Calculation of Analytical Expressions for Measured Percentage Depth Dose Data in Megavoltage Photon Therapy

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Abstract

Background: In external radiation therapy, the percentage depth dose (PDD) is an important factor for estimating patient dose and dose distribution in target volume; therefore, its exact measurement or calculation is important. The aim of this study was to evaluate analytically the dose received by different points in water phantom and to compare it with dosimetry measurement data.

Methods: To find the dose distribution throughout the tumor volume, first, the mathematical approach was performed for derivation of percentage depth dose photon beams of 6MV and 18MV Varian accelerator. Second, by dosimetry for different fields in different depths of water phantom, one can parameterize the obtained formula for percentage depth dose.

Results: By comparing the mathematical and dosimetry results, the parameters of PDD-expression were computed in terms of the dimension of equivalent square field in different depths. From this formula, one can find the PDD for any fields in different depths, surface skin dose, and depth of build-up region of dose distribution, which are in agreement with empirical results with $R^2 > 0.995$, showing a good agreement with the experimental data.

Conclusion: So one can measure the surface skin-dose, the depth of build-up region, and their variations in terms of square field size exactly; the measurement of these quantities have some technical problems in radiation dosimetry.

Keywords: Radiation therapy; Build-up region; Surface dose; Dosimetry

Introduction

Dose calculation and dosimetry of treatment beam are the most important part of radiation treatment planning since successful treatment requires an accurate delivery of dose to the tumor volume. Although computer treatment planning system which considerably lightens the dose calculation workload of a department is now well established, it is still necessary to be aware of the methods used to store the basic information required for individual radiation fields. Physicists are trying to develop new methods for the measurement of treatment factors in radiation therapy quantities which increase patient survival and minimize morbidity. Apart from accuracy of dose in a given point, the homogeneity of dose distribution in the tumor volume is also crucial for successful radiotherapy. Many researchers in radiotherapy try on dose measurements and calculation with introducing new methods for dose calculation.1,5

It is believed that decreasing 10-15% of dose delivery will result in a decrease in the chance of cure by a factor 2 or 3 while an increase in the dose will similarly increase the chance of irreversible damage.6 ICRU (International Commission on Radiation Units and Measurements) recommends an accuracy of $\pm 5\%$ in delivery of dose in radiotherapy.7 More recently, a tolerance of 3.5% has been suggested.8 Also in reported No-50 has recommended dose homogeneity of
between -5% and 7% of prescribed through the planning target volume (PTV).

The present research first aimed to find a reasonably accurate mathematical expression for depth dose distribution, skin surface dose, depth of buildup region, and photon attenuation coefficients in megavoltage photon beam radiation therapy based on physical aspects and analytical considerations.

**Material and Methods**

This work was carried out at the radiotherapy and oncology center in Golestan Hospital of Ahwaz, Iran. A Scantronix blue phantom (50 cm x 50 cm x 50 cm) was used for evaluating PDD of radiation fields 5x5 cm up to 40 x 40 cm in any point of irradiated volume. A 0.13 ml ionization chamber was used for measurement. This was installed on the robotic movable arms of blue phantom, moving step by step. Another ionization chamber was fixed on the head of radiation device as the reference chamber. The radiation device used was Varian 2100C/D accelerator with two types of photon energy 6 and 18 MV.

CU500E unit was used as computer interface to read the chambers’ output from two different channels and control blue phantom arms. Omni-Accept pro 6.5 software was connected to the interface and used for collecting and recording data on the computer. As mentioned above, two chambers were used. The first chamber could move and the other was fixed on the head of the radiation device. The fixed chamber was placed out of the lines on which the moving chamber moved. The outputs were read by electrometer and then a ratio of these readings was used to make PDD or profile data.

Dosimetry shows that for high energy photon, PDD initially increases rapidly below the surface until the depth of maximum dose is attained. Beyond this depth, the dose decreases slowly with depth. We aimed to find a simple yet reasonably accurate mathematical expression for depth dose distribution of a beam coming through depth based on physical aspects.

In clinical practice, the peak absorbed dose on the central axis is sometimes called the maximum dose that occurs at the end of the build up region. By definition, PDD is:

\[ PDD = \frac{D}{D_{\text{max}}} \]

where \( D \) is dose in depth of \( h \) in water phantom and \( D_{\text{max}} \) is dose in depth of \( x_{m} \) (depth of maximum dose). From mathematical point of view, one can write the differential equation governing the absorbed dose as the following:

\[ \frac{dD}{dx} = -\mu_{1}D + \mu_{2}D' \]

\[ D' = D'e^{-\mu_{1}x} \]

where \( D \) is the total absorbed dose due to absorption of photon in the medium and \( D' \) is the dose of secondary photons due to interaction of primary photons with medium and \( \mu_{1}, \mu_{2} \) are attenuation coefficients for these photons. By solving the Eqs. 2, one can find \( D \) and therefore PDD in the following way:

\[ PDD(x) = a_{1}e^{-\mu_{1}x} + a_{2}e^{-\mu_{2}x} \]

where the attenuation coefficients \( \mu_{1} \) and \( \mu_{2} \) are functions of treatment field size, \( L \) and the unknown coefficient \( a_{1} \) and \( a_{2} \) are also functions of \( L \) and can be written in terms of known function \( x \) and \( s = PDD(0) \) is the fractional surface dose.

