NUMERICAL SOLUTION OF PRICING OF EUROPEAN PUT OPTION WITH STOCHASTIC VOLATILITY

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Abstract

In this paper, European option pricing with stochastic volatility forecasted by well known GARCH model is discussed in context of Indian financial market. The data of Reliance Ltd. stock price from 3/01/2000 to 30/03/2009 is used and resulting partial differential equation is solved by Crank-Nicolson finite difference method for various interest rates and maturity in time. The sensitivity measures “Greeks” are also determined to validate the model. It is observed that the value of European put option increases with maturity time and decreases with interest rate.

Keywords European Option, Finite Difference Method, Stochastic Volatility, GARCH (1, 1), Greeks

1. INTRODUCTION

It is widely acknowledged by financial researchers Black, et al [1], Merton [2] that the valuation of options leads to mathematical model which has long been an intriguing problem in different ways. The most fundamental input into an option pricing model is volatility, a measure of how much the underlying asset price is likely to vary over time. In financial markets, volatility presents a strange paradox to the market participants, academicians and policy makers Nelson [3], even volatility estimation is by no means an exact science but a lot of efforts have been expended in improving volatility model since better forecasts transforms into better pricing of option Bollerslev, et al [4], and Loudon, et al [5].

Recently, Loudon, et al [5], Mc-Millan, et al [6], Yu [7], Klaassen [8], Vilasuso [9] and Balaban [10] investigated the forecasting models in various markets and found that ARCH class of models provide better forecast in terms of statistical error and evidence in favor of GARCH model over shorter intervals. In all these studies, various methods for the estimation of volatility and their performances were discussed in terms of statistical error but none of them used volatility forecasting in the valuation of option pricing governed by Black-Scholes partial differential equation.

Hull, et al [11] determined the numerical solution of Black-Scholes partial differential equation regarding constant volatility. Later on, an alternative approach has been proposed by Avellaneda, et al [12-13] in which it is assumed that volatility is uncertain but lies within a known range of values. Recently, Chawla, et al [14], Mayo [15], Tangman, et al [16] and Hu, et al [17] developed more efficient finite difference numerical methods for pricing of option giving better accuracy than normal difference method. But, the option prices obtained using Black-Scholes model with constant volatility is not
consistent with observed option prices. One possible remedy for this is to make the volatility to be a function of time and strike price, which leads to a model in terms of parabolic partial differential equation in two variables i.e. volatility and underlying asset value.

In this paper an alternative approach is developed for the pricing of option by Black-Scholes partial differential equation regarding variable volatility which is forecasted by GARCH (1, 1) method and the resulting one dimensional parabolic partial differential equation is solved by implicit finite difference method [18].

2. FORMULATION AND SOLUTION OF MODEL

In the modeling of option pricing, the physical system is the financial market place and the particular object of observation is the price of the option’s underlying asset. In order to develop a model tractable by mathematical and computational techniques, it is assumed that price of European option \( V \) is a function of current value of the underlying asset \( S \) and the time \( t \), i.e. \( V = V(S,t) \). The continuous-time, Black-Scholes model to European option is

\[
\frac{\partial V}{\partial t} + r S \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - r V = 0 \quad t \in [0,T], S \in [0,\infty]
\]

where, \( T \) is time of expiration, \( r \) is a risk-free interest rate and \( \sigma \) is volatility of stock returns.

Suppose \( K \) and \( S_T \) be strike price and price of underlying asset on the date of expiration \( T \), respectively. In case of \( S_T < K \), it has a financial sense (in-the-money for holder of put option) and encourage to the holder of put option for exercise, because holder can sell the asset of worth \( S_T \) at the cost of \( K \). Thus, the gain of holder from the call option is \( (K - S_T) \). In case of \( S_T \geq K \), the holder will forfeit the right to exercise the option because he can buy the asset at a cost, less than or equal to predetermined strike price \( K \). Similarly, in-the-money position for put option it is \( S_T < K \), under which, asset is sold at higher price of \( K \) instead of \( S_T \). Thus, the terminal payoffs from the long position in a European call and put options are defined as:

\[
V(S,T) = \max\{K-S,0\}
\]

In put options, the terminal payoffs are non-negative, which reflect the vary nature of the options. This condition is defined at a future point in time and we wish to determine values backwards to an earlier point in time.

