A Simple Approach to Static Analysis of Tall Buildings with a Combined Tube-in-tube and Outrigger-belt Truss System Subjected to Lateral Loading

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ABSTRACT

In this paper, an efficient technique is presented for static analysis of tall buildings with combined tube-in-tube and outrigger-belt truss system while considering shear lag effects. In the process of replacing the discrete structure with an elastically equivalent continuous one, the structure is modeled as two parallel cantilevered flexural-shear beams that are constrained at the outrigger-belt truss location by a rotational spring. Based on the principle of minimum total potential energy, simple closed form solutions are derived for stress and displacement distributions. Standard load cases, including uniformly distributed loads, triangularly distributed loads and point loads at top of the structure are considered. Results obtained from the proposed method for 50 and 60 story tall buildings are compared to those obtained using a standard finite element computer package. The approximate analyses are found to yield reasonable results and give a fairly good indication of actual structure’s response.

Keywords: Tube-in-tube
Outrigger-Belt Truss
Equivalent Continuum Model
Shear Lag
Stress Functions
Displacement Functions

1. INTRODUCTION

Today tall buildings are seen in cities all over the world. Tubular structures have been successfully utilized and are becoming a common feature in tall buildings. Basic forms of tubular systems are the framed tube, core tube, tube-in-tube and bundled tube. A tube-in-tube structure comprises of a peripheral framed tube and a core tube interconnected by floor slabs. For each of these vertical components, various simplified models have been developed that analyze structure’s behavior under lateral loads. Approximate techniques for a single tube and multi-tube systems have been developed by many researchers over the past decades. Chang analyzed tube-in-tube structures using a continuum approach in which the two beams are individually modeled by a tubular beam that accounts for flexural deformation, shear deformation, and shear lag effects [1]. Takabatake [4] developed an analytical method, suitable for the preliminary design stages of doubly symmetric single and double frame-tubes with braces. Lee et al. [5] proposed an analytical method for modeling the discrete tube structure with multiple internal tubes in obtaining deflection and stress distributions of the structure. In this model, stress field for each member of the structure is expressed in terms of linear functions dependent on member’s second moment of area, material property and geometry of the structure. Shape functions are assumed to describe displacements in flange and web frame panels for each tube [5]. Marsono and Wee studied the ultimate failure behavior of an overall reinforced concrete tube-in-tube tall building using a non-linear model [6].

The exterior and interior columns of a tube-in-tube structure are placed so closely together that they not only appear to be solid, but they act as a solid surface as well. The entire building acts as a huge hollow tube.
with a smaller tube in the middle of it. Lateral loads are shared between the inner and outer tubes. For structures may not rely on tube-in-tube interaction to resist lateral loads, girders are pin-connected to the columns, resulting in a substantial increase in stiffness and subsequent decrease in lateral drift, smoothening of stress distribution in columns is obtained by tying the exterior columns of outer tube to the core tube at one or several levels with one or two story stiff horizontal outrigger trusses [7, 8].

The outrigger-braced system is regarded as one of the most effective ways of increasing structural stiffness in tall buildings and has been widely studied over the past decades [9-14]. Taranath discussed the optimum locations for a two-outrigger system [15]. Gao investigated the effect of lintel beams in the outer tube, on elastic response of the outrigger structures [16]. Wu and Lee proposed the design criteria for reinforced concrete tall building structures with outriggers [17]. Wu and Lee presented detailed analyses on how the top drift, base moment in the core and fundamental vibration period are influenced by variations in outrigger location and structural stiffness when a multi-level outrigger-braced tall building structure is subjected to uniformly distributed forces or triangularly distributed loads along the building height [18].

Recently, static and dynamic analyses of combined system of outrigger-belt truss and shear core with single tube have been studied by Rahgozar et al. and Malekinejad and Rahgozar [19, 20]. In these works framed tube structure is modeled with four orthotropic membranes [21]. The effect of shear core and outrigger-belt trusses on framed tube system under lateral loads is modeled as a rotational spring at outrigger-belt truss location [15].

Most studies up to now have concentrated on static and dynamic analysis of tube-in-tube structures. Therefore work needs to be done on analysis of combined systems of tube-in-tube and outrigger-belt truss. Based on previous studies, in particular the method developed previously by Rahgozar et al. [19, 22], this paper presents detailed analysis of displacement and stress distribution functions in a combined system of tube-in-tube and multi-level outrigger-belt truss in a tall building structure that is subjected to uniformly distributed, triangularly distributed and concentrated loading along its building height considering shear lag effects.

