Modeling of TiO$_2$-water Nanofluid Effect on Heat Transfer and Pressure Drop

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**ABSTRACT**

This paper reports a numerical study on the force convection heat transfer and flow characteristics of a nanofluid containing water with various volume fractions of TiO$_2$ (0.002<φ<0.02). The heat transfer coefficient (Nusselt number) and pressure drop of the nanofluid in a horizontal copper tube, under turbulent flow conditions (8000<Re<51000) with a single phase approach were investigated. The Nusselt number was enhanced with increasing the Reynolds number and nanoparticles volume fractions. Besides, with improving the heat transfer coefficient, the pressure drop was increased by using nanofluid. Modeling outcomes were compared with the experimental data. This comparison showed good agreement between the experimental data and the simulated results. In addition, two equations were developed to estimate Nusselt number and friction factor based on the dimensionless numbers.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Specific heat (J/kg.K)</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat in constant pressure(J/kg.K)</td>
</tr>
<tr>
<td>D</td>
<td>Tube diameter (m)</td>
</tr>
<tr>
<td>$d_p$</td>
<td>Nanoparticle diameter(m)</td>
</tr>
<tr>
<td>F</td>
<td>Friction factor (dimensionless)</td>
</tr>
<tr>
<td>h</td>
<td>Heat transfer coefficient (W/m$^2$.K)</td>
</tr>
<tr>
<td>K</td>
<td>Thermal conductivity (W/m.K)</td>
</tr>
<tr>
<td>L</td>
<td>Tube length(m)</td>
</tr>
<tr>
<td>Nu</td>
<td>Nusselt number (dimensionless)</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number (dimensionless)</td>
</tr>
<tr>
<td>$\Delta P$</td>
<td>Pressure drop (Pa)</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number (dimensionless)</td>
</tr>
<tr>
<td>$T_{inlet}$</td>
<td>Fluid inlet temperature (K)</td>
</tr>
<tr>
<td>$T_{outlet}$</td>
<td>Fluid outlet temperature (K)</td>
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</table>

**Greek Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Density (kg/m$^3$)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Viscosity (kg/m.s)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Particle volume fraction</td>
</tr>
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</table>

**Subscripts**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>Fluid</td>
</tr>
<tr>
<td>$P$</td>
<td>Particle</td>
</tr>
<tr>
<td>$nf$</td>
<td>Nanofluid</td>
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</tbody>
</table>

1. INTRODUCTION

Conventional heat transfer fluids such as water or ethylene glycol, used in cooling or heating applications are characterized by poor thermal properties. In the past years, many different techniques were utilized to improve the heat transfer rate in order to reach a satisfactory level of thermal efficiency. The heat transfer rate can passively be enhanced by changing Flow geometry, boundary conditions or by improving thermo physical properties for example, increasing fluid thermal conductivity [1].
Convection heat transfer plays an important role in the design and performance of very diverse thermal systems such as power plants, domestic refrigerators and electronic devices. Great efforts are, therefore, expanded in R&D trying to increase the heat flux and reduce the size of heat exchangers. One promising avenue in the pursuit of this objective is the use of nano fluids, a new generation of solid–liquid mixtures containing nano size particles. In the last 15 years, many studies have been published reporting on their thermo physical properties and performance as heat transfer fluids [2]. Several studies have been conducted on the performances of nanofluids convective heat transfer in laminar flow [3-5]. The stability and heat transfer rate of suspension enhanced compared with the suspensions with particles of millimeter or even micrometer in size. Many experimental studies have been done by researchers. Their report indicates that nanofluids have shown special advantages, such as better stability, greater thermal conductivity, and lower pressure drop compared to the base fluids. However, all these benefits may not occur simultaneously. It is observed that many of these papers are limited to a small range of turbulent flow regime and Reynolds number [6].

A few studies have reported the influence of TiO$_2$ on the heat transfer of nanofluids. Other researchers measured the thermal conductivity of suspended TiO$_2$ nanoparticles in deionized water. The results showed that thermal conductivity of nanofluid increased remarkably with increasing the volume fraction of nanoparticles. Some researchers studied static thermal conductivity, heat transfer and flow behavior of stable aqueous TiO$_2$ nanofluid with different particle sizes and concentrations. They found that the convective heat transfer coefficient increased with increasing nanoparticle concentration. They showed that the improvement in heat transfer was more significant in turbulent flow regime [7].