The option pricing problem (1) is posed on the domain \([0,\infty] \times [0,T]\) with final condition (2). To complete the option pricing model, we prescribe two spatial boundary conditions at \( S = 0 \) and \( S = \infty \). But, in case of numerical solution, infinite grids cannot be represented in the computer so we truncate the solution domain artificially at point \( S = S_{\max} \) and replace the deleted portions with boundary conditions that minimize the deleterious effects of the truncation. The truncation point \( S_{\max} \) has to be sufficiently far from the region of interest in order to avoid the excessive error due to truncation and even if the imposed boundary conditions are imperfect, it does not materially affect the solution. On the other hand, unnecessarily large value of \( S_{\max} \) increases the computational cost. The choice of \( S_{\max} \) is consider in [19]. Hence, the solution value for the option pricing model (1) is \([0,S_{\max}] \times [0,T]\).

Boundary conditions for put option are as:

- When \( S = 0 \) for some \((t < T)\), \( S \) will stay at zero at all subsequent times so that the option is sure to expire in-the-money. Hence

\[
V(0,t) = Ke^{-(t-T)} - S_{\max}
\]

- When \( S = S_{\max} \), it becomes almost certain that the put value will be in out-of-the-money. Hence

\[
V(S_{\max},t) = 0
\]

More and more researchers have found that the option pricing with stochastic volatility is more
realistic than Black-Scholes model with constant volatility. Thus, volatility is an important issue to be addressed properly. From computational point of view, it is convenient to handle the pricing problem by forecasting the valuation by well known GARCH method than regarding volatility as function of strike price and time. Now we discuss the GARCH model.

2.1. GARCH (1, 1) Model In 1986, Bollerslev [4] proposed GARCH (1, 1) model as

$$y_t = \varepsilon_t \sigma_t$$

with conditional variance

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

with

$$\alpha_0 > 0, \ and \ \alpha_1, \ \beta_1 > 0,$$

(6)

Where $\sigma_t^2, \ \sigma_{t-1}^2$ are volatilities on the day $t$ and previous day, $y_{t-1}$ is return on the previous day and $\varepsilon_t$ denotes a real-valued discrete-time stochastic process as $\varepsilon_t = N(0, \sigma_t^2)$, $y_t$ is the dependent variable of return $x_t$ at a time $t$ and $\alpha_0, \ \alpha_1$ and $\beta_1$ are weighted assigned to conditional and unconditional variances. The GARCH (1,1) regression model is obtained by assuming the $\varepsilon, s$ be innovation in a linear regression

$$y_t = bx_t + \varepsilon_t$$

(7)

The GARCH (1,1) process as defined in Equations 5 and 6 is stationary with $E(y_t) = 0$, $var(y_t) = \frac{\alpha_0}{1-\alpha_1-\beta_1}$ and $cov(y_t, y_s) = 0$ for $t \neq s$ if and only if $\alpha_0 + \beta_1 < 1$ or characteristic roots of GARCH (1,1) process are outside the unit circle. The likelihood function for estimation of parameters is defined as:

$$l_0 = -\frac{1}{2} \left[ T \ln(2\pi) + \sum_{t=1}^{T} \left( \ln(\sigma^2_t) + \frac{y_t^2}{\sigma^2_t} \right) \right]$$

(8)

2.2. Discretization of Equation For numerical solution of partial differential Equation 1 the rectangular domain $[0, S_{\text{max}}] \times [0, T]$ is divided into $(N+1) \times (M+1)$ uniform grid points. The step width $\Delta S$ and $\Delta t$ are in general independent, where

$$S_{i+1} - S_i = \Delta S = \frac{S_{\text{max}} - 0}{M}; \ i = 0, 1, 2, \ldots, M$$

$$t_{i+1} - t_i = \Delta t = \frac{T}{N}; \ j = 0, 1, 2, \ldots, N$$

and $V_{i,j}$ denotes the numerical approximation of $V(i\Delta S, j\Delta t)$ i.e. at value of $V$ grid point $(i, j)$.

