Structural Vibration Reduction Using Gain-Scheduled Fuzzy Control of Magnetorheological Dampers

C.M.D. Wilson* and M.M. Abdullah

1. Assistant Professor, Civil and Environmental Engineering Department, New Mexico Institute of Mining and Technology, Socorro, New Mexico, USA
2. Professor and Dean, College of Engineering Sciences, Technology and Agriculture at Florida Agricultural and Mechanical University, USA

*Corresponding Author; email: cwilson@nmt.edu

ABSTRACT

Fuzzy control has recently been proposed to control the properties of magnetorheological (MR) dampers and therefore reduce vibrations of civil structures subjected to earthquake loads. These controllers have the advantage of not depending on system model, of being simple, and intrinsically robust. Their tuning, however, has shown to be a difficult task. This paper proposes a gain-scheduled fuzzy controller to regulate the damping properties of MR dampers and reduce structural responses of single degree-of-freedom seismically excited structures. Robustness of the algorithm to changes in seismic motions and structural characteristics were assessed by subjecting two different one-story buildings, one rigid and one flexible, to a wide range of earthquake records. Results show that the algorithm proposed effectively reduced responses of both structures to all twenty-four earthquake motions considered. In addition, results were compared to those of a fuzzy controller with constant scaling factors and to those of two passive systems: "passive on" and "passive off", where the current to the MR damper was set at maximum allowable value, and zero, respectively.

Keywords: Fuzzy control; Magnetorheological dampers; Structural vibration

1. Introduction

Because of their capability of varying the amount of damping provided to a structure, magnetorheological (MR) dampers have been proposed to reduce structural vibrations caused by seismic motions. These devices consist of hydraulic cylinders filled with MR fluid, a suspension of micron-sized, magnetically polarizable iron particles capable of reversibly changing from a free-flowing, viscous fluid to a semi-solid. This is done through the application of a magnetic field to the MR fluid, causing the iron particles to form chains and therefore increase the fluid’s viscosity. MR dampers are however highly nonlinear devices for which developing effective and practical control algorithms has proven to be a challenging task.

Several strategies proposed are model-based, that is, they require an accurate mathematical model of the system. The most commonly used controller in this category is the clipped-optimal algorithm developed by Dyke et al [1-2]. It consists of a bang-bang (on-off) controller that attempts to generate a desirable control force which is determined by an “ideal” active controller using $H_2/LQG$ strategies. The effectiveness of such controller has been extensively demonstrated [1-7]. Also in the model-based category are the optimal controllers [8-11], the control strategies based on Lyapunov stability theory [3, 5, 6, 12], as well as skyhook control and continuous sliding mode control [13].

Although model-based controllers have been successful in reducing structural vibrations, their performance is strongly affected by the accuracy of the model selected. Since models are based on assumptions and uncertainties, when designing model-based controllers, one must make sure that they will be robust enough to control the real life
structure [14]. Nonlinear and/or complex systems render the development of these controllers even more challenging. Models for such systems are often difficult to obtain and are usually very computationally intensive and therefore impractical for control applications. As an alternative, intelligent control strategies have been proposed for use with MR dampers. These controllers can be divided into the following three categories: neural network-based control [15-18], neuro-fuzzy based control [19-20], and fuzzy logic-based control [21-26].

Fuzzy controllers are an attractive alternative, not only because they are not based on a system model, but also because they are simple and intrinsically robust. These algorithms are based on intuitive understanding of the system and use “IF-THEN” rules instead of differential equations to relate the controller inputs to the desired outputs. There are three steps involved, namely:

1) Fuzzification, where membership functions convert crisp input values to fuzzy linguistic values.
2) Decision Making, which uses the “IF-THEN” rules created based on knowledge of the system to relate the linguistic input variables to linguistic output variables.
3) Defuzzification, where the fuzzy linguistic output variable is converted to a crisp control value [27].