2. SYSTEM DESCRIPTION AND ASSUMPTIONS

The design principle is to create two parallel hollow cantilevered box beams above ground that are constrained at outrigger-belt truss location by a rotational spring; as a result, the lateral loads are mainly or completely resisted by facades of the cantilever [19, 21]. Assumptions which are considered in this paper are same as those implemented by Kwan and Rahgozar et al. [19, 21]. Using continuous model, outer and inner tubes are comprised of two panels parallel to lateral load direction (web panels) and two panels perpendicular to lateral load direction (flange panels). The beams are forced to have equal lateral deflections, and the amount of load carried by each beam is a function of its relative stiffness [1]. Due to shear lag, plane sections will no longer remain plane after the structure is loaded. Herein, based on work which is carried out by Kwan [21], independent distributions for the axial displacements in the web and flange panels is made. The axial displacement distributions are assumed to be cubic in the web panels and parabolic in the flange panels and the principle of minimum total potential energy is employed for the formulation. Axial deformation in flange and web panels of outer and inner tubes are as follows:

\[ w_{ax} = \phi \left(1 - \alpha \frac{x}{a} + \alpha^2 \frac{x^2}{a^2}\right) \]  
\[ w_{ay} = \phi \left(1 - \beta \frac{y}{b} + \beta^2 \frac{y^2}{b^2}\right) \]  
\[ w_{ey} = \phi \left(1 - \gamma \frac{z}{c} + \gamma^2 \frac{z^2}{c^2}\right) \]  
\[ w_{fe} = \phi \left(1 - \gamma \frac{z}{c} + \gamma^2 \frac{z^2}{c^2}\right) \]

In which subscripts \( e \) and \( i \) are used for outer and inner tubes, respectively. \( w_{ax}, w_{ay}, w_{ey} \) and \( w_{fe} \) are axial deformation of outer and inner tube’s webs and flanges, respectively. \( \alpha, \beta, \alpha', \beta' \) are shear lag coefficients of outer and inner tube’s webs and flanges, respectively. Also, \( a, b, a', b' \) parameters are dimensions of outer and inner tubes’ panels as shown in Figure 1. The \( x, y \) direction coordinates are also shown in Figure 1 and \( z \) direction coordinate is along the structure’s height.

Figure 1. Tube-in-tube structure’s plan at outrigger-belt truss location.
Due to axial and shear strain of outer and inner tubes, the elastic strain energy of the tube-in-tube structure can be expressed as follows:

\[
\Pi = \int_0^H \int_0^L \left( E_\alpha \varepsilon_\alpha^2 + G_s \gamma_s^2 + E_t \varepsilon_t^2 + G_w \gamma_w^2 + E_f \varepsilon_f^2 + G_w \gamma_w^2 + E_i \varepsilon_i^2 + G_w \gamma_w^2 \right) \, dx \, dz
\]  
\[
+ \int_0^H \int_0^L \left( E_t \varepsilon_t^2 + E_f \varepsilon_f^2 + G_w \gamma_w^2 \right) \, dy \, dz
\]
\[
+ \int_0^H \int_0^L \left( E_i \varepsilon_i^2 + E_f \varepsilon_f^2 + G_w \gamma_w^2 \right) \, dy \, dz
\]
\[
+ \int_0^H \int_0^L \left( E_t \varepsilon_t^2 + E_f \varepsilon_f^2 + G_w \gamma_w^2 \right) \, dy \, dz
\]
\[
+ \int_0^H \int_0^L \left( 2 E_t A_t \varepsilon_t + 2 E_f A_f \varepsilon_f \right) \, dz + \int_0^H \int_0^L \left( 2 E_i A_i \varepsilon_i + 2 E_f A_f \varepsilon_f \right) \, dz
\]

In Equations (6-9), \( \varepsilon_\alpha, \varepsilon_f, \varepsilon_t, \) and \( \varepsilon_i \) are axial strains in flange and web frames of outer and inner tubes, respectively. \( \gamma_w, \gamma_w, \gamma_w, \gamma_w \) are shear strains of flange and web frames of outer and inner tubes, respectively. \( H \) is structure’s height as shown in Figure 2.

**Figure 2.** Equivalent tube-in-tube structure.

In a similar manner, total potential energy of the structure is minimized with respect to unknown parameter \( u \), yielding the governing equation in terms of \( u \) as [21]:

\[
S = 4 \left( G_w t_w b + G_s t_s b' \right) \left( \frac{\partial u}{\partial \phi} \right)
\]

Hence, \( u \) can be expressed as [21]:

\[
u = \int_0^H \left( \frac{S}{4 (G_w t_w b + G_s t_s b')} - \phi \right) \, dz
\]

where \( S \) is the shear related to external loads.