Keeping view for stability of finite difference scheme in mind, we used Crank and Nicolson (1947) method, which incorporates both explicit and implicit features [20]. The Crank-Nicolson scheme for Equation 1 is

$$aV_{i+1,j} + bV_{i,j} + cV_{i-1,j} = xV_{j,i+1} + yV_{j,i} + zV_{j,i-1}$$

(9)

where

$$a = \frac{1}{4}[r_i - \sigma^2_i] \Delta t, \ b = \frac{1}{2} \frac{\sigma^2_i \Delta t}{2} \Delta t, \ c = \frac{1}{4}[r_i + \sigma^2_i] \Delta t$$

$$x = \frac{1}{4}[\sigma^2_i - r_i] \Delta t, \ y = \frac{1}{2} \frac{\sigma^2_i \Delta t}{2} \Delta t, \ z = \frac{1}{4}[r_i + \sigma^2_i] \Delta t$$

2.3. Discretization of Boundary and Initial Conditions We need the discretization of boundary and initial conditions for the European option. For a put option, boundary conditions are

$$V^0_j = K \ and \ \ V^N_j = 0 \ for \ j = 0, 1, 2, \ldots, N$$

and initial condition is:

$$V^0_i = \max(K - i\Delta S, 0) \ for \ i = 0, 1, 2, \ldots, M$$

2.4. Stability of Scheme It is unconditionally stable and the amplification factor of Crank-Nicolson scheme for Equation 1 is:

$$G(\beta) = \frac{1 - \frac{1}{2} \sigma^2 S^2 \Delta(1-\cos \beta) - \frac{r}{2} \Delta t}{1 + \frac{1}{2} \sigma^2 S^2 \Delta(1-\cos \beta) + \frac{r}{2} \Delta t}$$
TABLE 1. 60 Days Forecasted Volatility of Reliance Ltd. using Parameters of GARCH (1,1) Model.

<table>
<thead>
<tr>
<th>No. Days</th>
<th>Volatility $\sigma_i$</th>
<th>No. Days</th>
<th>Volatility $\sigma_j$</th>
<th>No. Days</th>
<th>Volatility $\sigma_i$</th>
<th>No. Days</th>
<th>Volatility $\sigma_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4247</td>
<td>16</td>
<td>0.4332</td>
<td>31</td>
<td>0.4339</td>
<td>46</td>
<td>0.4339</td>
</tr>
<tr>
<td>2</td>
<td>0.4262</td>
<td>17</td>
<td>0.4333</td>
<td>32</td>
<td>0.4339</td>
<td>47</td>
<td>0.4339</td>
</tr>
<tr>
<td>3</td>
<td>0.4277</td>
<td>18</td>
<td>0.4334</td>
<td>33</td>
<td>0.4339</td>
<td>48</td>
<td>0.4339</td>
</tr>
<tr>
<td>4</td>
<td>0.4284</td>
<td>19</td>
<td>0.4335</td>
<td>34</td>
<td>0.4339</td>
<td>49</td>
<td>0.4339</td>
</tr>
<tr>
<td>5</td>
<td>0.4293</td>
<td>20</td>
<td>0.4336</td>
<td>35</td>
<td>0.4339</td>
<td>50</td>
<td>0.4339</td>
</tr>
<tr>
<td>6</td>
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<td>21</td>
<td>0.4336</td>
<td>36</td>
<td>0.4339</td>
<td>51</td>
<td>0.4339</td>
</tr>
<tr>
<td>7</td>
<td>0.4306</td>
<td>22</td>
<td>0.4337</td>
<td>37</td>
<td>0.4339</td>
<td>52</td>
<td>0.4339</td>
</tr>
<tr>
<td>8</td>
<td>0.4311</td>
<td>23</td>
<td>0.4337</td>
<td>38</td>
<td>0.4339</td>
<td>53</td>
<td>0.4339</td>
</tr>
<tr>
<td>9</td>
<td>0.4315</td>
<td>24</td>
<td>0.4338</td>
<td>39</td>
<td>0.4339</td>
<td>54</td>
<td>0.4339</td>
</tr>
<tr>
<td>10</td>
<td>0.4319</td>
<td>25</td>
<td>0.4338</td>
<td>40</td>
<td>0.4339</td>
<td>55</td>
<td>0.4339</td>
</tr>
<tr>
<td>11</td>
<td>0.4322</td>
<td>26</td>
<td>0.4338</td>
<td>41</td>
<td>0.4339</td>
<td>56</td>
<td>0.4339</td>
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<tr>
<td>12</td>
<td>0.4325</td>
<td>27</td>
<td>0.4338</td>
<td>42</td>
<td>0.4339</td>
<td>57</td>
<td>0.4339</td>
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<tr>
<td>13</td>
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<td>28</td>
<td>0.4338</td>
<td>43</td>
<td>0.4339</td>
<td>58</td>
<td>0.4339</td>
</tr>
<tr>
<td>14</td>
<td>0.4319</td>
<td>29</td>
<td>0.4338</td>
<td>44</td>
<td>0.4339</td>
<td>59</td>
<td>0.4339</td>
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<tr>
<td>15</td>
<td>0.4330</td>
<td>30</td>
<td>0.4338</td>
<td>45</td>
<td>0.4339</td>
<td>60</td>
<td>0.4339</td>
</tr>
</tbody>
</table>

