Reliability Analysis of Three Elements in Series and Parallel Systems under Time-varying Fuzzy Failure Rate

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Abstract

Reliability is the most important performance issue in engineering design process but in the real world problems, there are limitations for using conventional reliability. Fuzzy logic has proved to be effective in expressing uncertainties in different fields, including engineering reliability. In this paper, for both series and parallel systems composed of three identical or different elements, the reliability has been analyzed using the fuzzy concepts and some characteristics such as the mean time to failure have been evaluated, taking in our account that all the operating units have increasable time varying failure rates with fuzzy triangular membership functions. This paper includes a numerical example to illustrate the model and validate the analytical results.

1. INTRODUCTION

System reliability occupies important and significant role in design of power plants, manufacturing systems, industrial systems, standby systems, etc. By the reliability analysis, a manager can decide the optimal strategy, by setting the range of Mean Time to Failure (MTTF) to reflect the desired failure rates and this in turn will minimize the total cost involved. The reliability of an item is the probability that the item will perform a required function during a given interval of time under specified conditions. The system reliability depends on the reliability of its components, configuration of the system, and the system failure criteria. During the study of the reliability of any system, the goal is predicting the suitable system reliability features based on the component failure data and the system design. Series or parallel systems or both of them can represent the large engineering system with similar or different components having constant [1-4], or time-varying failure rates. Many studies handled the problem of time-varying failure and repair rates [5, 6].

In the real world applications as sensor information, data are vaguely specified so fuzzy theory can handle such vagueness using membership function. The fuzzy set theory has a realistic and practical means to describe the real world. It has been introduced firstly by Lotfi zadah [7] and many researcher [8-10] who used the fuzzy sets in the reliability analysis to get some reliability characteristics. But the system parameters are often imprecise due to incomplete and accurate information, so failure and repair rates may be expressed with some uncertainty by experiences of experts. In fact, the fuzzy sets make descriptions of the objective world more realistic, practical, and accurate so they have been widely applied in decision-making and logic programming. Sharma et al. [11] used a system containing three elements established with constant and increasable fuzzy failure rates in real time situations and Jiang et al. [12] constructd an algorithm to get the fuzzy reliability. Wang [13] made the analysis with fuzzy random variables. Mishra et al. and Krishnan et al. [14, 15] presented the real time study of a k-out-of-n system having n identical elements with constant fuzzy failure rates. In this paper, both series and parallel system composite of three similar or different components are analyzed when the failure distribution of each operating
2. NOTATIONS

\( \lambda_i \): Failure rate of the \( i \)-th element.

\( L_i \): Lower limit of the triangular fuzzy number related to the failure rate of the \( i \)-th element.

\( M_i \): Medium of the triangular fuzzy number related to the failure rate of the \( i \)-th element.

\( U_i \): Upper limit of the triangular fuzzy number related to the failure rate of the \( i \)-th element.

\( P_i(t) \): Probability that the system at the \( t \) moment to be in condition \( i \).

\( R_i(t) \): Probability of the functionality of the series system.

\( R_p(t) \): Probability of the functionality of the parallel system.

\( MTTF \): Mean time to failure.

3. SYSTEM RELIABILITY

3.1. System Reliability in Crisp Case

In this section, we discuss the reliability of the general unrepairable series system and parallel system in the crisp case. Figure 1 represents the configuration of the series and the parallel systems, with components \( P_1, P_2, \ldots, P_n \), assuming their reliabilities are \( R_1, R_2, \ldots, R_n \) (0 ≤ \( R_i \) ≤ 1; \( i = 1, 2, \ldots, n \)), respectively. For each component, the failure rate is not constant but linearly changes with time and equals (\( \lambda_i t \)), then:

\[ R_i(t) = e^{-\int_0^t \lambda_i \, dt} = e^{-\frac{1}{2} \lambda_i t^2} \] (1)

Then for the series system of three components we can get:

\[ R_s(t) = \prod_{i=1}^{3} R_i(t) = e^{-\frac{1}{2} (\lambda_1 + \lambda_2 + \lambda_3) t^2} \] (2)

Figure 1. (a) Series system configuration and (b) parallel system configuration.