### 3. SHEAR LAG PARAMETERS

Substituting \( \phi \) and \( u \) from Equations (15) and (17) into the total potential energy statement of the structure and minimizing it with respect to unknown shear lag coefficients, the governing equations for \( \alpha \) and \( \beta \) can be obtained. For simplicity, \( \alpha \) and \( \beta \) are estimated via polynomial functions with unknown coefficients \( \alpha_0, \alpha_1, \beta_0, \beta_1, \alpha_0', \alpha_1', \beta_0', \) and \( \beta_1' \). Applying boundary condition at top of the structure where axial stress is
zero gives: \( \frac{d\beta}{dz} = \frac{d\alpha}{dz} = \frac{d\beta'}{dz} = \frac{d\alpha'}{dz} = 0 \) and then minimizing the total potential energy with respect to parameters \( \alpha, \alpha', \beta, \beta', \alpha'' \), \( \alpha'' \), \( \beta' \), \( \beta'' \) and \( \beta''' \), yields the shear lag coefficients in web and flange panels of inner and outer tubes. Shear lag coefficients for three cases of force and concentrated moment are listed in Tables 1-4, where \( C \) is the location of outrigger-belt truss from structure’s base. \( \alpha, \alpha', \beta \) and \( \beta' \) are shear lag coefficients of web and flange panels for inner and outer tubes due to the moment created by outrigger-belt truss located at \( C \).

### Table 1. Shear lag coefficients for concentrated load at top of the structure

| \( \alpha_1 \) | \( \frac{7\alpha E_z (6\alpha E_z + 7H^2 G_s)}{24\alpha E_z + 112\alpha^3 H^4 E_z G_s + 42H^2 G_s^2} \) |
| \( \alpha_2 \) | \( \frac{7\alpha E_z (24\alpha E_z + 7H^2 G_s)}{8(12\alpha E_z + 56\alpha^3 H^4 E_z G_s + 21H^2 G_s^2} \) |
| \( \beta_1 \) | \( \frac{35\beta E_z (18\beta E_z + 5H^2 G_s)}{504\beta E_z + 56\beta^3 H^4 E_z G_s + 50H^2 G_s^2} \) |
| \( \beta_2 \) | \( \frac{35\beta E_z (72\beta E_z + 5H^2 G_s)}{8(252\beta E_z + 280\beta^3 H^4 E_z G_s + 25H^2 G_s^2)} \) |

### Table 2. Shear lag coefficients for uniformly distributed load along structure’s height

| \( \alpha_1 \) | \( \frac{126\alpha E_z (531185\alpha E_z + 1219427H^2 G_s)}{3.82 \times 10^5 a^2 E_z + 1.75 \times 10^2 a^5 H^4 E_z G_s + 5.96 \times 10^6 H^3 G_s^2} \) |
| \( \alpha_2 \) | \( \frac{63\alpha E_z (406370\alpha E_z + 24997H^2 G_s)}{3.82 \times 10^5 a^2 E_z + 1.75 \times 10^2 a^5 H^4 E_z G_s + 5.96 \times 10^6 H^3 G_s^2} \) |
| \( \beta_1 \) | \( \frac{810\beta E_z (106370\beta E_z + 5078H^2 G_s)}{6.89 \times 10^5 a^2 E_z + 7.52 \times 10^2 a^5 H^4 E_z G_s + 6.39 \times 10^6 H^3 G_s^2} \) |
| \( \beta_2 \) | \( \frac{405\beta E_z (21274\beta E_z + 119H^2 G_s)}{6.89 \times 10^5 a^2 E_z + 7.52 \times 10^2 a^5 H^4 E_z G_s + 6.39 \times 10^6 H^3 G_s^2} \) |

### Table 3. Shear lag coefficients for trianally distributed load along structure’s height

| \( \alpha_1 \) | \( \frac{231\alpha E_z (3061830\alpha E_z + 62583663H^2 G_s)}{4.04 \times 10^5 a^2 E_z + 1.86 \times 10^2 a^5 H^4 E_z G_s + 6.54 \times 10^6 H^3 G_s^2} \) |
| \( \alpha_2 \) | \( \frac{231\alpha E_z (53099115\alpha E_z + 1420204H^2 G_s)}{2.02 \times 10^5 a^2 E_z + 9.30 \times 10^2 a^5 H^4 E_z G_s + 3.27 \times 10^6 H^3 G_s^2} \) |
| \( \beta_1 \) | \( \frac{495\beta E_z (214328114\beta E_z + 104306105H^2 G_s)}{8.48 \times 10^5 a^2 E_z + 9.30 \times 10^2 a^5 H^4 E_z G_s + 7.78 \times 10^6 H^3 G_s^2} \) |
| \( \beta_2 \) | \( \frac{165\beta E_z (32149217\beta E_z + 7101020H^2 G_s)}{4.24 \times 10^5 a^2 E_z + 4.65 \times 10^2 a^5 H^4 E_z G_s + 3.89 \times 10^6 H^3 G_s^2} \) |