Figure 1. Stock price and return of Reliance Ltd. from 30/03/2009 to 01/01/2000.

2.5. Sensitivities by Finite Difference Scheme

Finite difference method provides the price of an
<table>
<thead>
<tr>
<th>(r In %)</th>
<th>6.00</th>
<th>12.00</th>
<th>18.00</th>
<th>24.00</th>
<th>30.00</th>
<th>36.00</th>
<th>42.00</th>
<th>48.00</th>
<th>54.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.00</td>
<td>23.7927</td>
<td>17.7927</td>
<td>11.7928</td>
<td>5.8632</td>
<td>1.4259</td>
<td>0.1322</td>
<td>0.0060</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>35.00</td>
<td>28.7595</td>
<td>22.7595</td>
<td>16.7595</td>
<td>10.7626</td>
<td>5.0242</td>
<td>1.2587</td>
<td>0.1551</td>
<td>0.0111</td>
<td>0.0006</td>
</tr>
<tr>
<td>40.00</td>
<td>33.7263</td>
<td>27.7262</td>
<td>21.7262</td>
<td>15.7263</td>
<td>9.7515</td>
<td>4.3471</td>
<td>1.1472</td>
<td>0.1757</td>
<td>0.0173</td>
</tr>
<tr>
<td>45.00</td>
<td>38.6930</td>
<td>32.6930</td>
<td>26.6930</td>
<td>20.6930</td>
<td>14.6947</td>
<td>8.7936</td>
<td>3.8167</td>
<td>1.0696</td>
<td>0.1909</td>
</tr>
</tbody>
</table>

**Maturity Time T = 30 Days**

<table>
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<tr>
<th>(r In %)</th>
<th>6.00</th>
<th>12.00</th>
<th>18.00</th>
<th>24.00</th>
<th>30.00</th>
<th>36.00</th>
<th>42.00</th>
<th>48.00</th>
<th>54.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.00</td>
<td>23.7669</td>
<td>17.7669</td>
<td>11.7670</td>
<td>5.8385</td>
<td>1.4129</td>
<td>0.1302</td>
<td>0.0058</td>
<td>0.0000</td>
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</tr>
<tr>
<td>35.00</td>
<td>28.7295</td>
<td>22.7295</td>
<td>16.7295</td>
<td>10.7327</td>
<td>4.9977</td>
<td>1.2463</td>
<td>0.1527</td>
<td>0.0108</td>
<td>0.0005</td>
</tr>
<tr>
<td>40.00</td>
<td>33.6922</td>
<td>27.6922</td>
<td>21.6922</td>
<td>15.6923</td>
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<td>4.3471</td>
<td>1.1472</td>
<td>0.1757</td>
<td>0.0173</td>
</tr>
<tr>
<td>45.00</td>
<td>38.6548</td>
<td>32.6548</td>
<td>26.6548</td>
<td>20.6548</td>
<td>14.6548</td>
<td>8.7936</td>
<td>3.8167</td>
<td>1.0696</td>
<td>0.1909</td>
</tr>
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</table>