Although fuzzy controllers are simple algorithms, their tuning is a complex and often difficult task due to the large number of parameters that define the membership functions and inference mechanisms [28-30]. Several methods have been developed for tuning these controllers [28, 31-38]. The most commonly used strategies involve the adjustment of the scaling factors [29-30] because these parameters are responsible for mapping the inputs and outputs to their universe of discourse and therefore have a large influence on the controller’s performance. Among the methodologies proposed for tuning scaling factors are: heuristic approaches [30, 39-41], neuro-like approaches [32, 42], genetic algorithms [35, 43], and gain-scheduling [36, 37, 44]. The latter consists in varying one or more of the scaling factors online, according to the changes in the input variables to the fuzzy controller or the input excitation to the system. Although gain-scheduling has been proposed to tune scaling factors of some fuzzy-controlled devices, it has not yet been employed with MR dampers, where all fuzzy control applications to date consist in selecting fixed values for all scaling factors and maintaining these values constant at all times.

Therefore, the objective of this paper is to develop a gain-scheduled fuzzy controller to regulate the damping properties of MR dampers and reduce structural responses of single degree-of-freedom (SDOF) seismically excited structures. Robustness of the algorithm to changes in seismic motions and structural characteristics will be assessed by subjecting two different one-story buildings, one rigid and one flexible, to a wide range of earthquakes.

2. System Description

The lumped mass model was selected to model the buildings, that is, the entire structural mass \( m \) was assumed to be located at the top of the building. Additional assumptions include constant stiffness \( k \) and damping \( c \). The equation of motion for the seismically excited SDOF structures equipped with a damper is as follows:

\[
mx'' + cx' + kx = -f - m\ddot{x}_g
\]

where \( x, x', x'' \) are the floor displacement, velocity, and acceleration, respectively, \( f \) is the control force, and \( \ddot{x}_g \) is the ground acceleration.

2.1. Structural Parameters

The first building selected for designing and tuning of the controller has a mass of 345,600 kg, a stiffness of 3.4 \( \times 10^7 \text{N/m} \) and a damping ratio of 0.02 [45]. Its natural frequency is 9.92 rad/s and its period is 0.63 s, therefore, according to Sadek and Mohaz [46], it is classified as a rigid structure. To test the robustness of the controller to changes in structural characteristics, a more flexible structure was also considered. Since damage to the building columns during seismic events causes a loss of structural stiffness, it would be important for the algorithm employed to still effectively control this less stiff structure. Thus, the second structure has the same mass and damping coefficient as the original building, but a reduced stiffness of 5.3 \( \times 10^6 \text{N/m} \). This value was chosen to obtain a flexible structure, that is, one with a period equal to or greater than 1.5 s [46]. The resulting natural frequency is 3.92 rad/s and the period is 1.6 s.
2.2. MR Damper Model

Each building in this study was equipped with two large-scale 20-ton MR dampers. Since numerical simulations will be employed in this paper, a damper model will be necessary, but it is important to note that this model is only used to simulate structural responses, not to design or run the gain-scheduled fuzzy controller proposed.

The damper model selected was the phenomenological model proposed by Spencer Jr. et al [47], a modification of the commonly used Bouc-Wen model to improve the reproduction of the force-velocity behavior of the damper. It consists of the addition of a dashpot in series with the original Bouc-Wen model and a spring in parallel with the accumulator, and viscous damping at large velocities, as can be expressed as follows [47]:

\[
f = \alpha z + c_o(\dot{x} - \dot{y}) + k_o(x - y) + k_i(x - x_o) = c_1\dot{y} + k_i(x - x_o)
\]  

(2)

\[
\ddot{z} = -\gamma|\dot{x} - \dot{y}|\varepsilon^m - \beta(\dot{x} - \dot{y})|\varepsilon^m + A(\dot{x} - \dot{y})
\]  

(3)

\[
\dot{y} = \frac{1}{c_o + c_1}\{\alpha z + c_o \dot{x} + k_o(x - y)\}
\]  