### Table 4. Shear lag coefficients for concentrated moment

| \( \alpha_1 \) | \( \frac{35\alpha E_z (6\alpha E_z - 15C^2 G_s)}{160\alpha E_z^2 + 680\alpha C^2 E_z G_s + 273C^3 G_s^2} \) |
| \( \alpha_2 \) | \( \frac{35\alpha E_z (4a^2 E_z + 3C^2 G_s)}{160\alpha E_z^2 + 680\alpha C^2 E_z G_s + 273C^3 G_s^2} \) |
| \( \beta_1 \) | \( \frac{15\beta E_z (6\beta^2 E_z - 25C^2 G_s)}{672\beta E_z^2 + 680\beta C^2 E_z G_s + 65C^3 G_s^2} \) |
| \( \beta_2 \) | \( \frac{30\beta E_z (28\beta^2 E_z + 5C^2 G_s)}{672\beta E_z^2 + 680\beta C^2 E_z G_s + 65C^3 G_s^2} \) |

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### 4. GOVERNING PARAMETERS FUNCTIONS

#### 4.1. Parametric Stress Functions

Flange and web frames’ deflection functions may be calculated using shear lag coefficients that were presented earlier. In the derivation process of these functions, first, axial strain functions are calculated. Then axial strain functions are multiplied by modulus of elasticity for each panel and considering outrigger-belt truss effects, equations of axial stress distribution for the structure are obtained in terms of the elevation of outrigger-belt truss. Amount of load carried by each inner and outer tube is a function of its relative stiffness. Therefore, axial stress distribution functions of web and flange panels for inner and outer tubes can be stated as follows:

For \( z > C \):

\[
\sigma_{wz} = \left[ \frac{E_l}{E_i} + \frac{E_i}{E_l} \right] \frac{d\delta}{dz} \left[ \alpha \right] + \frac{1}{b} \left[ \beta \right]
\]

(18)

\[
\sigma_{wz} = \left[ \frac{E_l}{E_i} + \frac{E_i}{E_l} \right] \frac{d\delta}{dz} \left[ \alpha \right] + \frac{1}{a} \left[ \beta \right]
\]

(19)

\[
\sigma_{wz} = \left[ \frac{E_l}{E_i} + \frac{E_i}{E_l} \right] \frac{d\delta}{dz} \left[ \alpha \right] + \frac{1}{b} \left[ \beta \right]
\]

(20)

\[
\sigma_{wz} = \left[ \frac{E_l}{E_i} + \frac{E_i}{E_l} \right] \frac{d\delta}{dz} \left[ \alpha \right] + \frac{1}{a} \left[ \beta \right]
\]

(21)

For \( z \leq C \):

\[
\sigma_{wz} = \left[ \frac{E_l}{E_i} + \frac{E_i}{E_l} \right] \frac{d\delta}{dz} \left[ \alpha \right] + \frac{1}{b} \left[ \beta \right]
\]

(22)

\[
\sigma_{wz} = \left[ \frac{E_l}{E_i} + \frac{E_i}{E_l} \right] \frac{d\delta}{dz} \left[ \alpha \right] + \frac{1}{a} \left[ \beta \right]
\]

(23)

\[
\sigma_{wz} = \left[ \frac{E_l}{E_i} + \frac{E_i}{E_l} \right] \frac{d\delta}{dz} \left[ \alpha \right] + \frac{1}{b} \left[ \beta \right]
\]

(24)
\[ \varepsilon = \left( \frac{E}{E_1} + \frac{E}{E_1} \right) \left[ E_2 \frac{d^2 \varepsilon}{dx^2} \right] + \beta \left( \frac{d^2 \varepsilon}{dx^2} \right) \]

where \( \varepsilon_1 \), \( \varepsilon_2 \), \( \varepsilon_n \), and \( \varepsilon_b \) are axial stress in web and flange frames for outer and inner tubes along structure's height while considering the outrigger-belt truss effects. \( E_1 \) and \( E_2 \) are flexural stiffness of outer and inner tubes, respectively.