Duangthongsuk and Wongwises studied the heat transfer coefficient and the friction factor of TiO$_2$–water nanofluids in a horizontal double tube counter-flow heat exchanger at $3 \times 10^3 < \text{Re} < 1 \times 10^4$ [8]. Titanium dioxide nanoparticles of 21 nm diameter dispersed in water with different concentrations ranging from 0.2 to 2 vol.%. The heat transfer coefficient of nano fluids with 1.0 vol. % was approximately 26% greater than that of base fluid, while for that with 2.0 vol.%, it was approximately 14% lower. The pressure drop of nanofluids was slightly higher than that of the base fluid and increased with increasing the volume concentrations [6].

According to the literature, the effects of the Peclet number (between 20,000 and 60,000) and volume concentration of c-Al$_2$O$_3$ (25 nm)–water and TiO$_2$ (10 nm)–water nanofluids in a shell and tube heat exchanger were investigated. The maximum nanoparticle volume fraction of Al$_2$O$_3$ was 2%, and 0.75% for TiO$_2$. Based on their results, adding nanoparticles to the base fluid causes significant enhancement of heat transfer characteristics. Two different optimum nanoparticle concentrations exist; for TiO$_2$ nanoparticles, it was 0.3 vol.% [6].

Sajadi and Kazemi investigated the heat transfer behavior of TiO$_2$ (30 nm)–water nano fluid up to 0.25 vol.% in a circular pipe at $5 \times 10^3 < \text{Re} < 3 \times 10^4$ [7]. The results indicated that the addition of small amounts of nanoparticles to the base fluid augmented heat transfer remarkably, and heat transfer enhancement has a small variation with increasing the volume fraction of nanoparticles. The pressure drop of nanofluid was slightly higher than that of the base fluid and increased with increasing volume concentration.

This study attempted to examine numerically the convective heat transfer coefficient (Nusselt number) and pressure drop using computational fluid dynamic (CFD) tools in high Reynolds numbers ($8 \times 10^3 < \text{Re} < 5 \times 10^4$) and a wide range of nanoparticles volume fraction (0.002 < φ < 0.02). Further, numerical results were compared with the experimental data [6], and validation of code was assessed. Influences of Reynolds number and volume fraction of nanoparticles on the convective heat transfer coefficient and pressure drop were evaluated. Finally, the correlations of Nusselt number and friction factor based on the dimensionless numbers were obtained.

### 2. MATHEMATICAL MODELING

In this investigation, two-dimensional computational fluid dynamic (CFD) (version: 6.3.26) [9] was developed based on steady state fluid system. The nanofluid as a single phase fluid with complete different physical properties such as density, thermal conductivity and viscosity was considered. A continuous phase assumption was applied for the fluid phase. Governing equations (continuity, momentum and energy) are presented as follows [10]:

**Continuity equation:**
\[
\nabla \cdot (\rho_{nf} V_m) = 0
\]  
(1)

**Momentum equation:**
\[
\nabla \cdot (\rho_{nf} V_m V_m) = -\nabla p + \nabla \cdot (\mu_{nf} \nabla V_m)
\]  
(2)

**Energy equation:**
\[
\nabla \cdot (\rho_{nf} c_v V_m T) = \nabla \cdot (k_{nf} \nabla T)
\]  
(3)

In this numerical analysis, the well-known $\kappa$-$\epsilon$ turbulent model was used. $\kappa$-$\epsilon$ turbulent model brought into use two additional equations namely turbulent kinetic energy ($\kappa$) and rate of dissipation ($\epsilon$) [11]. The equations for turbulent kinetic energy ($\kappa$) and rate of dissipation ($\epsilon$) were given by:
\[ \nabla \left( \rho \vec{v} k \right) = \nabla \left( \frac{\kappa + \mu_k}{\sigma_{\text{grad} k}} \right) + G_k - \rho \varepsilon \]  

(4)

\[ \nabla \left( \rho \varepsilon \right) = \nabla \left( \mu + \left( \frac{\sigma_\varepsilon}{\sigma_\varepsilon} \mu_k \right) \text{grad} \varepsilon \right) + C_1 \varepsilon \left( \frac{\varepsilon}{\rho} \right) G_k - \frac{C_2 \rho \varepsilon^2}{\mu_t} \]  