(4)

where \(f\) is the control force of the MR damper, \(x\), the damper displacement, \(y\), an internal displacement of the damper, \(\alpha\), the Bouc-Wen parameter describing the MR fluid yield stress, \(c_o\) represents the viscous damping at large velocities, \(k_o\) the stiffness at large velocities, \(k_i\) models the damper force due to the accumulator, and \(c_1\) reproduces the roll-off occurring in the experimental data when velocities are close to zero. Values for the parameters of the 20-ton MR damper were obtained experimentally by Yang [48] and Yang et al [49]:

\[
\alpha(i) = 16566i^3 - 87071i^2 + 168326i + 15114
\]  

(5)

\[
c_o(i) = 437097i^3 - 1545407i^2 + 1641376i + 457741
\]  

(6)

\[
c_1(i) = -9363108i^3 + 5334183i^2 + 48788640i - 2791630
\]  

(7)

To accommodate the dynamics of the MR fluid reaching rheological equilibrium, the following first order filter is also provided by Yang [48] and Yang et al [49]:

\[
H(s) = \frac{31.4}{s + 31.4}
\]  

(8)

2.3. Seismic Excitations

The north-south acceleration of four seismic motions with very different characteristics were used to tune the gain-scheduled fuzzy controller: El Centro, Hachinohe (Takochi-oki), Northridge, and Kobe (Hyogo-ken Nanbu), as shown in Table (1).

To test the robustness of the controller to changes in seismic excitations, the structures were subjected to 20 additional seismic motions. These were selected by Sadek and Mohraz [46], to include a wide range of earthquake magnitudes, epicentral distances, peak ground accelerations and soil conditions, see Table (2).