Equations (18-25) depend on the unknown parameter \( \delta \phi / \delta z \), which can be determined by placing axial stress of the framed tube into equilibrium equation of the structure using Equation (14) as follows:

\[ M = E_1 \delta \phi / \delta z \left[ \sum \alpha \varepsilon \right] + \sum \alpha \varepsilon \]

where \( \alpha \varepsilon \) and \( \beta \varepsilon \) are axial stress of the corner columns of outer and inner tubes, respectively. Equivalent flexural stiffness values \( E_1 \) and \( E_2 \) can be determined by equating the amount of \( E_1 \delta \phi / \delta z \) from Equation (26) to each of the external moments due to the three load cases of concentrated load at top of the structure, triaxially and uniformly distributed loads along the height of the structure. \( E_1 \) and \( E_2 \) become:

\[ E_2 = \frac{4}{3} E_1 \alpha \varepsilon \]

In a similar manner, equivalent stiffness due to concentrated moment can be expressed as follows:

\[ E_1 = \frac{4}{3} E_1 \alpha \varepsilon \]

where \( E_i \) and \( E_o \) are equivalent flexural stiffness of outer and inner tubes, respectively.

Axial stress in the outer and inner tubes can be calculated from the amount of equivalent stiffness in outrigger-belt truss, \( K \), and the amount of structure's rotation at outrigger-belt truss location.

4.2. Parametric Displacement Function and Rotation Function along Structure's Height

Lateral displacement functions of the structure due to each of the three types of load cases considered here (concentrated load at top of the structure, uniformly and triaxially distributed loads along the height of the structure) can be obtained by first substituting \( E_1 \) and \( E_2 \) from Equations (31) and (32) into Equation (15) and solving for \( \phi \) and then calculating Equation (17) with the computed \( \phi \). If the rigidity factor \( E_1 \), is variable along the height of the structure, calculation of \( \phi \) and lateral displacement become complicated. Hence, \( E_1 \) and \( E_2 \) are assumed to be constant along the height of the structure and are equal to their values at structure's base [21]. Based on this assumption, lateral displacement functions of the structure due to lateral loads are expressed as follows:

Concentrated load at top of the structure:

\[ u = \frac{P}{E_1} \left( H^2 - \frac{1}{6} + \frac{1}{z} \right) + \frac{P}{E_1} \left( \frac{K0}{E_1} \right) \left( C - C^2 \right) \]

for \( z > C \):

\[ u = \frac{P}{E_1} \left( H^2 - \frac{1}{6} + \frac{1}{z} \right) + \frac{P}{E_1} \left( \frac{K0}{E_1} \right) \left( C - C^2 \right) \]

for \( z \leq C \):

Uniformly distributed load along structure's height:

\[ u = \frac{U}{E_1} \left( H^2 - \frac{1}{6} + \frac{1}{z} \right) + \frac{U}{E_1} \left( \frac{K0}{E_1} \right) \left( C - C^2 \right) \]

for \( z > C \):

\[ u = \frac{U}{E_1} \left( H^2 - \frac{1}{6} + \frac{1}{z} \right) + \frac{U}{E_1} \left( \frac{K0}{E_1} \right) \left( C - C^2 \right) \]

for \( z \leq C \):

Triangularly distributed load along structure's height:

\[ u = \frac{T}{E_1} \left( H^2 - \frac{1}{6} + \frac{1}{z} \right) + \frac{T}{E_1} \left( \frac{K0}{E_1} \right) \left( C - C^2 \right) \]

for \( z > C \):

\[ u = \frac{T}{E_1} \left( H^2 - \frac{1}{6} + \frac{1}{z} \right) + \frac{T}{E_1} \left( \frac{K0}{E_1} \right) \left( C - C^2 \right) \]

for \( z \leq C \):

\[ u = \frac{T}{E_1} \left( H^2 - \frac{1}{6} + \frac{1}{z} \right) + \frac{T}{E_1} \left( \frac{K0}{E_1} \right) \left( C - C^2 \right) \]

Relations for rotation of the combined system, \( \theta \), at outrigger-belt truss location at height \( C \) for the three loading cases can be derived by considering compatibility of deformations. Rotation for each load case becomes:

Concentrated load at top of the structure:

\[ \theta = \frac{EL}{EL + KC} \left( \frac{P}{4Gz^3f_0 + 4Gz^3f_0} \right) \left( CH - C^2 \right) \]
\[ \theta_{e} = \frac{EI}{EI + kC} \left\{ \frac{U}{EI} \left( \frac{H^2}{2} + \frac{H}{6} \right) + \frac{U}{4EI} \left( \frac{H^2}{12} + \frac{H}{6} \right) \right\} \]  

(40)

Triangulated distributed load along structure’s height:

\[ \theta_{e} = \frac{EI}{EI + kC} \left\{ \frac{T}{2EI} \left( \frac{H}{2} \right) + \frac{T}{2EI} \left( \frac{H}{2} \right) + \frac{T}{2EI} \left( \frac{H}{2} \right) \right\} \]  

(41)

Equivalent stiffness of the rotational spring at outrigger-belt truss location is given by Malekinejad and Rahgozar as follows [20, 24]:

\[ K = \frac{C}{2b'^4A_{f}} + \frac{C}{2b'^4A_{w}} + \frac{b'^2}{6EI} + \frac{1}{kGA} \]  

(42)

where \( A_{c}, A_{b}, k, G, A \) and \( l \) are sum of columns cross sectional areas of flange frames of outer and inner tubes, shear coefficient, shear modulus, sectional area, sectional area of an outrigger-belt truss and height of outrigger-belt truss, respectively. \((EI)\) is the effective flexural stiffness of the outrigger.

