Upper Limit to Compression Reinforcement Ratio in Flexural HSRC Members

A.A. Maghsoudi1, H. Akbarzadeh Bengar2

1Assistant Professor, Civil Engineering, Kerman University, Kerman, Iran.
2PhD. Candidate, Kerman University, Kerman, Iran, academic member of Shomal University.

Abstract: limit to the tension reinforcement ratio in flexural high strength reinforced concrete (HSRC) members is based on the requirement that tension failure as sufficient rotation capacity are ensured at ultimate limit state. However, the provisions for the maximum amount of compression reinforcement ratio ($\rho_{\text{max}}'$) in a doubly reinforced section are not associated with any rational derivation. A quantitative measure to evaluate an upper limit to the compression reinforcement ratio ($\rho_{\text{max}}'$) of such flexural HSRC members is proposed. The quantitative criterion to $\rho_{\text{max}}'$ can be derived from i) steel congestion and ii) considerations that are related to the diagonal compression bearing capacity of the members.

In this paper it is shown that, when flexural loading is dominant the shear loading, the limit to $\rho'$ is set by the steel congestion criterion. Parameters that affect this limit such as, $f_c'$, beams geometry (cross sectional dimension and concrete cover), and bars diameter are deeply investigated and the expressions were derived to provide an additional tool for a better design and assessment of the flexural capacity of HSRC members considering different end conditions and loading arrangement.

Keyword: Steel Congestion Criterion, End Conditions and Loading, HSRC and Upper Limit to $\rho'$.

1. Introduction

RC and HSRC design codes set an upper limit to the bending resistance of flexural members with only tension reinforcement (considered here as $M_{cd,\text{max}}$). The limit is set in order to ensure tension failure as well as sufficient rotational capacity at ultimate limit state, either by maximum permissible tension reinforcement ratio [1] or by a maximum depth of the concrete compression area [2&3]. It is also possible an additional resistance moment $\Delta M$ added to the resistance moment $M_{cd,\text{max}}$ by adding compression reinforcement and an additional amount of tension reinforcement for equilibrium. Thus, while the upper limit to the tension reinforcement ratio is derived from the “proficient-design” considerations described above, the limitation on the total amount of longitudinal reinforcement ratio (and, therefore, on the compression reinforcement ratio) in flexural RC and HSRC members is not associated with any rational derivation.

Lin and Furlong [4] proposed rational derivations for longitudinal steel limits of RC columns. Dancygier and Eid [5, 6] proposed an upper limit to the amount of longitudinal compression reinforcement in flexural RC simply supported beams. No report was observed for an upper limit to compression reinforcement ratio in flexural HSRC members.

This paper proposes an upper limit to the amount of longitudinal compression reinforcement ratio in flexural HSRC members considering different end conditions and loading arrangement based on construction requirements or steel...
congestion, plus the known proficient-design requirement (i.e., tension failure and sufficient rotation capacity at ultimate limit state). This criterion is referred to in the text as the congestion criterion. Another criterion is associated with the prevention of diagonal compression failure due to shear loading which is not further considered.

2. Maximum Steel Ratio Based On Congestion Criterion for HSRC Members

As stated, for RC columns and for RC flexural elements [4, 5, 7], perhaps the most obvious reason for an upper limit to compression reinforcement in flexural members is the congestion of space if too many bars are used. Consider a rectangular beam (for positive and negative moment zone) with a width $b$, for which construction requirements determine the longitudinal steel cover (note that $c_1$ includes both the clear concrete cover plus the stirrup diameter (Fig.1) and the minimal spacing between horizontal and vertical longitudinal bars are respectively $S_h$ and $S_v$. An upper limit to the number of tension and compression reinforcement bars in HSRC members is attained when they are placed respectively, in
a maximum number of rows, \( n \), and \( n' \) (Fig.1). The maximum numbers, \( m \), and \( m' \), of tension and compression bars that can be placed per row are given by

\[
m = \text{ROUND DOWN} \left( \frac{b - 2C_z + S_h}{S_h + \phi} \right) \quad (1a)
\]

\[
m' = \text{ROUND DOWN} \left( \frac{b - 2C_z + S_h}{S_h + \phi'} \right) \quad (1b)
\]

where \( \phi \) and \( \phi' \) are the tension and compression bar diameters, respectively.