3. Gain-Scheduled Fuzzy Controller

As can be seen in the system schematic presented in Figure (1), the input variables to the fuzzy controller were chosen as floor displacement (\(x\)) and floor velocity (\(\dot{x}\)), while the output as the current applied to the MR damper (\(i\)). Membership functions for both input variables are shown in Figure (2a) and consist of seven identical triangles with 50% overlap, defined on the universe of discourse [-1, 1]. Figure (2b) presents membership functions for the output, which were defined on the universe of discourse [0, 1] and consist of four identical triangles.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Date</th>
<th>Magnitude</th>
<th>Component</th>
<th>Peak Acceleration (m/s²)</th>
<th>RMS Acceleration (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro, CA</td>
<td>May 18, 1940</td>
<td>7.1</td>
<td>N-S</td>
<td>3.4170</td>
<td>0.4764</td>
</tr>
<tr>
<td>Hachinohe, Japan</td>
<td>May 16, 1968</td>
<td>7.9</td>
<td>N-S</td>
<td>2.2500</td>
<td>0.3956</td>
</tr>
<tr>
<td>Northridge, CA</td>
<td>January 17, 1994</td>
<td>6.8</td>
<td>N-S</td>
<td>8.2676</td>
<td>0.7219</td>
</tr>
<tr>
<td>Kobe, Japan</td>
<td>January 17, 1995</td>
<td>6.9</td>
<td>N-S</td>
<td>8.1782</td>
<td>0.5909</td>
</tr>
</tbody>
</table>
Table 2. Earthquake records used for evaluation of controller's effectiveness.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Station Name</th>
<th>Magnitude</th>
<th>Component</th>
<th>Peak Acceleration (m/s²)</th>
<th>RMS Acceleration (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Fernando</td>
<td>Pacoima Dam</td>
<td>6.4</td>
<td>S16 E</td>
<td>11.4806</td>
<td>1.5954</td>
</tr>
<tr>
<td>California</td>
<td>Pacoima Dam</td>
<td>6.4</td>
<td>S 74 W</td>
<td>10.5495</td>
<td>1.0744</td>
</tr>
<tr>
<td>02/09/1971</td>
<td>250 E. First Street Basement, Los Angeles</td>
<td>6.4</td>
<td>N 36 E</td>
<td>0.9781</td>
<td>0.1592</td>
</tr>
<tr>
<td></td>
<td>250 E. First Street Basement, Los Angeles</td>
<td>6.4</td>
<td>N 54 W</td>
<td>1.2273</td>
<td>0.1584</td>
</tr>
<tr>
<td>Loma Prieta</td>
<td>Corralitos Eureka Canyon Road</td>
<td>7.1</td>
<td>0°</td>
<td>6.1770</td>
<td>0.7126</td>
</tr>
<tr>
<td>California</td>
<td>Corralitos Eureka Canyon Road</td>
<td>7.1</td>
<td>90°</td>
<td>4.6938</td>
<td>0.6333</td>
</tr>
<tr>
<td>10/17/1989</td>
<td>Capitol Fire Station</td>
<td>7.1</td>
<td>0°</td>
<td>4.6292</td>
<td>0.8267</td>
</tr>
<tr>
<td></td>
<td>Capitol Fire Station</td>
<td>7.1</td>
<td>90°</td>
<td>3.9079</td>
<td>0.6084</td>
</tr>
<tr>
<td>Northridge</td>
<td>Arleta Nordhoff Ave. Fire Station</td>
<td>6.7</td>
<td>90°</td>
<td>3.3732</td>
<td>0.3985</td>
</tr>
<tr>
<td>California</td>
<td>Arleta Nordhoff Ave. Fire Station</td>
<td>6.7</td>
<td>360°</td>
<td>3.0205</td>
<td>0.3496</td>
</tr>
<tr>
<td>01/17/1994</td>
<td>Pacoima Dam Down Stream</td>
<td>6.7</td>
<td>175°</td>
<td>4.0711</td>
<td>0.4406</td>
</tr>
<tr>
<td></td>
<td>Pacoima Dam Down Stream</td>
<td>6.7</td>
<td>265°</td>
<td>4.2555</td>
<td>0.3908</td>
</tr>
<tr>
<td>Northwest</td>
<td>Ferndale City Hall</td>
<td>5.8</td>
<td>S 44 W</td>
<td>1.0200</td>
<td>0.1001</td>
</tr>
<tr>
<td>California</td>
<td>Ferndale City Hall</td>
<td>5.8</td>
<td>N 46 W</td>
<td>1.0950</td>
<td>0.1122</td>
</tr>
<tr>
<td>10/07/1951</td>
<td>San Francisco Golden Gate Park</td>
<td>5.3</td>
<td>N10 E</td>
<td>0.8180</td>
<td>0.0663</td>
</tr>
<tr>
<td>San Francisco</td>
<td>San Francisco Golden Gate Park</td>
<td>5.3</td>
<td>S 80 E</td>
<td>1.0280</td>
<td>0.0885</td>
</tr>
<tr>
<td>California</td>
<td>03/22/1957</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Helena Montana</td>
<td>Helena Montana Carrol College</td>
<td>6.0</td>
<td>S 00 W</td>
<td>1.4350</td>
<td>0.0947</td>
</tr>
<tr>
<td>10/31/1935</td>
<td>Helena Montana Carrol College</td>
<td>6.0</td>
<td>S 90 W</td>
<td>1.4250</td>
<td>0.1190</td>
</tr>
<tr>
<td>Parkfield</td>
<td>Temblor, California # 2</td>
<td>5.6</td>
<td>N 65 W</td>
<td>2.6430</td>
<td>0.2504</td>
</tr>
<tr>
<td>California</td>
<td>Temblor, California # 2</td>
<td>5.6</td>
<td>S 25 W</td>
<td>3.4080</td>
<td>0.3043</td>
</tr>
</tbody>
</table>

Figure 1. System diagram.

Figure 2. Membership functions.
also with 50% overlap. Labels NL, NM, NS, ZO, PS, PM, and PL refer to negative large, negative medium, negative small, zero, positive small, positive medium, and positive large, respectively.

Since the universes of discourse selected are normalized, scaling factors are required to map the variables. They are labeled normalized, scaling factors are required to map the medium, and positive large, respectively. Labels negative small, zero, positive small, positive medium, and positive large, respectively.