(5)

where, \( G_k \) represented the generation of turbulent kinetic energy due to mean velocity gradients, \( \sigma_k \) and \( \sigma_\varepsilon \) were effective Prandtl numbers for turbulent kinetic energy and rate of dissipation, respectively; \( C_{1t} \) and \( C_{2t} \) were constants and \( \mu_t \) was the eddy viscosity and was modeled as:

\[ \mu_t = \frac{(\rho C_p \kappa^2)}{C_\mu} \]  

(6)

\( C_\mu \) was a constant and its value was 0.09. \( C_{1c} = 1.44; \ C_{2c} = 1.92; \) \( \sigma_k = 1.0 \) and \( \sigma_\varepsilon = 1.3 \). Further information is available in [12, 13]. The effective density of the nanofluid was given by:

\[ \rho_{nf} = (1 - \varphi) \rho_f + \varphi \rho_p \]  

(7)

The heat capacity of the nanofluid was defined as:

\[ (\rho C_p)_{nf} = (1 - \varphi)(\rho C_p)_f + \varphi(\rho C_p)_p \]  

(8)

where, \( \varphi \) was the particle volume fraction, \( \rho_{nf} \) was density and \( C_{nf} \) specific heat of nanofluid. Corcione’s correlation [13] was used for determination of nanofluid effective thermal conductivity:

\[ \frac{k_{nf}}{k_f} = 1 + 4.4Re_p^0.4Pr_f^{0.66} (\frac{T}{T_f})^{-0.1} (\frac{k_p}{k_f})^{0.3} \varphi^{0.66} \]  

(9)

where, \( T_f \) was the freezing point of the base liquid (about 273.16 K). Reynolds was the nanoparticle Reynolds number, defined as:

\[ Re_p = \frac{2 \rho_k u h T}{\eta_p} \]  

(10)

\( k_B \) was Boltzmann’s constant. Another correlation [14] was used to calculate dynamic viscosity of nanofluid:

\[ \frac{\mu_{nf}}{\mu_u} = \frac{1}{1 + 34.97 (\frac{\mu_u}{\eta_p})^{2.5}} \varphi^{1.03} \]  

(11)

where, \( d_\text{e} \) defined as the equivalent diameter of a molecule of base fluid and was equal to 3.85×10^{-10} m.

The temperature dependent variables: viscosity, thermal conductivity, density and specific heat of water, thermal conductivity, specific heat and density of TiO\(_2\) were calculated as reference paper formulas [6]. As shown in Figure 1, two-dimension horizontal pipe (with 1.288 m total length, 0.988 m heated section and 0.00818 m inner diameter) was spotted in our modeling. The tube wall was partially exposed to hot water. The nanofluid consisted of TiO\(_2\) nanoparticles of 30 nm average size. In this study, distilled water was used as liquid medium with different concentrations of nanofluid [0.002 (0.2%), 0.005 (0.5%), 0.01 (1.0%), 0.015 (1.5%) and 0.02 (2.0%)] and different Reynolds numbers (8000<Re<51000). Governing equations have been discretized using the finite volume method (FVM). The solver specification of the domain was involved the first-order upwind for momentum and energy; and SIMPLE (Semi-Implicit Method for Pressure-Linked Equations) algorithm was used to link the pressure and velocity fields. The linear systems resulting from discretization scheme were solved using Gauss-Seidel linear equation solver. During the iterative process, the residuals were carefully monitored and convergence criteria for mass, velocity, and temperature parameters was restricted to below 10^{-5}. At the tube inlet, different velocities depending on the values of Reynolds number, extracted from reference experimental data [6], were applied (u=V_{inlet}, v=w=0). At the domain outlet, the velocity and temperature profile were assumed to be fully developed; the outlet boundary condition was assumed zero pressure with no viscous stress. The no-slip boundary condition at the wall was appropriate for this study. In the heated section of the tube, convection heat transfer boundary condition was considered \([h=2000 (w/K.m^2)]; \) convective heat transfer coefficient was calculated using Dittos-Bolter equation, with free stream temperature of hot water as 333 K]; the unheated sections were thermally insulated. Only half of the tube was modeled due to the symmetry; on the lower section of the modeled domain, the axis boundary condition was applied. A non-uniform grid was considered spotting for our modeling domain.