To determine the maximum total number of longitudinal compression bars based on a concrete standard, it is necessary to consider geometrical construction limitations on placement and spacing of the reinforcing steel Eqn. (1) as well as the following further requirements:

**Equilibrium**, which for rectangular cross sections is giving by

\[
\rho = \rho' + \frac{\beta_3 y}{\alpha} \frac{f_c'}{f_y}
\]  
(2)

where \( \rho \) and \( \rho' \) are the tension and compression reinforcement ratios, respectively; \( f_c' \) is the characteristic cylinder compressive strength of concrete; \( f_y \) is the reinforcement steel yield stress; \( y \) is the height of the concrete compression area; \( \beta_3 \) is the ratio between the height of the stress block and \( y \); \( \alpha \) is the stress block coefficient.

**Tension failure mode** (i.e., at ultimate limit state the tension reinforcement strain \( \varepsilon_s \) has reached its yield strain \( \varepsilon_y \)). Here it is further by use of the Dancygier’s assumption in which they assumed an efficient design is one in which the compression reinforcement’s stress at ultimate limit state is at its design level (e.g., yield stress):

\[
\varepsilon_{s,min} = \frac{\varepsilon_{cu}}{y} \left( h - y - \varepsilon_y - n'(\phi' + \varepsilon_y) - 0.5\phi' \right) \geq \varepsilon_y
\]  
(3)

\[
\varepsilon_{s,min} = \frac{\varepsilon_{cu}}{y} \left( h - y - \varepsilon_y - n'(\phi' + \varepsilon_y) - 0.5\phi' \right) \geq \varepsilon_y
\]  
(4)

where \( \varepsilon_{s,min} \) and \( \varepsilon_{s,min}' \) are the steel strains (both positive) of the longitudinal tension and compression bars, located closest to the neutral axis (Fig. 1); \( \varepsilon_{cu} \) is the ultimate concrete strain; and \( \varepsilon_y \) is the reinforcement yield strain.

**Proficient design** requires sufficient rotation capacity at the ultimate limit state [1,2, 3]. It is ensured by the known provisions, either to the relation between the tension and compression reinforcement ratios with respect to the balance condition [1], or to a minimum tensile steel strain (ACI 2002, Appendix B). Alternatively, the requirement for sufficient rotation capacity is set by a provision to the concrete compression area acted by an equivalent stress block \( A'_{c} \) [2]. In the later case \( A'_{c} \) is limited to \( A'_{c,max} \). For rectangular cross sections \( A'_{c,max} \) can be written as follows:

\[
K_{max} = \beta_{3} h d
\]  
(5)

The coefficient \( \beta_3 \) is set either according to an empirical database [8] or by resolving the conditions that are set for the ratio or strain of the tension reinforcement [1, 2, 9]. For RC members both approaches give, for rectangular cross sections, \( \beta_3 = 0.3-0.5 \). Where as, for HSC members based on ACI, \( \beta_3 \) is given by

\[
\beta_3 = 0.75 \beta_1 \left( \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y} \right)
\]  
(6)

Using these terms, and assuming linear strain distribution along the cross section’s height (section A and B in Fig. 1), this requirement for rectangular cross sections is given by

\[
y \leq y_{max} = \frac{\beta_{3} h d}{\beta_3}
\]  
(7)

