design, Shape Reconstruction and Image Reconstruction in Electrical Impedance Tomography (EIT) and Optical Tomography (OT), Neural Network Application to Field Theory.

Prof. J. Sikora is a member of the Process Tomography Association and a member of Polish Electrical Association (S.E.P.).