A New Strategy for Unsubsidized Transmission Loss Allocation

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Abstract—This paper proposes a new method for transmission loss allocation. In this method, initially, the share of each bus in the transmission line losses is determined by using loss equations for only one transmission line. Then, it is applied to the other transmission lines of the network and for each line the share of all buses is found. The total share of each bus is concluded from the sum of shares. To be preferred over other methods, the proposed method is modified to remove the negative loss allocation. This method is based on the electric relations between network elements and the injected power in various buses considering the network topology. The proposed algorithm is studied on a typical 3-bus topology and applied to IEEE 14-bus network. In comparison to other methods, the modified solution removes negative loss allocation which makes it more reasonable and applicable.

Index Terms—Transmission loss allocation, line losses, injected power, network topology.

I. INTRODUCTION

In power networks, a few percentage of the transmission power is always lost. The main part of these losses is due to the flow of current in ohmic resistance of the transmission lines. In traditional power systems, all attempts are done in order to minimize the network losses in terms of costs. The overall cost of losses is added to other generation and transmission costs and forms the total operation cost of the network. But in deregulated power systems, every player of the system posses the separate operation cost of the network. The overall cost of losses is added to income and costs. Thus, determining their share in total network costs including the losses is unavoidable [1]. On the other hand, in deregulated power systems, regardless of losses optimization, another serious question is posed that how the total cost of losses should be paid by the power market players. In the pool-based electricity market, the loss allocation helps to recognize the share of each generation or consumption unit from the total network losses. So, the Independent System Operator (ISO) could receive the losses costs from each of the market participants and could return it to the generation companies [2].

In the markets which are based on bilateral contracts, the losses of each contract should be specified in the contract content and its support source should be determined. In spite of the high importance of loss allocation to the participants, technically and economically, due to complexity, nonlinear nature and high dependence of loss function on different variables, no comprehensive and precise method which can be practically employed has been presented hitherto. But due to significance of this issue, the various methods have been published in the papers which most of them have used simple assumptions. In Pro rata method [3], that is the most popular one, the loss is allocated to each generator or load, regarding their power injection to network, rather than total network power injection. In fact, this method doesn’t consider the location of losses or network topology. So, a remote generator or load that certainly causes more power losses treats the same as other near network players.

Proportional sharing principle is based on a non-provable theorem that assumes the inflow powers are proportionally shared between the outflows power at each network bus [4]-[5]. This method uses an additional assumption, which losses of each branch allocate in 50 percent to its sending and ending nodes.

Reference [6] suggests a radial equivalent network for transmission system that each generator may have an individual connection to all loads and in this way makes it possible to allocate system loss. But in this method, the total losses may not equal to real system loss and also it is too complicated for real power systems.

References [7]-[9] trace losses back from the network branch to the load. These strategies generally involve an algorithm to determine how the losses are attributed to generators/loads as one traverse through the network. Either the algorithm allows loss attribution to be specified according to a user-defined formula, or a loss sharing formula is implicitly included.

Cooperative game theory was utilized to allocate transmission costs to wheeling transactions in [10]. A method, based on circuit theory, has also been proposed to trace power from either the seller's and/or the buyer's point of view [11]. In [12], line power flows are first unbundled into a sum of components, each corresponding to a bilateral transaction. In these schemes, the coupling terms among the components appeared in the line losses could be allocated to individual bilateral transactions. Reference [13] uses a process whereby individual bilateral transactions are gradually incremented along a given path of variation. Each bilateral transaction might elect to have its losses supplied by a separate slack generator.

In [9] starting from an ac load flow solution, the contributions of all generators to the flow in each circuit are evaluated and the same proportion is used to share circuit losses among them. The Z-bus loss allocation uses the total system loss formula and tries to write it in the
A three-bus test system is employed to show the main steps of the proposed technique. In addition, utilizing the numerical results obtained from the IEEE 14-bus test system, the quality of the loss allocation determined via the proposed methodology is illustrated.

II. PROPOSED METHOD

In power networks, the total loss is due to the power flow in transmission lines and is sum of the losses of all transmission lines. Assuming that the power flow results of the network are available, the connected transmission line between \( i \)-th and \( j \)-th buses is considered as shown in Fig. 1.