\[ MTTF_s = \int_0^\infty R_s(t) \, dt = \frac{\pi}{\sqrt{2(\lambda_1 + \lambda_2 + \lambda_3)}} \] (3)

and for the parallel system:

\[ R_p(t) = 1 - (1 - e^{-\frac{1}{2} \lambda_1 t^2})(1 - e^{-\frac{1}{2} \lambda_2 t^2})(1 - e^{-\frac{1}{2} \lambda_3 t^2}) \] (4)

\[ MTTF_p = \int_0^\infty R_p(t) \, dt = \frac{\pi}{\sqrt{2 \lambda_1}} + \frac{\pi}{\sqrt{2 \lambda_2}} + \frac{\pi}{\sqrt{2 \lambda_3}} - \frac{\pi}{\sqrt{2(\lambda_1 + \lambda_2 + \lambda_3)}} \] (5)

3.2. System Reliability under Fuzzy Failure Rates

Most important consideration is that the values of the failure rates \( \lambda_i ; i = 1, 2, 3 \) are not fixed, since they are driven from collected data or the opinions of the experts. Uncertainty of these values is an undeniable fact. Most of the times, these failure rates are considered as known values or have known distribution function. In this paper, we assume that failure rates, \( \lambda_i ; i = 1, 2, 3 \) in the form of triangular fuzzy numbers are as follows:

\[ \lambda_i = (L_i, M_i, U_i) ; i = 1, 2, 3 \] (6)

in which \( M_i \) is the point estimation and \( L_i, U_i \) are the lower and the upper limits of (1−β)100% confidence interval of \( \lambda_i \) where the \( \alpha \)-cuts of these failure rates can be calculated as follows:

\[ \lambda_i(\alpha) = [\lambda_i L_i - \alpha(M_i - L_i), U_i - \alpha(U_i - M_i)] ; \alpha = 1, 2, 3 \] (7)

By using the extension principle, the reliability function and MTTF can be calculated in each case as follows:

\[ R_s(t, \alpha) = [RS_1(t, \alpha), \ldots, RS_n(t, \alpha)] \]

\[ RS_i(t, \alpha) = e^{-\frac{1}{2}(U_i + U_j - \alpha(U_i + U_j - L_i - L_j))t^2} \] (8.1)
\[ R_S(t, \alpha) = e^{-\frac{1}{2} (t_1 + t_2 + \alpha (M_1 + M_2 + M_3 - t_1 - t_2 - t_3))} \]  
\[ MTTFS_0[a] = [MTTFS_0(a), MTTFS_0(t, a)] \]
\[ MTTFS_0(\alpha) = \int_0^\infty R_S(t, \alpha) \, dt = \frac{1}{\sqrt{2(\alpha^2 + (t_1 + t_2 - \alpha (M_1 + M_2 + M_3 - t_1 - t_2 - t_3))^2)}} \]
\[ MTTFS_0(t, \alpha) = \int_0^\infty R_S(t, \alpha) \, dt = \frac{1}{\sqrt{2(t_1 + t_2 - \alpha (M_1 + M_2 + M_3 - t_1 - t_2 - t_3))^2}}} \]

\[ \bar{R}_P[a] = [R_P(t, a), R_P(t, a)] \]
\[ R_P(t, \alpha) = 1 - \frac{1}{2} (1 - e^{-\frac{1}{2} (t_1 - \alpha (M_1 - t_1))} + (1 - e^{-\frac{1}{2} (t_2 - \alpha (M_2 - t_2))} + (1 - e^{-\frac{1}{2} (t_3 + \alpha (M_3 - t_3))}) \]  
\[ MTTFP[a] = [MTTFP(a), MTTFP(t, a)] \]
\[ MTTFP(a) = \int_0^\infty R_P(t, \alpha) \, dt = \frac{1}{\sqrt{2(t_1 - \alpha (M_1 - t_1)) + \sqrt{2(t_2 - \alpha (M_2 - t_2)) + \sqrt{2(t_3 + \alpha (M_3 - t_3))}}} \]
\[ MTTFP(t, \alpha) = \int_0^\infty R_P(t, \alpha) \, dt = \frac{1}{\sqrt{2(t_1 - \alpha (M_1 - t_1)) + \sqrt{2(t_2 - \alpha (M_2 - t_2)) + \sqrt{2(t_3 + \alpha (M_3 - t_3))}}} \]

\[ M_i = \frac{n_i}{\sum T_i} \]
\[ L_i = \frac{\sigma^2 n_i (1 - \beta/2)}{2 \sum T_i} \]
\[ U_i = \frac{\sigma^2 n_i (1 - \beta/2)}{2 \sum T_i} \]

where the data is synthesized for each component \( i \) : \( i = 1, 2, 3 \) in two statistics: a total test time is denoted as \( \sum T_i \) and a sample size is denoted as \( n_i \).