5. COMPARING RESULTS OF THE PROPOSED METHOD TO SAP2000 ANALYSES

In this section, simplicity and accuracy of the proposed method are illustrated through numerical examples for 50 and 60 story reinforced concrete buildings with combined systems of tube-in-tube and outrigger-belt truss systems. Location of outrigger-belt truss is moved along the height of structure.

All beams, columns and outrigger-belt truss members have constant sizes along the height of structure as \( 0.8 \text{ m} \times 0.8 \text{ m} \) and other geometrical dimensions are illustrated in Table 5. Also, actual and equivalent properties of the structure are listed in Table 6. The structure is subjected to three external load cases separately as follows:

- Concentrated load at top of the structure:
  \[ P = 18 \times 10^3 \text{ kN} \]

- Uniformly distributed load along the structure’s height:
  \[ U = 120 \text{ kN/m} \]

- Triangulated distributed load along the structure’s height:
  \[ T = 240 \text{ kN/m} \]

Based on continuum model presented by Kwan [21] first, equivalent properties of the structure are calculated and then by calculating shear lag coefficients for different load cases, axial stresses in the inner and outer tubes and lateral displacement of the structure are calculated using Equations (18-25) and (33-38). Outrigger-belt truss is placed at different elevations (\( H/6, H/4, H/2, 3H/4 \) and \( H \)).

Displacement and axial stresses from the proposed method are compared with those obtained using SAP2000. Results for the three load cases are listed in Tables 7-12. Lateral displacement of the structure along its height, and axial stress in web and flange of the inner and outer tubes are shown in Figures 3-17 for 50-story tall building. Lateral displacement, and axial stress in web and flange of the inner and outer tubes of 60-story tall building are similar to those which are plotted for 50-story tall building. Therefore, these graphs are not repeated here.

The results presented here show that displacement values at top of the structure calculated via the proposed method are in good agreement with those obtained from SAP2000 analysis (differences are about 18 percent). Also, the proposed functions for stress distribution in web and flange panels of outer and inner tubes are accurate enough.

For example, in 50-story tall building when outrigger-belt truss is placed at height \( H/6 \), axial stresses in corner columns at structure’s base are underestimated by 10-17 percent, axial stress at structure’s base in middle columns is overestimated by 2-7 percent and the lateral displacement at top of the structure is underestimated by 2-12 percent for the three load cases considered here.

Estimating axial deformation in web and flange with cubic and quadratic functions based on research by Kwan [21]; equivalent stiffness for outrigger-belt trusses; assuming constant value for shear and flexural stiffness of the structure and using continuum modeling for the structure are sources of error in the proposed method when comparing to finite element analysis.

### Table 5. Geometrical dimensions of the 50 and 60 story tall buildings

<table>
<thead>
<tr>
<th>Height of story</th>
<th>Total height</th>
<th>Outer tube dimensions</th>
<th>Inner tube dimensions</th>
<th>Center to center columns distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{(\text{no})} )</td>
<td>( H_{(\text{no})} )</td>
<td>( 2b_{(\text{no})} )</td>
<td>( 2a_{(\text{no})} )</td>
<td>( 2b'_{(\text{no})} )</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>30</td>
<td>50</td>
<td>10</td>
</tr>
</tbody>
</table>

### Table 6. Actual and equivalent properties of the 50 and 60 story tall buildings

<table>
<thead>
<tr>
<th>Actual elastic properties</th>
<th>Equivalent elastic properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>Flange</td>
</tr>
<tr>
<td>( E_{(\text{web})} )</td>
<td>( G_{(\text{web})} )</td>
</tr>
<tr>
<td>( E_{(\text{flange})} )</td>
<td>( G_{(\text{flange})} )</td>
</tr>
<tr>
<td>( t_{(\text{web})} )</td>
<td>( t_{(\text{flange})} )</td>
</tr>
<tr>
<td>20</td>
<td>8</td>
</tr>
<tr>
<td>20</td>
<td>8</td>
</tr>
</tbody>
</table>
### TABLE 7. Comparison between results for 50-story tall building subjected to concentrated load at top of the structure