The equation of the line current flow with respect to the network impedance and admittance matrix, the voltage across the line and injected currents to buses can be written as follows

\[
I_y = y_{yj} \times (V_i - V_j) \\
I_y = R(I_y) + j \mathcal{Z}(I_y)
\]

also, the bus voltage equations with respect to the injected bus currents can be calculated as

\[
V_i = \sum_{k=1}^n z_{ik} I_k \\
V_j = \sum_{k=1}^n z_{jk} I_k
\]

inserting the above equation in (1) yields

\[
I_y = \sum_{k=1}^n y_{yk} (z_{ik} - z_{jk}) I_k
\]

the real and imaginary parts of the line current can be written as (4)

\[
\Re[I_y] = \sum_{k=1}^n (a_{yk} I_k - b_{yk} I_k) \\
\Im[I_y] = \sum_{k=1}^n (a_{yk} I_k + b_{yk} I_k)
\]

the share of bus \( k \) from the line admittance current is as follows

\[
\Re[I_y]^i = a_{ik} I_y - b_{ik} I_y \\
\Im[I_y]^i = a_{ik} I_y + b_{ik} I_y
\]

also, the transmission line loss can be written as

\[
P_{loss,y} = r_y |I_y|^2 \\
I_y = \Re[I_y] + j \Im[I_y]
\]

according to the real and imaginary parts of the line current, we have
\[ |I_t|^2 = |R(I_t)|^2 + |\Im(I_t)|^2 \]
\[ |R(I_t)|^2 = \left| \sum_{i=1}^{n} (a_{ij} I_{ip} - b_{ij} I_{iq}) \right|^2 \]
\[ = \sum_{i=1}^{n} (a_{ij} I_{ip} - b_{ij} I_{iq}) \times \left( \sum_{i=1}^{n} (a_{ij} I_{ip} - b_{ij} I_{iq}) \right) \]
\[ = \sum_{i=1}^{n} \left( (a_{ij} I_{ip} - b_{ij} I_{iq})^2 + (a_{ij} I_{ip} - b_{ij} I_{iq})^2 \right) \times \sum_{i=1}^{n} \left( (a_{ij} I_{ip} - b_{ij} I_{iq}) \right) \]
\[ = \sum_{i=1}^{n} \left( (a_{ij} I_{ip} - b_{ij} I_{iq}) \right) \times \sum_{i=1}^{n} \left( (a_{ij} I_{ip} - b_{ij} I_{iq}) \right) \]
\[ = \sum_{i=1}^{n} \left( (a_{ij} I_{ip} + b_{ij} I_{iq}) \right) \times \sum_{i=1}^{n} \left( (a_{ij} I_{ip} + b_{ij} I_{iq}) \right) \]
\[ |\Im(I_t)|^2 = \left| \sum_{i=1}^{n} (b_{ij} I_{ip} + a_{ij} I_{iq}) \right|^2 \]
\[ = \sum_{i=1}^{n} (b_{ij} I_{ip} + a_{ij} I_{iq}) \times \left( \sum_{i=1}^{n} (b_{ij} I_{ip} + a_{ij} I_{iq}) \right) \]
\[ = \sum_{i=1}^{n} \left( (b_{ij} I_{ip} + a_{ij} I_{iq})^2 + (b_{ij} I_{ip} + a_{ij} I_{iq})^2 \right) \times \sum_{i=1}^{n} \left( (b_{ij} I_{ip} + a_{ij} I_{iq}) \right) \]
\[ = \sum_{i=1}^{n} \left( (b_{ij} I_{ip} + a_{ij} I_{iq}) \right) \times \sum_{i=1}^{n} \left( (b_{ij} I_{ip} + a_{ij} I_{iq}) \right) \]
\[ further \] for bus \( k \), we have
\[ S_i = V_i I_i \Rightarrow I_i = \frac{P_i - jQ_i}{V_i} \]
\[ = \frac{(P_i - jQ_i) \times (\cos \Delta_i + j \sin \Delta_i)}{|V_i|^2} \]
\[ I_p = \frac{P_i \cos \Delta_i + Q_i \sin \Delta_i}{|V_i|^2} \]
\[ I_q = \frac{P_i \sin \Delta_i - Q_i \cos \Delta_i}{|V_i|^2} \]

so, (6) can be written as
\[ P_{loss} = r_g \times |I_t|^2 = r_g \left| R(I_t) \right|^2 + r_g \left| \Im(I_t) \right|^2 \]
\[ = r_g \sum_{i=1}^{n} \left( (a_{ij} I_{ip} - b_{ij} I_{iq})^2 + (a_{ij} I_{ip} - b_{ij} I_{iq})^2 \right) \times \sum_{i=1}^{n} \left( (a_{ij} I_{ip} - b_{ij} I_{iq}) \right) \]
\[ = r_g \sum_{i=1}^{n} \left( (a_{ij} I_{ip} + b_{ij} I_{iq}) \right) \times \sum_{i=1}^{n} \left( (a_{ij} I_{ip} + b_{ij} I_{iq}) \right) \]
\[ = r_g \sum_{i=1}^{n} \left( (a_{ij} I_{ip} + b_{ij} I_{iq}) \right) \times \sum_{i=1}^{n} \left( (a_{ij} I_{ip} + b_{ij} I_{iq}) \right) \]
according to (10), some buses may have a negative loss allocation. In the next section a solution for removing the negative loss allocation is proposed.