A solution to Eqn. (2) and to the geometric
conditions in Eqn. (1) is sought, subject to the constraints in Eqn. (3), (4) and (7). It yields \( n, n', m, m', m_1 \) and \( m'_1 \), where \( m_1 \) and \( m'_1 \) are the number of longitudinal tension and compression bars, respectively, in the rows that are located closest to the cross section’s neutral axis. If both \( m_1 \) and \( m'_1 \) are less than \( m \) and \( m' \) Eqn. (1), then these rows can be further “filled” with an additional equal number of tension and compression bars (of equal diameters) \( m_2 \), which is given by

\[
m_2 = \min(m - m_1, m' - m'_1) \\end{align}
(8)

Hence, the total numbers of tension and compression bars in the rows that are located closest to the neutral axis are, respectively, \( m_1 + m_2 \) and \( m'_1 + m_2 \). Thus, the maximum reinforcement ratios (for rectangular cross sections A or B) that comply with construction and with proficient-design requirements are given by

\[
\rho_{\text{max}} = \frac{[(n - 1)m + m_1 + m_2] \pi \phi^2}{4bd} \tag{9a}
\]

\[
\rho'_{\text{max}} = \frac{[(n' - 1)m' + m'_1 + m_2] \pi \phi'^2 + m_2 \pi \phi^2}{4bd} \tag{9b}
\]

Note that the effective depth \( d \), which is the distance from the extreme compression fiber to the centroid of the tension reinforcement, is given by

\[
d = h - c_t - 0.5\phi - \frac{1}{2}(\phi + S_z)(n - 2)(n - 1)m + [(m_1 + m_2)(n - 1)(\phi + S_z)]}{m(n - 1) + m_1 + m_2} \tag{10}
\]

3. Examples

The solution that yields \( \rho_{\text{max}} \) and \( \rho'_{\text{max}} \) in Eqn. (9) was obtained for various problem parameters as well as the different beams end condition and loading type as shown in Table 1 through an iterative procedure (Fig. 2). A computer program based on the Fortran language was developed and the results obtained are drawn by using the Excel software as shown in Figs. 3-5. The calculation were carried out for the following assumptions used in ACI for HSC; \( e_{cu} = 0.003, \alpha = 0.85, \beta_1 = 1.09 - 0.008 f'_c \) where \( 0.65 \leq \beta_1 \leq 0.85 \). The values of \( S_h = 25 \) and \( S_v = 25 \) mm if \( \phi \) and \( \phi' \) are less than 25 mm, otherwise their values are taken equal to the bar diameter. The \( c_f \), is considered to be 38 mm for the case of the concrete subjected to unexposed conditions (ACI). Finally, the value of \( \beta_3 \) is calculated based on the Eqn. (6).

The results are shown in Figs. 3-5. As can be seen, the effects of the bars diameter, the cross section’s dimensions, and the concrete strength on \( \rho'_{\text{max}} \) are shown. Figs 3-5 also reflect the discrete nature of the solution of Eqn. (9) for the number of rows and the number of bars per row, and of bar diameters. The effect of steel bars diameter on \( \rho'_{\text{max}} \), for different cross sections dimension, is shown in Fig. 3. The general trend shows an increase of the derived upper limit with an increase of the bar diameter. The increase is more pronounced for wider beam widths \( b \) (Fig. 4). However, the influence of the ratio (is the height of the cross section) on \( \rho'_{\text{max}} \) is relatively moderate, as can also be seen in Fig. 4. The effect of concrete strength on \( \rho'_{\text{max}} \) is also relatively moderate and that as the beam width increases its dependency on the cross section dimensions or its ratio diminishes (Fig. 5).