<table>
<thead>
<tr>
<th>C</th>
<th>Axial stress of middle columns of flange frames (MPa)</th>
<th>Axial stress of corner columns of flange frames (MPa)</th>
<th>Displacement at top of the structure (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SAP</td>
<td>proposed method</td>
<td>% of error</td>
</tr>
<tr>
<td>H/6</td>
<td>3.97</td>
<td>4.22</td>
<td>6</td>
</tr>
<tr>
<td>H/4</td>
<td>3.98</td>
<td>4.12</td>
<td>3</td>
</tr>
<tr>
<td>H/2</td>
<td>3.52</td>
<td>3.87</td>
<td>9</td>
</tr>
<tr>
<td>3H/4</td>
<td>3.27</td>
<td>3.77</td>
<td>15</td>
</tr>
<tr>
<td>H</td>
<td>3.13</td>
<td>3.73</td>
<td>19</td>
</tr>
</tbody>
</table>

### TABLE 8. Comparison between results for 50-story tall building subjected to uniformly distributed load along structure’s height

<table>
<thead>
<tr>
<th>C</th>
<th>Axial stress of middle columns of flange frames (MPa)</th>
<th>Axial stress of corner columns of flange frames (MPa)</th>
<th>Displacement at top of the structure (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SAP</td>
<td>proposed method</td>
<td>% of error</td>
</tr>
<tr>
<td>H/6</td>
<td>1.78</td>
<td>1.81</td>
<td>1</td>
</tr>
<tr>
<td>H/4</td>
<td>1.65</td>
<td>1.76</td>
<td>6</td>
</tr>
<tr>
<td>H/2</td>
<td>1.36</td>
<td>1.47</td>
<td>8</td>
</tr>
<tr>
<td>3H/4</td>
<td>1.19</td>
<td>1.37</td>
<td>15</td>
</tr>
<tr>
<td>H</td>
<td>1.13</td>
<td>1.38</td>
<td>22</td>
</tr>
</tbody>
</table>

### TABLE 9. Comparison between results for 50-story tall building subjected to triangularly distributed load along structure’s height

<table>
<thead>
<tr>
<th>C</th>
<th>Axial stress of middle columns of flange frames (MPa)</th>
<th>Axial stress of corner columns of flange frames (MPa)</th>
<th>Displacement at top of the structure (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SAP</td>
<td>proposed method</td>
<td>% of error</td>
</tr>
<tr>
<td>H/6</td>
<td>2.47</td>
<td>2.65</td>
<td>7</td>
</tr>
<tr>
<td>H/4</td>
<td>2.34</td>
<td>2.58</td>
<td>10</td>
</tr>
<tr>
<td>H/2</td>
<td>1.99</td>
<td>2.14</td>
<td>7</td>
</tr>
<tr>
<td>3H/4</td>
<td>1.76</td>
<td>2.04</td>
<td>15</td>
</tr>
<tr>
<td>H</td>
<td>1.67</td>
<td>2.03</td>
<td>21</td>
</tr>
</tbody>
</table>

### TABLE 10. Comparison between results for 60-story tall building subjected to concentrated load at top of the structure

<table>
<thead>
<tr>
<th>C</th>
<th>Axial stress of middle columns of flange frames (MPa)</th>
<th>Axial stress of corner columns of flange frames (MPa)</th>
<th>Displacement at top of the structure (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SAP</td>
<td>proposed method</td>
<td>% of error</td>
</tr>
<tr>
<td>H/6</td>
<td>3.99</td>
<td>4.72</td>
<td>18</td>
</tr>
<tr>
<td>H/4</td>
<td>4.12</td>
<td>4.93</td>
<td>19</td>
</tr>
<tr>
<td>H/2</td>
<td>4.27</td>
<td>5.07</td>
<td>18</td>
</tr>
<tr>
<td>3H/4</td>
<td>4.67</td>
<td>5.10</td>
<td>9</td>
</tr>
<tr>
<td>H</td>
<td>4.51</td>
<td>4.80</td>
<td>6</td>
</tr>
</tbody>
</table>

### TABLE 11. Comparison between results for 60-story tall building subjected to uniformly distributed load along structure’s height

<table>
<thead>
<tr>
<th>C</th>
<th>Axial stress of middle columns of flange frames (MPa)</th>
<th>Axial stress of corner columns of flange frames (MPa)</th>
<th>Displacement at top of the structure (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SAP</td>
<td>proposed method</td>
<td>% of error</td>
</tr>
<tr>
<td>H/6</td>
<td>1.74</td>
<td>1.92</td>
<td>10</td>
</tr>
<tr>
<td>H/4</td>
<td>1.69</td>
<td>2.05</td>
<td>21</td>
</tr>
<tr>
<td>H/2</td>
<td>2.27</td>
<td>2.63</td>
<td>15</td>
</tr>
<tr>
<td>3H/4</td>
<td>2.53</td>
<td>2.65</td>
<td>4</td>
</tr>
<tr>
<td>H</td>
<td>2.44</td>
<td>2.54</td>
<td>4</td>
</tr>
</tbody>
</table>
TABLE 12. Comparison between results for 60-story tall building subjected to triangularly distributed load along structure’s height