A. A Solution for Removing Negative Loss Allocation

In the previous section, according to the network topology and the real and imaginary parts of the injected currents to buses, a new method for loss allocation has been proposed. In the proposed method some buses might have the negative loss allocation. The negative loss allocation to some buses is due to flowing of their currents in opposite direction with respect to the dominant flow in some transmission lines. In fact, in an appointed transmission line the injected current to a bus may flow in the opposite direction with respect to the total injected current resultant of the other buses. So, the loss allocation equation of these buses has a negative result. Here, the proposed method is modified to remove the negative share of buses. In this modification the decreasing role of such buses that reduce the line losses is considered. According to (4) and (5), the new solution to remove the negative values of loss allocation is introduced

if \( R(I_g) \times R(I_g) \leq 0 \) then \( P_{loss}^{k} = 0 \) (11)

The above equation shows that if the real part of the current in line \( i-j \), contributed by bus \( k \) is in the opposite direction with respect to the real part of current contributed by other buses, the allocated loss to the bus \( k \) will be zero. On the other hand, if the real part of the current in line \( i-j \), contributed by bus \( k \) is in the same direction with respect to the real part of current contributed by other buses, the modified real part of line \( i-j \) current should be expressed with respect to the real part of the currents contributed by buses which are in the same direction. In fact, the real parts of such currents that are in opposite direction with respect to the real part of the line \( i-j \) current are negligible (see (12))

if \( R(I_g) \times R(I_g) \geq 0 \Rightarrow R(I_g)_{\text{modified}} = \sum_{k=1}^{N} R(I_g)^k \) (12)

so, the transmission loss due to the real part of the bus currents in line \( i-j \) can be written as the following

\[ P_{loss_{i-j}}^{k} = r_g \times \left| R(I_g)_{\text{modified}} \right|^2 \]
\[ = r_g \sum_{k=1}^{N} \left( \sum_{k=1}^{N} \left( R(I_g)_{\text{modified}} \right)^k \right) \times \sum_{k=1}^{N} \left( R(I_g)_{\text{modified}} \right)^k \]
\[ = r_g \sum_{k=1}^{N} \left( \sum_{k=1}^{N} \left( R(I_g)_{\text{modified}} \right)^k \right) \times \sum_{k=1}^{N} \left( R(I_g)_{\text{modified}} \right)^k \]

therefore, the contribution of bus \( k \) in the losses of line \( i-j \) due to the real part of currents is as follows

\[ P_{loss_{i-j}}^{k} = \left( \sum_{k=1}^{N} \left( R(I_g)_{\text{modified}} \right)^k \right) \times \left( \sum_{k=1}^{N} \left( R(I_g)_{\text{modified}} \right)^k \right) \]

also, the above equations can be obtained in a similar way for imaginary part of the line \( i-j \) current. So, the contribution of bus \( k \) in the losses of line \( i-j \) due to the imaginary part of current can be calculated as

\[ P_{loss_{i-j}}^{k} = \left( \sum_{k=1}^{N} \left( R(I_g)_{\text{modified}} \right)^k \right) \times \left( \sum_{k=1}^{N} \left( R(I_g)_{\text{modified}} \right)^k \right) \]

therefore, the allocated loss value to the bus \( k \) is as follows

\[ P_{loss_{i-j}}^{k} = \left( \sum_{k=1}^{N} \left( R(I_g)_{\text{modified}} \right)^k \right) \times \left( \sum_{k=1}^{N} \left( R(I_g)_{\text{modified}} \right)^k \right) \]
according to (12) we have
\[ P_{\text{loss}_{ij}} \neq P_{\text{loss}_{ij}} \]

finally, by normalizing the allocated loss values, the normalized contribution of bus \( k \) can be calculated as follows
\[ P_{\text{loss}_{ij}}^{\text{normalized}} \frac{\sum_{i,j} P_{\text{loss}_{ij}}^{\text{normalized}}}{P_{\text{loss}_{ij}}^{\text{normalized}}} \times P_{\text{loss}_{ij}} \]

\[ (18) \]

### III. NUMERICAL RESULTS

A simple example without fixed losses is selected to show the application of the proposed allocation method. Fig. 2 and Table I show a 3-bus system and its transmission line data, respectively. The generator (located at buses 1) supplies the power demand located at buses 2 and 3.