4. Conclusions

Studies of upper limit to compression
<table>
<thead>
<tr>
<th>Span Type</th>
<th>Diagram</th>
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<tbody>
<tr>
<td>Simple supported span, carrying one concentrated load</td>
<td><img src="image1" alt="Diagram" /></td>
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<tr>
<td>Continuous supported span, carrying one concentrated load</td>
<td><img src="image2" alt="Diagram" /></td>
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<td>Cantilever supported span, carrying one concentrated load</td>
<td><img src="image3" alt="Diagram" /></td>
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<td>One end fixed and one end simply supported span, carrying one concentrated load</td>
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<td>Fixed supported span, carrying one concentrated load</td>
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<td>Continuous supported span, carrying two or more concentrated load</td>
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<td>Simple supported span, carrying two or more concentrated load</td>
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<td>Cantilever supported span, carrying two or more concentrated load</td>
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<td>One end fixed and one end simply supported span, carrying two or more concentrated load</td>
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<td>Fixed supported span, carrying two or more concentrated load</td>
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<td>Fixed supported span, carrying combination of uniform load and one or more concentrated load</td>
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<td>Continuous supported span, carrying uniform load</td>
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Table—1 Beams end condition and loading type
Fig. 2. Flowchart of computer program.

\[ m = \text{int} \left( \frac{b - 2c + z}{x + \phi} \right) \quad m' = \text{int} \left( \frac{b - 2c + z}{x' + \phi'} \right) \]

Solution (2), (4), (5) (with equal sing) for \( n_1, n'_1, y \)

\[ n = \text{roundup}(n_1), \quad n' = \text{roundup}(n'_1), \quad m_1 = \text{int}(n_1 + 1 \cdot n) m, \]
\[ m'_1 = \text{int}(n'_1 + 1 \cdot n') m', \quad \mu_2 = \min(m - m_1, m' - m'_1) \]

Calculate "d" of formula (10)

Calculate "y_{max}" of formula (7)

If inequality (4) satisfy NO Change the \( m_1, m'_1 \) and solve again until (4) satisfy

Yest

If \( y \leq y_{max} \) YES

Solution (2), (7), (5) (with equal sing) for \( n_1, n'_1, y \)

that \( d = h - c, -n\phi + 0.5\phi \)

Calculate \( p'_{max} \) of formula (9b)

end
Fig. 3. Effect of bar diameter on different cross section based on congestion criteria (b=300, 400, 600 mm).

Fig. 4. Effect of cross section size on $\rho_{\text{max}}'$ for different bar diameter based on the congestion criteria (b=300, 400, 600 mm).

Fig. 5. Effect of concrete strength on $\rho_{\text{max}}'$ for different cross section based on congestion criteria (b=300, 400, 600 mm).
reinforcement ratio in flexural HSCR members considering different end conditions and loading type were undertaken and the results of the studies led to the following conclusion:

A quantitative measure to evaluate an upper limit to the compression reinforcement ratio $\rho'_{\text{max}}$ of flexural HSRC members is proposed. It is shown that a quantitative criterion to $\rho'_{\text{max}}$ can be derived from steel congestion and proficient-design consideration (Eqn. (9)). Parameters that affect this limit include the concrete and steel strengths, the beam’s geometry (cross section’s dimensions, and concrete cover), the reinforcement diameter are deeply studied and reported.

The expressions that were derived provide additional tool for a better design and assessment of the flexural capacity of HSRC members with compression reinforcement for different boundary conditions or load pattern.

With an increase of the bar diameter increase the derived upper limit. The increase is more pronounced for wider beam widths b.

The influence of the ratio (is the height of the cross section) on $\rho'_{\text{max}}$ is relatively moderate. While using the HSC, by increasing the concrete strength, the values of $\rho'_{\text{max}}$ which is obtained by congestion criterion of equation (9), are not in a regular manner.

5. Acknowledgements

The authors are grateful to the Dancygier, A.N., who had checked the developed computer program, and also the management and planning organization of Kerman province for he financial supports.