![Figure 1. Numerical domain of pipe](www.SID.ir)
It was finer near the wall and entrance of mini-channel where the temperature and velocity gradients were large. Grid independency was carried out four mesh sizes on pure water. As can be seen in Figure 2, the 15×240 grids (15 in radial direction and 240 in axial direction) provided sufficiently precise results. Also, further fining of grids did not make a significant influence on the accuracy of results.

3. RESULTS AND DISCUSSION

3.1. Code Validation

The geometry configuration and boundary conditions are the same as experimental work reported by [6]. In order to validate the CFD model, numerical results were compared with the experimental reference data [6]. Figure 3 (a, b) displays the comparison between experimental reference data and simulated results. Also, Table 1 shows detailed deviation [%error = (experimental-calculated)/100/experimental data] about present simulation. As it is shown in Figure 3 (a, b) and Table 1, the Nusselt number and friction factor coincide closely with the experimental [6].

3.2. Heat Transfer Characteristics

The convective heat transfer coefficient and Nusselt number for nanofluid were calculated from the following equations:

\[
\dot{Q}_{nf} = \dot{m}_{nf} c_{nf} (T_{out} - T_{in})_{nf}
\]

\[
\overline{h}_{nf} = \frac{\dot{Q}_{nf}}{A(T_{wall} - T_{nf,in})}
\]

\[
\overline{Nu}_{nf} = \frac{\overline{h}_{nf} D}{k_{nf}}
\]

where, \(\dot{Q}_{nf}, \overline{h}_{nf}, A\), \(T_{wall}\) are average heat transfer rate, nanofluid mass flow rate, the heat transfer surface and average temperature of tube wall, respectively. Figure 4 represents the average Nusselt number versus Reynolds number for different concentrations of nanoparticles. It is shown that increasing Reynolds number and volume concentration of nanofluid improve the Nusselt number. Previous investigators have mentioned that the reasons for heat transfer enhancement of nano fluids included many issues, such as the mixing effects of particles near the wall, thermal conductivity enhancement, Brownian motion of particles, particle shape, particle migration and reduction of boundary layer thickness [8, 15]. Difference between augmentation in laminar & turbulent regime is due to difference in two flow conditions. At higher concentrations, more particles are taking part in heat transport and there is higher surface area of particles interacting with the base fluid, thereby enhancing the heat transfer process [6].
TABLE 1. Deviations in simulation

<table>
<thead>
<tr>
<th>Concentration%</th>
<th>Min error%</th>
<th>Average error%</th>
<th>Max error%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nusselt Friction factor</td>
<td>Nusselt Friction factor</td>
<td>Nusselt Friction factor</td>
</tr>
<tr>
<td>ϕ=0</td>
<td>0.6522 0.3048</td>
<td>1.8730 5.6952</td>
<td>5.1202 10.6667</td>
</tr>
<tr>
<td>ϕ=0.2</td>
<td>2.1580 1.5089</td>
<td>5.7376 5.3913</td>
<td>11.257 10.7143</td>
</tr>
<tr>
<td>ϕ=0.5</td>
<td>0.2930 0.4944</td>
<td>5.9531 6.1960</td>
<td>14.5314 10.8696</td>
</tr>
<tr>
<td>ϕ=1</td>
<td>3.0031 0.2653</td>
<td>6.9759 6.6927</td>
<td>12.8891 10.8383</td>
</tr>
<tr>
<td>ϕ=1.5</td>
<td>2.5646 0.5556</td>
<td>8.3562 6.2982</td>
<td>19.8485 7.0116</td>
</tr>
<tr>
<td>ϕ=2</td>
<td>5.3667 0.0270</td>
<td>9.4690 13.4996</td>
<td>10.5882 10.5882</td>
</tr>
</tbody>
</table>

For instance, convective heat transfer coefficient for \((\phi=0\% \text{ at } Re=8000, h=3616.387469), (\phi=0.02\% \text{ at } Re=8000, h=4847.714072) \) & \((\phi=2\% \text{ at } Re=8000, h=7114.053844) \) also \((\phi=0\% \text{ at } Re=46000, h=17533.28327), (\phi=0.02\% \text{ at } Re=46000, h=18396.15508) \) and \((\phi=2\% \text{ at } Re=46000, h=31452.36276) \).