4. NUMERICAL AND GRAPHICAL EXAMPLE

The Maple software is used to analyze the reliability and MTTF of the series and parallel systems.

4.1. For Three Different Components

Assume a system consisting of three major units. These are continuously monitored by a failure detection device. For efficiency, the manager wants to analyze the system and get fuzzy reliability and MTTF. Due to uncertainty or the imprecision of data, the failure rates of the three operating units can be represented by triangular fuzzy numbers.

Assume each unit is tested with a sample size 25, 35, 30 and total failure test times are 920, 700, 1200. Based on that statistical data with a confidence level 95\%, the values of the medium, lower and upper limits of these three triangular fuzzy numbers related to failure rates \( \lambda_1, \lambda_2, \lambda_3 \) are calculated according to Equation (12) as follows:

\[ \lambda_1 = (0.0176/0.0272/0.0388) \]
\[ \lambda_2 = (0.0231/0.05/0.051) \]
\[ \lambda_3 = (0.01348/0.025/0.02975) \]

It is easy to find that the fuzzy reliability function for both the series and parallel systems using Equations (8) and (10) and represent them graphically at different values of \( \alpha \) as shown in Figure 2 and according to Equations (9) and (11), we can get the crisp intervals of the failure rates of the three operating units and the mean-time to failure of both series and parallel systems at different possibility \( \alpha \) levels as shown in Table 1 and their overall shapes are represented in Figure 3.

4.2. For Three Identical Components

If each component has the same fuzzy failure rate \( \lambda_1 = \lambda_2 = \lambda_3 = \lambda \), we test with a sample size of 25 and a total test time of 920. Based on that statistical data with a confidence level 95\%, the triangular fuzzy number related to that failure rate is \( \lambda = (0.0176/0.0272/0.0388) \). The fuzzy reliability function for both the series and parallel systems can be represented graphically at different values of \( \alpha \) as shown in Figure 4 and the crisp intervals of that failure rate and the mean-

Because the fuzzy rate of each component linearly changes with time (\( \beta, t \)) and has exponential distribution with unknown fuzzy parameter (\( \lambda \)) which can be calculated, see [17], using the point estimation and (1-\( \beta \))100\% confidence intervals (two ways) of this parameter as:
time to failure of both series and parallel systems at different $\alpha$-cuts are shown in Table 2 and their overall shapes are represented in Figure 5.

![Figure 2](image2.png)

Figure 2. (a) Fuzzy reliability of parallel system and (b) Fuzzy reliability of series system with different three units

![Figure 3](image3.png)

Figure 3. (a) Fuzzy MTTF of parallel system and (b) Fuzzy MTTF of series system with different three units

![Figure 4](image4.png)

Figure 4. (a) Fuzzy reliability of parallel system and (b) Fuzzy reliability of series system with similar units

![Figure 5](image5.png)

Figure 5. (a) Fuzzy MTTF of parallel system and (b) Fuzzy MTTF of series system with similar units
TABLE 1. \(\alpha\)-cuts of the failure rates, the fuzzy mean-time to failure of the series and parallel system with different three units

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(\lambda_1(\alpha))</th>
<th>(\lambda_2(\alpha))</th>
<th>(\lambda_3(\alpha))</th>
<th>(\lambda_{\text{MTTF}_\text{S}}(\alpha))</th>
<th>(\lambda_{\text{MTTF}_\text{P}}(\alpha))</th>
<th>(\lambda_{\text{MTTF}_\text{U}}(\alpha))</th>
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</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0176</td>
<td>0.03882</td>
<td>0.0231</td>
<td>0.01348</td>
<td>0.02976</td>
<td>3.6244</td>
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<tr>
<td>0.1</td>
<td>0.0185</td>
<td>0.0377</td>
<td>0.0258</td>
<td>0.0146</td>
<td>0.0288</td>
<td>3.678</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0195</td>
<td>0.0365</td>
<td>0.0285</td>
<td>0.01578</td>
<td>0.0288</td>
<td>3.678</td>
</tr>
<tr>
<td>0.3</td>
<td>0.0205</td>
<td>0.0353</td>
<td>0.0312</td>
<td>0.01694</td>
<td>0.0283</td>
<td>3.706</td>
</tr>
<tr>
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<td>0.0214</td>
<td>0.0342</td>
<td>0.03386</td>
<td>0.0181</td>
<td>0.02785</td>
<td>3.7347</td>
</tr>
<tr>
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<td>0.0365</td>
<td>0.0258</td>
<td>0.02738</td>
<td>3.7639</td>
</tr>
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<td>0.0233</td>
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<td>0.0272</td>
<td>0.05</td>
<td>0.025</td>
<td>0.025</td>
<td>3.9209</td>
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</table>