Values considered for these variables were estimated based on the structural displacement and velocity, respectively. Values for these variables were estimated based on the largest uncontrolled responses of the rigid structure to the following four earthquakes: El Centro, Hachinohe, Northridge, and Kobe, see Table (3). Since there are no systematic methods for creating rule-bases for fuzzy logic controllers, standard rule-bases are often selected as a starting point [29] and if necessary, modified to better achieve the control objectives. In this paper, the rule-base developed by MacVicar-Whelan [50] and a modified version of such standard, developed by Liu et al [22], were selected as starting points because they seemed to well represent the desired control strategy for the different displacement and velocity combinations. For example: if displacement was “large” and velocity was also “large” and in the same direction, the rules dictated that current applied should be “large” to increase the amount of damping provided. Since variations to these standards did not appreciably modify the results obtained, the Liu et al [22] rule-base was ultimately selected and is presented in Table (4).

Results of this study showed that the controller was not sensitive to changes in the magnitude of \( K_d \) or \( K_u \). Therefore, these scaling factors will remain constant and equal to the values that yielded the best results for the cases considered: \( K_d = 7 \), and \( K_u = 4 \). Results of these simulations also showed that the smaller the earthquake, the smaller the value of \( K_v \) that would produce greater structural reductions. A linear equation relating the ground acceleration (\( \ddot{x}_g \)) and scaling factor \( K_v \) was therefore obtained and is presented below:

\[
K_v(t) = 23.37|\ddot{x}_g(t)| - 0.68
\]  

Table 3. Uncontrolled responses of rigid structure to four different earthquakes.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Maximum Displacement (m)</th>
<th>Maximum Velocity (m/s)</th>
<th>Maximum Acceleration (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro, CA</td>
<td>0.0870</td>
<td>0.8126</td>
<td>10.4105</td>
</tr>
<tr>
<td>Hachinohe, Japan</td>
<td>0.0514</td>
<td>0.4968</td>
<td>5.2095</td>
</tr>
<tr>
<td>Northridge, CA</td>
<td>0.1436</td>
<td>1.4306</td>
<td>19.8497</td>
</tr>
<tr>
<td>Kobe, Japan</td>
<td>0.2204</td>
<td>2.1732</td>
<td>24.5961</td>
</tr>
</tbody>
</table>
To determine both the effectiveness and the robustness of the gain-scheduled fuzzy algorithm developed to changes in seismic motions, the rigid structure was subjected to all 24 earthquakes mentioned previously. The robustness of the controller to changes in structural characteristics was evaluated by subjecting the flexible structure to all 24 earthquakes while controlled by the gain-scheduled algorithm developed and tuned using responses of the rigid building.

### 4. Results and Discussion

Matlab and Simulink were used for the numerical simulation of structural responses of the gain-scheduled fuzzy control strategy proposed. Four criteria were used to evaluate the effectiveness of the control system: the first two \( J_1 \) and \( J_2 \) were based on root mean square (RMS) displacement and acceleration, respectively, while the last two \( J_3 \) and \( J_4 \) were based on peak displacement and acceleration, also respectively. All evaluation criteria were obtained by dividing the controlled responses by the respective uncontrolled responses, as shown in Eqs. (15) to (18):

\[
J_1 = \frac{\text{RMS} (x(t))}{\text{RMS} (x_{\text{unc}}(t))} \\
J_2 = \frac{\text{RMS} (\ddot{x}(t))}{\text{RMS} (\ddot{x}_{\text{unc}}(t))} \\
J_3 = \frac{\max |x(t)|}{\max |x_{\text{unc}}(t)|} \\
J_4 = \frac{\max |\ddot{x}(t)|}{\max |\ddot{x}_{\text{unc}}(t)|}
\]

where \( x \) and \( \ddot{x} \) are the controlled displacement and acceleration, respectively, whereas \( x_{\text{unc}} \) and \( \ddot{x}_{\text{unc}} \) are the uncontrolled displacement and acceleration, also respectively.