It means that at \(Re=8000 \) for \(\phi=0.02\% \) and \(\phi=2\% \) in comparison with pure water, convective heat transfer coefficient are \(34\% \) and \(96.7\% \) greater respectively; and at \(Re=46000 \), for \(\phi=0.02\% \) and \(\phi=2\% \) convective heat transfer coefficient are \(4.9\% \) and \(80\% \) higher than that of the base fluid.

Figure 5 indicates the ratio of convective heat transfer coefficient in nanofluid to that of pure water as a function of Reynolds number. As can be seen from Figure 5, appreciable enhancement of heat transfer characteristics could be achieved by suspending nanoparticles in water.

The suspended particles increased the thermal conductivity of the two phase mixture. Furthermore, chaotic movement of these particles accelerated energy exchange process in the fluid [7].

As an example, at \(Re=8000 \) values of \(h_{nf}/h_w \) are \(1.33 \) and \(1.98 \) for \(\phi=0.2\% \) and \(\phi=2\% \), respectively. It means that \(1.8\% \) in concentration enhancement causes \(49\% \) improvement in \(h_{nf}/h_w \) ratio.

According to the simulation results, Nusselt number in horizontal tube as a function of the Reynolds number, Prandtl number and particles volume fraction was correlated.

\[
Nu = 0.011692Re^{0.875151}Pr^{0.316722}(1 + \phi)^{22.959211}[0.002 < \phi < 0.02, 8000 < Re < 51000] \tag{15}
\]

The above correlation can be used for TiO\(_2\) nanoparticle in the same condition same as the boundary conditions. Figure 6a illustrates the accuracy of the correlated results from this equation in comparison with the simulation results. Deviation was between \(+8\% \) & \(-10\% \), which was acceptable.

Figure 6b depicts correlated Nu number versus experimental, and comparison between this work with the literature [7, 16]. It shows that the correlated Nu number can cover the experimental data better than other works.
3. Effect of Nanoparticle Volume Concentration on the Pressure Drop

Figure 7 depicts the values of the pressure drop of nanofluid for different volume fractions of nanoparticles as a function of Reynolds number. It is observed that by increasing the Reynolds number or $\phi$, the pressure drop increases. The nanofluid showed higher pressure drop compared with water. Obviously, pressure drop rises with increasing concentration of nanoparticles due to intensity of working fluid viscosity. The pressure drop enhanced at higher Reynolds number by increasing $\phi$ [6]. In Reynolds numbers below 20000, changing $\phi$ does not have a significant influence on pressure drop. But, for higher Reynolds numbers, pressure drop showed more variation by changing the $\phi$.

Darcy friction factor $f$ and pressure drop ($\Delta p$) through the tube were obtained by these equations:

$$\Delta p = f \frac{L \rho u^2}{2}$$

$$f = (1.58 \ln Re - 3.82)^{-2}$$

For our modeling, we have correlated the equation and consider the nanoparticles volume fraction. Equation (18) shows the correlated equation as:

$$f = (1.14647 \ln Re - 4.550779)^{-1.912786}(1 + \phi)^{4.4058}[(0.002<\phi<0.02)8000<Re<51000]$$

The deviation of our correlation was around 11%. Figure 8 demonstrates correlated friction factors versus modeling friction factors, and correlated friction factors versus experimental friction factors (deviation was between $+18\%$ & $-11\%$).
4. CONCLUDING REMARKS

In this article, the Nu number and pressure drop in horizontal copper tube containing TiO$_2$ as nanoparticle with water as base fluid were calculated using CFD tools. The good agreement between CFD results and the referenced experimental data has been demonstrated. The results showed that the heat transfer coefficient increased with increasing the nanoparticles concentration and Reynolds number. By using the nanofluid at high Reynolds number, more power consumption compared to low Reynolds number was needed to make up for the pressure drop of nanofluid. Two equations for Nu number and friction factor based on the dimensionless numbers were correlated.

5. REFERENCES


Modeling of TiO$_2$-water Nanofluid Effect on Heat Transfer and Pressure Drop

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Abstract:

This study investigates the effect of TiO$_2$-water nanofluid on heat transfer and pressure drop in a nanofluid with volume fractions of 0.002 < φ < 0.02. The nanofluid was prepared using TiO$_2$ as a dispersed phase in water. The experimental results were compared with the numerical predictions using the commercial CFD software, and the correlation coefficients were calculated to validate the model. The results showed that the nanofluid increased the convective heat transfer coefficient and decreased the pressure drop significantly compared to pure water.

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