**Maturity Time T = 60 Days**

<table>
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<th>(r In %)</th>
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<th>12.00</th>
<th>18.00</th>
<th>24.00</th>
<th>30.00</th>
<th>36.00</th>
<th>42.00</th>
<th>48.00</th>
<th>54.00</th>
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<tbody>
<tr>
<td>30.00</td>
<td>23.5868</td>
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<td>11.5924</td>
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<td>0.4259</td>
<td>0.0671</td>
<td>0.0086</td>
<td>0.0010</td>
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<tr>
<td>35.00</td>
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<td>22.8505</td>
<td>16.5209</td>
<td>10.5695</td>
<td>5.2658</td>
<td>1.8848</td>
<td>0.4948</td>
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<tr>
<td>40.00</td>
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<td>4.8000</td>
<td>1.8450</td>
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<tr>
<td>45.00</td>
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<td>32.3881</td>
<td>26.3881</td>
<td>20.3889</td>
<td>14.2658</td>
<td>8.8368</td>
<td>4.4562</td>
<td>1.8209</td>
<td>0.5631</td>
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</table>

**TABLE 2. Option Price for Different Stock Prices S, Strike Prices K = $30, $35, $40 and $45, Interest Rate r = 8 %, 9 %, 10 %, 11 % with One and Two Months Maturity Period.**
stock price (S) and present a concavity upward. The point S = K (i.e. at-the-money) the curve are more concave which refered that option lead to either in-the-money or out-of-the-money immediately. In case of in-the-money-(S<K) the value of option are changing in constant way with respect to the stock price (S) decrease, while the exponential change occurs when option become at-the-money as well as out-of-the money (Table 2, Figures 2-5).

The downward concavity conformed that a decrease in the asset price (S) will increase probability of a positive terminal payoff, resulting in a higher value of option. By keeping r small, t constant and increase of strike price from K = $30 to K = $45, the increment in option price has same trends with respect to stock price (S) and option is become in-the-money. But, as interest rate increases from = 8 % to r = 9 %, 10 % and 11 % the interval of positive terminal payoff reduces and
the option prices also reduce that means the possibility of positive terminal payoff decreases and portfolio become more sensitive with respect to interest rate increment, resulting a less option price. Hence, as the interest rate (r) increases, the option price decreases and the positive terminal payoff increase at only deep in-the-money. But, comparison of maturity life i.e. a short lived option with a long lived option depicts very strange results (Table 2). If option become deep in-the-money, the option price decreases when maturity time increases, while in case of at-the-money and out-of the money option price increases with maturity time increase. Intuitively, in case of put option, probability of positive payoff increases as maturity time and interest rate increases.

The delta ($\Delta_p$) of put option shows the negative values and lies (-1, 0). The negativity of delta ($\Delta_p$) for put option function confirmed the decreasing nature of the function with stock price ($S$) i.e. the
increment in the asset price causes a decrease in the value of put option prices and probability of a positive terminal payoff and then a long position for put option will be hedged by a continuously varying long position in the underlying asset.

The effect of strike price is also significant to hedge to the portfolio. Figures 6-9 show the curves of delta ($\Delta_p$) against stock price ($S$), change in concavity appears where option is at-the-money ($S = K$) so that the curve concave upward for in-the-money ($0 < S < K$) and downward for out-of-the-money ($K < S < \infty$). When option values become out-of-the-money, the concavity increases as strike price ($K$) increases, which means the option price for a higher strike price ($K$) has a small changes in corresponding delta ($\Delta_p$) i.e. the delta ($\Delta_p$) hedge dynamic for a higher strike price ($K$) is less than as compared with low strike price.
The effect of interest rate on delta ($\Delta_p$) also represents an interesting result. It is observed that when strike price is sufficiently small ($K=$30) the interest rate ($r$) has very small changes as $r = 8\%$ to $9\%, 10\%$ and $11\%$ then the delta ($\Delta_p$) increases only when option become at-the-money, but it remains same for option being deep in-the-money and out-of-the-money. If we peer in the data of delta for $K=$35, $40$ and $45$ for different interest rate, it has enough change with respect to interest rate.

The impact of maturity life of option is countable in this problem as maturity life increases then the absolute values of delta decreases for each case of stock price, strike price and interest rate as result option is less sensitive i.e. increments in strike price, interest rate and maturity time increase the probability of positive terminal payoff increase. Further, the delta can be shown easily that
\[
\lim_{t \to 0} \frac{\partial P}{\partial S} = 0
\]
for all values of stock price (S).