For succinctness, results to only 4 of the 24 earthquakes selected are presented in graphical form; nevertheless, discussions will be considering responses to all 24 earthquakes. Figures (3) to (6) show displacement and acceleration responses of the rigid structure to the following four earthquakes: El Centro, Hachinohe, Northridge, and Kobe. Average values for the evaluation parameters of all 24 earthquakes, along with their respective 95% confidence intervals were found to be: \( \bar{J}_1 = 0.292 \) (± 0.063), \( \bar{J}_2 = 0.768 \) (± 0.249), \( \bar{J}_3 = 0.453 \) (± 0.106),

<table>
<thead>
<tr>
<th>( \dot{x} )</th>
<th>NL</th>
<th>NM</th>
<th>NS</th>
<th>ZO</th>
<th>PS</th>
<th>PM</th>
<th>PL</th>
</tr>
</thead>
<tbody>
<tr>
<td>NL</td>
<td>PL</td>
<td>PL</td>
<td>PL</td>
<td>PM</td>
<td>ZO</td>
<td>ZO</td>
<td>ZO</td>
</tr>
<tr>
<td>NM</td>
<td>PL</td>
<td>PL</td>
<td>PL</td>
<td>PS</td>
<td>ZO</td>
<td>ZO</td>
<td>PS</td>
</tr>
<tr>
<td>NS</td>
<td>PL</td>
<td>PL</td>
<td>PL</td>
<td>ZO</td>
<td>ZO</td>
<td>PS</td>
<td>PM</td>
</tr>
<tr>
<td>ZO</td>
<td>PM</td>
<td>PL</td>
<td>PS</td>
<td>ZO</td>
<td>PL</td>
<td>PL</td>
<td>PL</td>
</tr>
<tr>
<td>PS</td>
<td>PL</td>
<td>ZO</td>
<td>PS</td>
<td>ZO</td>
<td>PL</td>
<td>PL</td>
<td>PL</td>
</tr>
<tr>
<td>PM</td>
<td>ZO</td>
<td>ZO</td>
<td>ZO</td>
<td>PM</td>
<td>PL</td>
<td>PL</td>
<td>PL</td>
</tr>
<tr>
<td>PL</td>
<td>ZO</td>
<td>ZO</td>
<td>ZO</td>
<td>PM</td>
<td>PL</td>
<td>PL</td>
<td>PL</td>
</tr>
</tbody>
</table>

![Figure 3. Rigid building responses to El Centro earthquake.](Image)

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**Table 4. Control rule base [22].**
Figure 4. Rigid building responses to Hachinohe earthquake.

Figure 5. Rigid building responses to Northridge earthquake.

Figure 6. Rigid building responses to Kobe earthquake.
and $J_4 = 0.824 \pm 0.125$, showing that peak and RMS displacements and accelerations were reduced with the gain-scheduled fuzzy controlled MR dampers for a very wide range of seismic motions. As expected, displacement reductions were larger than those obtained for acceleration because control rules were based on floor displacement.

RMS displacements were reduced for all excitations and $J_1$ values ranged from 0.068 for the N46W component of the Northwest earthquake to 0.556 for the S25W component of the Parkfield earthquake. Peak displacements were reduced for all but one seismic motion, with $J_3$ values reaching 0.093 for the N54W component of the San Fernando earthquake. RMS accelerations were reduced for almost all of the excitations considered, with the lowest value for $J_2$, 0.216, obtained for the FS90 component of the Loma Prieta earthquake at the Capitola station. Peak accelerations were also reduced for most earthquakes, and $J_4$ reached 0.353 for the N54W component of the San Fernando earthquake.

These results were compared to those obtained with the fuzzy control strategy presented in [24, 26] which uses constant scaling factors: $K_d = 32$, $K_v = 3.3$, and $K_d = 4$. For simplicity, in this paper, this strategy will be referred to as “fuzzy control”, while the gain-scheduled fuzzy control system will simply be called “gain-scheduled”. Average results for these two control strategies ($J$) are presented in Figure (7) and Table (5), along with those obtained with two passive systems: “passive on” and “passive off”, where the current to the MR damper was set at maximum allowable value, and zero, respectively. Paired difference two-tailed t-tests were conducted to compare the average value of the evaluation criteria of each of the control strategies against each other. Observed significance levels ($P$-values) for these tests are presented in Table (6). Results show that the gain-scheduled algorithm performed better than the fuzzy controller that maintained constant all scaling factors. For evaluation criteria $J_1$ and $J_3$, the gain-scheduled controller was more effective.