TABLE 2. \(\alpha\)-cuts of the failure rate, the fuzzy mean-time to failure of the series and parallel system with similar three units

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(\lambda_1(\alpha))</th>
<th>(\lambda_2(\alpha))</th>
<th>(\lambda_{\text{MTTF}_\text{S}}(\alpha))</th>
<th>(\lambda_{\text{MTTF}_\text{P}}(\alpha))</th>
<th>(\lambda_{\text{MTTF}_\text{U}}(\alpha))</th>
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<tbody>
<tr>
<td>0.0</td>
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<td>0.03882</td>
<td>3.6726</td>
<td>9.26195</td>
<td>13.7554</td>
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<tr>
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<td>9.4041</td>
<td>13.39589</td>
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<tr>
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<td>0.0272</td>
<td>4.3896</td>
<td>11.07</td>
<td>11.07</td>
</tr>
</tbody>
</table>

5. RESULTS AND DISCUSSIONS

From the numerical example that introduced for analyzing a model for three items un-repairable system, as shown in Table 1, if the items have different triangular shaped fuzzy parameters, it is impossible for the value of the fuzzy mean-time to failure to fall below 3.6244 or exceed 5.3843 in the series connection and its most possible value is 3.9209. In the parallel connection, it can’t fall below 9.42249 or exceed 13.997 and the most possible value is 10.4806. As shown in Table 2, if we have similar triangular shaped fuzzy parameters, the value of the fuzzy mean-time to failure in the series connection falls in the interval [3.6726, 5.454] and the most possible value is 4.3896. In the parallel connection, it falls in [9.26195, 13.7554] with the most possible value 11.07. After drawing the results using MAPLE software program, we find that the \(\text{MTTF}\) in all cases can be approximated to be a triangular shaped fuzzy number too and the fuzzy reliability for each value of \(\alpha\) has a two reliability curves which represent the lower and the upper bounded curves. These are around the main reliability curve in the crisp case evaluated at \(\alpha=1\) and all those curves are bounded by the lower and upper limit curves at \(\alpha=0\).

6. CONCLUSION

Different reliability models can represent most electronic and electrical systems. In many practical situations, we can deal with them as series or parallel systems consisting of number of operating units with parameters driven from gathering data or the opinions of the experts. Due to uncertainty or imprecision of data in calculations of these parameters, they are estimated
through triangular fuzzy numbers. In this paper, we establish a model for a system consisting of three independent units connected in parallel or in series. We introduce a reliability analysis for the series and the parallel systems in the presence of fuzziness and model the uncertainty of their parameters values. For the three operating units having time-varying failure rates, we use triangular fuzzy numbers which are estimated using random samples to represent their parameters. In addition, we introduce a procedure to estimate the fuzzy reliability and the mean-time to failure. This procedure can be applied to similar reliability models. Finally, we introduce a numerical example for illustration and we use MAPLE program to show the results graphically. Further researches can focus on other un-repairable systems as parallel-series systems, series-parallel systems, \( k \)-out-of-\( n \) systems or standby systems to evaluate the reliability and mean time of failure.

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8. REFERENCES

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چکیده
قابلیت اطمینان مهم تصمیم‌گیری در فرآیند طراحی و مهندسی است. اما در مسائل و مشکلات دنیای واقعی، محدودیت‌های استفاده از قابلیت اطمینان تجویز وجود دارد. مطالعه نتایج این مقاله در حین عدم قطعیت در زمانی که اثرات اطمینان قابل بروز در این مقاله برای هر دو سری و سیستم‌های موزایی مشکل از سه عصر پیکان با میل به قابلیت اطمینان با استفاده از مفاهیم فارا و برخی از ویژگی‌های از قبل زمان متوسط برای تشکیل ارگانی شده است تحقیق درک نشان می‌دهد. معرفی شده در حساب ما که همه عامل واحد واقعی تفریش نرخ مختلف مشکات با توالی عضویت مشی‌های این مقاله شامل یک مثال عدي برای نشان دادن مدل و اعتبار نتیجه‌گیری است.