While
\[
\lim_{t \to 0} \frac{\partial P}{\partial S} = \begin{cases} 
0 & \text{if } S > K \\
-\frac{1}{2} & \text{if } S = K \\
-1 & \text{if } S < K 
\end{cases}
\]

Figures 10-13 represent the values of theta (\(\Delta_p\)) for a put option against stock prices (S) are negative and positive. The curve of theta (\(\Delta_p\)), tends asymptotically as option become deep in-the-money. This means the results discussed above through Table 2 (i.e. the value of option decreases when option becomes out-of-the-money (S>\(K\)) and increases if option is in-the-money (S<\(K\)). While option becomes deep in-the-money then the change in option price appears in a constant way) are validated from these figures. The negative sign of

**Figure 10.** Theta (\(\Delta_p\)) of the put option for different stock prices S, strike prices \(K = \$30, \$35, \$40, \$45,\) interest rates \(r = 8\%\), 9\% with one month maturity.

**Figure 11.** Theta (\(\Delta_p\)) of the put option for different stock prices S, strike prices \(K = \$30, \$35, \$40, \$45,\) interest rates \(r = 10\%\), 11\% with one month maturity.
$\frac{\partial P}{\partial t}$ confirms the long lived counterparts (Table 2).

The positive sign of $\frac{\partial P}{\partial t}$ depicts the put values should be below their intrinsic values (K-S) and option should be grown to (K-S) at expiry. The tendency of asymptotic of curve also validates the change in option price, when option become deep in-the-money, constant way. It is also clear that the theta ($\Theta_p$) has its greatest absolute value when the option is at-the-money ($S<K$) since the option may become in-the-money ($S<K$) or out-of-the-money ($S>K$) at an instant later. Also, the theta ($\Theta_p$) has a minimum absolute value when the option is sufficiently out-of-the-money ($S>K$). Hence, the absolute value of theta ($\Theta_p$) increases as strike price increases but decreases as interest rate ($r$) and maturity time increase (Figures 10-13).

The gamma ($\gamma_p$) for put option are positive, this
explain why the curves of the option price function with respect to stock price (S) are concave upward. From the figures, it is clear that, generally the absolute values of gamma are maximum as option become at-the-money (S=K) for each strike price (K), interest rate (r) and maturity time (T). This implies that the concavity of the curve for the option prices should be maximum as option become at-the-money (S=K) (Figures 2-6). The absolute values of the gamma ($\gamma_p$) decreases as strike price increases i.e. the change of delta ($\Delta_p$) is small when strike price increases, this means a portfolio could be hedged by taking long position in put option. From Figures 14-17, it is clear that the gamma ($\gamma_p$) shows a bell shape curve which have a left long tail and right long tail when option becomes in-the-money (S<K) and out-of-the-money (S>K),

![Figure 14](image1.png)  
**Figure 14.** Gama ($\gamma_p$) of the put option for different stock prices S, strike prices K = $30, $35, $40, $45, interest rates r = 8 % , 9 % with one month maturity.

![Figure 15](image2.png)  
**Figure 15.** Gama ($\gamma_p$) of the put option for different stock prices S, strike prices K = $30, $35, $40, $45, interest rates r = 10 %, 11 % with one month maturity.
respectively. From the figures of gammas ($\gamma_p$) it is quite clear that the area of left long tail is less than the area of right long tail. This means the changes of deltas ($\Delta_p$) with respect to stock price ($S$) should be more dynamic when option becomes in-the-money whenever, option becomes out-of-the-money. This also shows that option has a positive terminal payoff in the case of in-the-money.

The results depict that the change of interest rate is insignificant on gamma values (Figures 14 and 15). But increase in the maturity time i.e. long lived option results in less values of gamma as compared to short lived option (Figures 14 and 16). So, the curvature of long lived option is less than short lived option, while the option prices for an option become at-the-money and out-of-the-money, long lived option are greater than short lived option which has a financial sense that hedging is less dynamic for long lived put option.
with short lived option.

4. CONCLUSION

Due to stochastic nature of financial market, the volatility is a crucial variable in option pricing and hedging strategies. The alternative approach for the stochastic volatility used in this paper form a one dimensional partial differential equation, where volatility is regarded as a function of stock price \(S\) and time \(t\), leads to a partial differential equation in two variables. From the computational point of view, the present model is more economic as compared to model with \(\sigma(S, t)\). The results may be useful for the financial engineers in order to understand the effect of stochastic volatility on the hedging movement.

5. REFERENCES

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