![Figure 7. Comparison of average evaluation criteria values for the rigid structure's responses obtained with different control strategies (Error bars indicate 95% intervals).](image)

**Table 5.** Average values of evaluation criteria for rigid structure's responses obtained with different control strategies (95% intervals presented in parenthesis).

<table>
<thead>
<tr>
<th></th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$J_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive Off</td>
<td>0.445 (± 0.064)</td>
<td>0.667 (± 0.124)</td>
<td>0.575 (± 0.068)</td>
<td>0.818 (± 0.098)</td>
</tr>
<tr>
<td>Passive On</td>
<td>0.268 (± 0.062)</td>
<td>1.548 (± 0.752)</td>
<td>0.423 (± 0.101)</td>
<td>0.902 (± 0.144)</td>
</tr>
<tr>
<td>Fuzzy Controller</td>
<td>0.374 (± 0.075)</td>
<td>0.652 (± 0.151)</td>
<td>0.507 (± 0.097)</td>
<td>0.843 (± 0.129)</td>
</tr>
<tr>
<td>Gain-Scheduled</td>
<td>0.292 (± 0.063)</td>
<td>0.768 (± 0.249)</td>
<td>0.453 (± 0.106)</td>
<td>0.824 (± 0.125)</td>
</tr>
</tbody>
</table>

**Table 6.** Average values of evaluation criteria for rigid structure's responses obtained with different control strategies (95% intervals presented in parenthesis).

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$J_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive Off vs. Passive Off</td>
<td>&lt; 0.01</td>
<td>0.01</td>
<td>&lt; 0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>Passive Off vs. Fuzzy Controller</td>
<td>&lt; 0.01</td>
<td>0.41</td>
<td>0.01</td>
<td>0.27</td>
</tr>
<tr>
<td>Passive Off vs. Gain-Scheduled</td>
<td>&lt; 0.01</td>
<td>0.18</td>
<td>&lt; 0.01</td>
<td>0.82</td>
</tr>
<tr>
<td>Passive On vs. Fuzzy Controller</td>
<td>&lt; 0.01</td>
<td>0.01</td>
<td>&lt; 0.01</td>
<td>0.07</td>
</tr>
<tr>
<td>Passive On vs. Gain-Scheduled</td>
<td>&lt; 0.01</td>
<td>&lt; 0.01</td>
<td>&lt; 0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Fuzzy Controller vs. Gain-Scheduled</td>
<td>&lt; 0.01</td>
<td>0.08</td>
<td>0.01</td>
<td>0.32</td>
</tr>
</tbody>
</table>
than the fuzzy controller ($P < 0.01$ and $P = 0.01$, respectively), while for evaluation criteria $\bar{J}_2$ and $\bar{J}_4$, paired difference t-test results ($P = 0.08$ and $P = 0.32$, respectively) indicate that there might be no difference in the values of the parameters obtained with the different fuzzy controllers.

As anticipated, the passive on scheme was found more effective in reducing $RMS$ and peak displacements than the passive off strategy ($P < 0.01$ for $\bar{J}_1$ and $\bar{J}_3$), while the passive off was found more effective than its passive counterpart in reducing $RMS$ and peak accelerations ($P = 0.01$ for $\bar{J}_2$, $P = 0.05$ for $\bar{J}_4$). Both fuzzy control strategies (with constant and gain-scheduled scaling factors) were found to improve on the performance of the passive on system with respect to acceleration reduction. Paired difference t-tests showed that, like the passive on system, these two controllers reduced $RMS$ and peak displacements more effectively than the passive off scheme, but instead of a worse performance with respect to acceleration, these tests indicated that there might be no difference in $RMS$ and peak acceleration results obtained with the passive off and the other two fuzzy systems. Finally, comparisons were made between the passive on system and the two fuzzy systems. For these cases, it was found that, although the passive on system was more effective than the fuzzy systems in reducing displacements, these control schemes performed better with respect to $RMS$ acceleration ($P = 0.01$ for fuzzy control, $P < 0.01$ for gain-scheduled control). In addition, while there seemed to be no difference in peak acceleration reduction with the fuzzy and the passive on schemes ($P = 0.07$), the gain-scheduled controller outperformed the passive on system in this aspect ($P = 0.01$).