<table>
<thead>
<tr>
<th>C</th>
<th>Axial stress of middle columns of flange frames (MPa)</th>
<th>Axial stress of corner columns of flange frames (MPa)</th>
<th>Displacement at top of the structure (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SAP proposed method % of error</td>
<td>SAP proposed method % of error</td>
<td>SAP proposed method % of error</td>
</tr>
<tr>
<td>H/6</td>
<td>2.34 2.85 21</td>
<td>8.82 7.13 19</td>
<td>40.10 45.12 12</td>
</tr>
<tr>
<td>H/4</td>
<td>2.15 2.47 14</td>
<td>9.04 7.25 19</td>
<td>39.75 44.53 12</td>
</tr>
<tr>
<td>H/2</td>
<td>1.69 2.04 20</td>
<td>9.27 7.32 21</td>
<td>39.47 43.41 9</td>
</tr>
<tr>
<td>3H/4</td>
<td>1.51 1.82 20</td>
<td>9.35 7.39 20</td>
<td>38.81 43.10 11</td>
</tr>
<tr>
<td>H</td>
<td>1.47 1.81 23</td>
<td>9.44 7.44 21</td>
<td>39.51 42.23 6</td>
</tr>
</tbody>
</table>

Figure 3. Displacement of 50-story combined system under uniformly distributed load along the height of structure (C = H/6).

Figure 4. Axial stress distribution of half length of flange in outer tube due to distributed load along the height of 50-story tall building (C = H/6).

Figure 5. Axial stress distribution of half length of flange in inner tube due to distributed load along the height of 50-story tall building (C = H/6).

Figure 6. Axial stress distribution of half length of web in outer tube due to distributed load along the height of 50-story tall building (C = H/6).

Figure 7. Axial stress distribution of half length of web in inner tube due to distributed load along the height of 50-story tall building (C = H/6).

Figure 8. Displacement of 50-story combined system under uniformly distributed load along the height of structure (C = H/2).
Figure 9. Axial stress distribution of half length of flange in outer tube due to distributed load along the height of 50-story tall building (C = H/2).

Figure 10. Axial stress distribution of half length of flange in inner tube due to distributed load along the height of 50-story tall building (C = H/2).

Figure 11. Axial stress distribution of half length of web in outer tube due to distributed load along the height of 50-story tall building (C = H/2).

Figure 12. Axial stress distribution of half length of web in inner tube due to distributed load along the height of 50-story tall building (C = H/2).

Figure 13. Displacement of 50-story combined system under uniformly distributed load along the height of structure (C = 3H/4).

Figure 14. Axial stress distribution of half length of flange in outer tube due to distributed load along the height of 50-story tall building (C = 3H/4).

Figure 15. Axial stress distribution of half length of flange in inner tube due to distributed load along the height of 50-story tall building (C = 3H/4).

Figure 16. Axial stress distribution of half length of web in outer tube due to distributed load along the height of 50-story tall building (C = 3H/4).
6. CONCLUSION

This paper presents parametric functions for static analysis of tall buildings with combined system of tube-in-tube and outrigger-belt truss system subjected to three separate load cases of concentrated load at top of the structure, uniformly and triangularly distributed loads along the height of the structure. Approximate formulas for estimating stress and displacement of the structure have been proposed by means of minimizing potential energy of the structure including the bending deformation; transverse shear deformation and shear lag effects in web and flange panels. The structure has been modeled by two continuous cantilever beams which are restrained at outrigger-belt truss location by a rotational spring. The formulas proposed here have been validated by comparing them to the computer static analysis results obtained from three-dimensional studies using the finite element method. It has been shown that results computed by the energy method correlate well with those obtained by means of SAP2000 analysis. Application of the method is versatile. The method is simple and the accuracy is acceptable. Also it can reduce the computational work drastically as compared to SAP2000 analysis for static analysis of combined system of tube-in-tube and outrigger-belt truss tall buildings; hence rendering it as a simple and efficient tool suitable for early stages of tall building design.

7. REFERENCES


Figure 17. Axial stress distribution of half length of web in inner tube due to distributed load along the height of 50-story tall building (C = 3H/4).
A Simple Approach to Static Analysis of Tall Buildings with a Combined Tube-in-tube and Outrigger-belt Truss System Subjected to Lateral Loading

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Keywords: Tube-in-tube, Outrigger-Belt Truss, Equivalent Continuum Model, Shear Lag, Stress Functions, Displacement Functions

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