6. References


Appendix A: Notation

\( A'_c \) = Concrete compression area acted by equivalent stress block;

\( b \) = width of rectangular cross section;

\( c_1 \) = concrete cover of longitudinal reinforcing bar (including concrete clear cover and stirrup’s diameter);

\( d \) = effect depth;

\( d' \) = distance from extreme compression fiber to centroidion reinforcement;

\( f'_c \) = characteristic cylinder compressive strength of concrete;

\( f_y \) = reinforcing steel yield stress;

\( h \) = height of cross section;

\( m \) = maximum number of tension reinforcing bars per row;

\( m_1 \) = number of longitudinal bars in rows that are located closest to cross section’s neutral axis;

\( m_2 \) = additional equal number of tension and compression bars to fill rows that are located closest to cross section’s neutral axis;

\( m' \) = maximum number of compression reinforcing bars per row;

\( n \) = maximum number of tension reinforcing bars rows;

\( n' \) = maximum number of compression reinforcing bars rows;

\( y \) = height of concrete compression area;

\( \beta_1 \) = height of stress block -to- \( y \) ratio;

\( \beta_3 \) = coefficient relating maximum height of stress block width;

\( \varepsilon_{cu} \) = ultimate concrete strain;

\( \varepsilon_{s,min} \) = steel strain of longitudinal tension bars, located closest to neutral axis;

\( \varepsilon_{s,min}' \) = steel strain of longitudinal compression bars, located closest to neutral axis;

\( \varepsilon_y \) = reinforcement yield strain;

\( \alpha \) = stress block coefficient;

\( \rho \) = longitudinal tension reinforcement ratio;

\( \rho' \) = longitudinal compression reinforcement ratio;

\( \phi, \phi' \) = tension and compression bar diameter.

Appendix B: Examples

Example 1:
A simply supported reinforced concrete beam is shown in Fig. 6 with the following properties:

\( b = 300 \text{ mm} \)
\( h = 600 \text{ mm} \)
\( f'_c = 80 \text{ MPa} \)
\( f_y = 400 \text{ MPa} \)
Fig. 6. Beam dimensions for section A-A.

Fig. 7. Beam dimensions for section B-B.

Fig. 8. Beam dimensions for section A-A and B-B.
Use $\phi_{22}$ as tensile and compressive bars, calculate the maximum amount of $\rho'$ for the section A-A.

The example can be solved using the diagram shown in Fig. 3.

It is easily possible to find out the maximum amount of $\rho'$ while, $b = 300$ mm, $h/b = 2$, $f_y = 400$ MPa, $f'_c = 80$ MPa and assuming $\phi_{22}$ for $\rho$ and $\rho'$. Hence, the maximum amount of $\rho'$ is found out Fig. 3 as 1.35%.

**Example 2:**
A cantilever reinforced concrete beam is shown in Fig. 7 with the following properties:

$b = 600$ mm  
$h = 1200$ mm  
$f'_c = 80$ MPa  
$f_y = 400$ MPa

Use $\phi_{28}$ as tensile and compressive bars, calculate the maximum amount of $\rho'$ for the section B-B.

The example can be solved using the diagram shown in Fig. 3.

It is easily possible to find out the maximum amount of $\rho'$ while, $b = 600$ mm, $h/b = 2$, $f_y = 400$ MPa, $f'_c = 80$ MPa and assuming $\phi_{28}$ for $\rho$ and $\rho'$. Hence, the maximum amount of $\rho'$ is found out Fig. 3 as 2.2%.

**Example 3:**
A continuous reinforced concrete beam is shown in Fig. 8 with the following properties:

$b = 400$ mm  
$h = 800$ mm  
$f'_c = 50$ MPa  
$f_y = 400$ MPa

Use $\phi_{25}$ as tensile and compressive bars, calculate the maximum amount of $\rho'$ for the section A-A and B-B.

The example can be solved using the diagram shown in Fig. 5.

It is easily possible to find out the maximum amount of $\rho'$ while, $b = 400$ mm, $h/b = 2$, $f_y = 400$ MPa, $f'_c = 50$ MP and assuming $\phi_{25}$ for $\rho$ and $\rho'$. Hence, the maximum amount of $\rho'$ is found out from Fig.5 as 2.25%.