The flexible structure previously described was also subjected to the 24 seismic motions to test the robustness of the gain-scheduled control algorithm to changes in structural characteristics. From these results, some of which are presented in Figures (8) to (11), it can be seen that, although tuned for a different structure, this algorithm was still able to effectively reduce responses of this flexible structure to the earthquakes. $RMS$ and peak displacements were all reduced: with $\bar{J}_1$ values ranging from 0.319 to 0.041 and $\bar{J}_3$ values ranging from 0.748 to 0.087. Peak and $RMS$ accelerations were reduced for most excitations considered. Average results for this control strategy are presented in Figure (12) and Table (7), along with those obtained with the fuzzy controller with fixed scaling factors and the two passive systems. Paired difference two-tailed t-tests were also conducted to compare the average evaluation criteria of each of the control strategies against each other and the P-values obtained are presented in Table (8). As with the rigid structure, the gain-scheduled controller was more effective in reducing $RMS$ displacements than the fuzzy controller ($P < 0.01$) and as effective as the fuzzy controller in reducing both peak displacements ($P = 0.09$) and peak accelerations ($P = 0.51$). However,
Figure 9. Flexible building responses to Hachinohe earthquake.

Figure 10. Flexible building responses to Northridge earthquake.

Figure 11. Flexible building responses to Kobe earthquake.
with respect to RMS accelerations, the fuzzy controller outperformed the gain-scheduled scheme ($P = 0.01$). Yet, it is important to note that the gain-scheduled controller was not tuned to the responses of this flexible structure and that the aim of this exercise is only to test the robustness of the control system to changes in structural stiffness. Comparing results of the gain-scheduled controller and the passive systems, it can be seen that, as expected, gain-scheduled control outperforms the passive off system with respect to displacement reductions ($P < 0.01$ for $J_1$ and $J_3$) while the inverse is observed with acceleration reductions ($P = 0.02$ for $J_2$, $P < 0.01$ for $J_4$). Finally, the gain-scheduled strategy was more effective than the passive on with respect to acceleration reductions ($P < 0.01$ for $J_2$ and $J_4$), as effective as the passive on scheme for RMS displacement reduction ($P = 0.13$ for $J_1$), but less effective for peak displacements ($P = 0.02$ for $J_3$).

5. Conclusions

In this paper, a gain-scheduled fuzzy controller was developed to regulate the damping properties of MR dampers, thereby reducing structural responses of SDOF seismically excited structures. Simulations were conducted in Matlab and Simulink to demonstrate the effectiveness and robustness of the control algorithm. Results showed that not only is this algorithm capable of reducing structural responses to a very wide range of earthquakes, being therefore robust to changes in external excitations, but it is also more effective in reducing structural displacements than the fuzzy controller proposed in [24, 26], which maintained all scaling factors constant. When compared to passive control strategies, see Figure (7), it was shown that the gain-scheduled controller performed similarly to the passive on system with respect to displacement reductions, but more effectively with respect to acceleration reductions. Statistical analysis also
showed that the overall performance of the gain-scheduled controller was better than that of the passive off system.

Finally, the robustness of the proposed gain-scheduled fuzzy control algorithm to changes in structural characteristics was demonstrated by subjecting a more flexible structure to the same seismic motions. Results of these analyzes showed that although tuned for a different structure, this algorithm was still able to effectively reduce responses of the flexible structure to the seismic excitations.

Because the extension of these algorithms to multi-degree-of-freedom structures would be especially important for their practical implementation, the first author of this paper is currently evaluating this possibility. Variations in the number, the size, and the placement of the dampers are being considered as are centralized and decentralized controllers. Preliminary results have shown to be very promising.

References