Table II summarizes the power flow solution by the Newton-Raphson method. Columns 2, 3, 4, 5, 6, and 7 show respectively, bus voltage magnitude, bus voltage angle, active generated power, reactive generated power, demand active power, and demand reactive power.

Loss allocation to each bus of the typical 3-bus network is illustrated in Table III. As shown in Fig. 2, bus 3 injects the current in the opposite direction with respect to the resultant current of the network in line 2-3. So, the allocated loss of the line 2-3 to the bus 3 has a negative value. The negative allocated loss to the bus 3 is due to its decreasing role in reduction of the network losses. On the other hand, if this bus increases the network losses, it receives the positive loss allocation cost.

To study the performance of the proposed method, the active load of all buses has been increased. For each case, using the proposed method, the loss allocation has been done. The variations of allocated loss to each bus and the network lines losses due to the load increase in bus 2 from zero to 1000 MW have been illustrated in Figs. 3 and 4 respectively.

As shown in Figs. 3 and 4, by increasing the load in bus 2, the power flow in lines and proportionally the network lines losses have increased. Thus, the allocated losses to buses 1 and 2 have been increased. By increasing the load of bus 2, the power flow in line 3-2 from bus 3 toward bus 2 has increased. Therefore, the load of bus 3 has a decreasing role in flowing power of line 3-2. So the share of bus 3 in the allocated loss should be constant that has been yielded by the proposed method.

Table IV shows the allocated losses to the buses of typical 3-bus network suggested by the modified method.

As illustrated in row 3 of Table IV, the allocated loss to the bus 3 is zero. This zero allocated loss to bus 3 is due to the opposite direction of bus 3 injected current to line 3-2.
with respect to the current contributed by other buses. Although the share of bus 3 is zero in loss allocation process, due to the usage of the network to transmit the power in line 3-2, it should pay the transmission service costs.

The proposed method has been tested on a set of networks with different sizes, and has been compared to some of the most well-known alternative algorithms described in the literatures. For this purpose, the IEEE 14-bus is used to show the result of presented method in comparison with pro-rata method (PR) based on complex power injection, incremental transmission loss method (ITL) as the most referenced loss allocation methods and Z-bus method.

As can be seen in Fig. 5, the IEEE 14-bus system has five controlled buses including two generator buses and bus 1 is considered as the slack bus. According to the power flow results in Table V, bus 1 provides 13.54 MW that should be divided between market players. Table VI shows the results of proposed method compared to other methods.

The proposed method, similar to the impedance matrix method, emphasizes on the location of buses and network topology. According to Table VI, bus 1 that provides about 85 percent of the total generation always has the highest contribution to the loss allocation in the all methods. Also, bus 3 which comprises 36 percent of system load, after bus 1, receives the highest loss allocation by all methods. Bus 2, because of the appropriate location in the network has the least loss allocation value.

In order to analyze the effect of generation and consumption distribution, a 100 MW generator is added to the bus 8.

Table VII shows the main variation of transmission system losses, which has been led to 50 percent decrease in total network losses. Therefore, the allocated loss to the buses has changed and share of bus 1 decreased from 62 percent in previous state to 42 percent in this condition. But due to the high distance of bus 3 from generation center, its contribution to the allocated loss has no major variation.

When the generation of bus 8 is raised from zero to 300 MW, the network losses firstly decrease and after reaching a minimum point, as illustrated in Fig. 6, it rises. Also, the slight reduction and then the growth of allocated loss shows the introduced method has considered the network topology and the injected currents to the network buses.
The loss allocation results of modified method, compared to other ones, for IEEE 14-bus system have been shown in Table VIII.

As can be seen in Table VII, bus 8 with negative loss allocation, has 0.0296 MW of total losses determined by the modified method. In addition to considering the decreasing role of generators in the network, the modified method considers the location of generators in the network and removes the negative loss allocation values as well.

In practical power systems, there are some loads with low power factor causing the network losses to increase and the transmission line capacity to decrease. This situation is also taken into account by the presented method.

Assuming the increment of reactive load of bus 14 from 5 MVAr to 50 MVAr, the effect of this growth on the connected line, namely line 13-14, and the allocated loss to bus 14 has been studied. As it can be seen from Fig. 7, due to this growth, the losses of line 13-14 and the allocated loss to the bus 14 is increased. This study results show that the proposed method can consider the effect of loads with low power factor in the increment of losses in network.

IV. CONCLUSION

In this paper, using the relations of a transmission line loss with respect to bus injected currents, a new method for loss allocating was proposed. In addition, the proposed method was modified and a new solution was presented to remove the negative loss allocation. This algorithm is based on the network main relations and the injected power in various buses. According to the results this method does not produce negative share of loss for buses which makes it more reasonable and applicable than other methods.

REFERENCES


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