\[ \frac{dPDD(x)}{dx} \bigg|_{x=0} = 0 \]

\[ S = PDD(0) \]

Substituting in Eq. 3 yields

\[ PDD(x) = \frac{1}{2}\left[ \mu_{1}e^{\mu_{1}(x-x_{s})} - \mu_{2}e^{\mu_{2}(x-x_{s})} \right] \]

By comparing the experimental dosimetry data and the Eq. 3 by Tblcurve2D software, one can find the functional form of \( S, x_{s}, \mu_{1} \) and \( \mu_{2} \).

For statistical evaluation, Tblcurve2D software was used, that has 4 common goodness of fit statistics. In the following formulae descriptions, SSM is the sum of squares about the mean, SSE is the sum of squared errors (residuals), \( n \) is the total number of data values, and \( m \) is the number of coefficients in the model. DOF, the degree of freedom, is \( n-m \).

**Coefficient of Determination (r-squared)**

\[ r^{2} = 1 - \frac{\text{SSE}}{\text{SSM}} \]

Degree of Freedom Adjusted Coefficient of Determination

\[ \text{DOF}r^{2} = \frac{(1-\text{SSE} \times (n-1))}{\text{SSM} \times (\text{DOF} - 1)} \]

**Fit Standard Error (Root MSE)**

\[ \text{StdErr} = \sqrt{\frac{\text{SSE}}{\text{DOF}}} \]
F-statistic

\[
F - \text{stat} = \frac{(SSM - SSE)}{m-1} \frac{SSE}{DOF}
\]  
(9)

As a fit becomes more ideal, the \( R^2 \) values approach 1.0 (0 represents a complete lack of fit), the standard error decreases toward zero, and the F-statistic goes toward infinity.

Results

By using the Tblcurve2D on experimental dosimetry results based on TRS398, one can parameterize the Eq.5 with coefficient determination \( R^2 > 0.99 \). Figures 1-3 shows the variation of PDD versus different depths in energy of 6 and 18 MV Varian accelerator system 2100C/D for treatment fields size 5x5cm\(^2\), 10x10cm\(^2\), and 20x20cm\(^2\) from dosimetry data and Eq.5. The functional form of \( s, \mu_1, \mu_2, \) and \( x_n \) in terms of treatment field size \( L \) (in centimeter) was obtained by Tblcurve2D statistics as the following:

\[
\mu_1 = a_1 + b_1 \times L \quad (10)
\]

\[
\mu_2 = a_2 + b_2 \times L \quad (11)
\]

\[
S = a + b \times L \quad (12)
\]

\[
x_n = A + B \times \arctan \left( \frac{L - C}{D} \right) \quad (13)
\]

where the coefficients \( a_1, a_2, b_1, b_2, a, b, A, B, C, D \) for 6\&18MV photon beam tabulated with \( R^2 > 0.96 \)

<table>
<thead>
<tr>
<th></th>
<th>6MV</th>
<th>18MV</th>
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<tbody>
<tr>
<td>( a_1 )</td>
<td>0.005</td>
<td>0.004</td>
</tr>
<tr>
<td>( b_1 )</td>
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<td>0.0</td>
</tr>
<tr>
<td>( a_2 )</td>
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<td>0.08</td>
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<tr>
<td>( b_2 )</td>
<td>0.0</td>
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<td>( a )</td>
<td>4.77</td>
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</tr>
<tr>
<td>( b )</td>
<td>1.16</td>
<td>1.258</td>
</tr>
<tr>
<td>( A )</td>
<td>1.93</td>
<td>3.46</td>
</tr>
<tr>
<td>( B )</td>
<td>0.14</td>
<td>0.79</td>
</tr>
<tr>
<td>( C )</td>
<td>18.84</td>
<td>9.71</td>
</tr>
<tr>
<td>( D )</td>
<td>-7.85</td>
<td>-10.14</td>
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Discussion

In comparison of the dosimetry results with the prediction of formula\(^5\) for any fields in different depths, one can see a good agreement and accuracy. Furthermore, one can obtain the important quantities like skin surface, the depth of maximum dose and coefficient attenuation of any energy device for different field sizes in any depth. This has an important role in treatment planning and also protection of patients.\(^\text{10-15}\)

It seems that in Eq.5, \( \mu_1 \) is the attenuation coefficient of photons and \( \mu_2 \) is the attenuation coefficient of photons.
electrons. These parameters are very important in radiation therapy and their measurement has some difficulties. The Eq. 11 for high energy photon (18MV) the attenuation coefficient in function of field size. That this a new result and it’s average is in agreement with experimental data.\(^{16}\)

Therefore, the behavior of electrons in the buildup region in this method can be understood completely by viewing the Eq.5 and then the measuring of skin dose can be done exactly. Moreover, this study shows that any increase in the size of fields results in a decrease in the depth of maximum dose and an increase in the skin surface dose, that are in complete agreement with the reported data.

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References


