A PROPOSED METHOD TO EVALUATE ULTIMATE RESISTANCE OF PLATE GIRDERS SUBJECTED TO SHEAR AND PATCH LOADING

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Abstract Experimental investigation of the ultimate resistance of slender, steel-plate girder web panels to combined shear-and-patch loading indicates significant interaction between shear loading and patch loading. However, an existing interaction formula is based on experimental results. Herein, an improved design procedure for slender plate girders subjected to combined shear and patch loading is proposed. A modified formula to evaluate shear resistance of plate girders in presence of patch loading is proposed that shows satisfactory correlation with the available test data and is acceptable for practical purposes.

Key Words Plate Girders, Combined Shear and Patch Loading, Modified Shear Resistance

1. INTRODUCTION

Slender steel plates are used in a variety of structural engineering applications owing to their high strength-to-weight ratio and post-buckling reserve of stiffness and strength. Typical structural forms are slender-plate and hog girders, in which the flanges primarily resist bending and the web primarily, resist shear and localized, in-plane, compressive edge loading, often described as patch loading.

Shear loading and localized edge or patch loading of slender plate girders are frequently encountered in practice. Significant shear loading arises from nonuniform bending and support reactions, while examples of patch loading are wheel loads on gantry girders, loads from purlins onto the mainframe members of buildings, and roller loads during the launching of plate girder bridges. Loading conditions that result in pure patch loading, pure shear loading, and combined shear-and-patch loading are illustrated in Figure 1.

During the past three decades, numerous tests have been performed on slender plate girders to provide a better understanding of their modes of failure. In the majority of these tests, shear loading and patch loading have been considered separately [1,2,3]. There are only few experimental studies of the interaction between shear and patch loading [4,5]. While these tests indicated significant interaction between the two forms of loading. To date the only solution, which is available for predicting the resistance of plate girders to combined shear-and-patch loading is based on experimental results, in the form of an interaction formula and not as a rigorous solution [5].
Herein, details of a theoretical investigation of the ultimate shear resistance of slender web panels in the presence of patch loading is presented. Theoretical predictions are compared with available experimental results and a modified formula for combined shear-and-patch loading is proposed.

2. THEORETICAL PREDICTION OF ULTIMATE SHEAR RESISTANCE

Theoretical predictions of the ultimate shear resistance of slender web panels can be made in accordance with tension field theory, developed by Porter et al. [6] and Evans [1]. For a web panel having width $b_w$, depth $d_w$, thickness $t_w$, and similar top and bottom flanges (Figure 2), the ultimate shear resistance $V_u$ is given by

$$V_u = \tau_{cr} d_w t_w + \sigma_y t_w \sin^2 \theta (d_w \cot \theta - b_w) + 4d_w t_w \sin \theta \left( \sigma_{yw} M_p^* \right)$$

(1)

$\tau_{cr}$ is the critical shear stress of an assumed simply supported web plate, given by

$$\tau_{cr} = \frac{K_s \pi^2 E t_w^2}{12(1-v^2)d_w^2}$$

(2)

where $K_s$ is buckling coefficient. $\sigma_y$ is the web tension field membrane stress, defined by the equation

$$\sigma_y = \sqrt{\left[ 1 - \left( \frac{\tau_{cr}}{\tau_{yw}} \right)^2 \left( 1 - \frac{3}{4} \sin^2 2\theta \right) \right] \frac{\sqrt{3}}{2} \frac{\tau_{cr}}{\tau_{yw}} \sin 2\theta} \sigma_{yw}$$

(3)

where $\tau_{yw}$ is shear yield stress of the web. Which is related to the uniaxial yield stress $\sigma_{yw}$ by the equation

$$\tau_{yw} = \frac{\sigma_{yw}}{\sqrt{3}}$$

(4)

$\theta$ is the inclination of the web tension field, assumed to be approximately two-thirds of the inclination of the web panel diagonal, i.e.,

$$\theta = \frac{2}{3} \tan^{-1} \left( \frac{d_w}{b_w} \right)$$

(5)

$M_p^*$ is a nondimensional flange strength parameter defined as

$$M_p^* = \frac{M_{ref}}{d_w^2 t_w \sigma_{yw}}$$

(6)
where \( M_{pf} \) is fully plastic moment of the flange, which for a rectangular flange having width \( b_f \), thickness \( t_f \), and yield stress \( \sigma_y \) is given by

\[
M_{pf} = 0.25b_f t_f^2 \sigma_y \tag{7}
\]

\[ P = \left[ 1.1t_w^2 \left( E \sigma_{yw} \right)^{0.5} \left( \frac{t_f}{t_w} \right)^{0.25} \left( 1 + \frac{c_e t_w}{d_u t_f} \right) \right] \frac{l}{F} \tag{8}
\]

\[
P_{uy} = \left( 16M_{pf} \sigma_{yw} t_w \right)^{0.5} + \sigma_{yw} t_w c_e \tag{9}
\]

where \( F \) is a numerical factor to be taken as 1.45 for a lower bound 95% confidence limit and 1.0 for a mean prediction and \( c_e \) is the effective length of the patch load, which is related to the actual length.

3. THEORETICAL PREDICTION OF ULTIMATE PATCH RESISTANCE

Theoretical predictions of the ultimate patch resistance of slender web panels can be made in accordance with the theory developed by Roberts and Newark [3]. The ultimate patch resistance \( P_u \) should be taken as the lesser of the resistance to web-crippling \( P_{wb} \) and web-yielding \( P_{wy} \) which are given by

\[
P_{ub} = \left[ 1.1t_w^2 \left( E \sigma_{yw} \right)^{0.5} \left( \frac{t_f}{t_w} \right)^{0.25} \left( 1 + \frac{c_e t_w}{d_u t_f} \right) \right] \frac{l}{F} \tag{10}
\]

\[
P_{uy} = \left( 16M_{pf} \sigma_{yw} t_w \right)^{0.5} + \sigma_{yw} t_w c_e \tag{11}
\]

4. INTERACTION BETWEEN SHEAR AND PATCH RESISTANCE

When a girder web subjected to patch loading in addition to shear, the determination of the ultimate load capacity becomes more complex. Analysis of test results for combined shear and patch loading conducted by first author indicates significant interaction between the two forms of loading [7]. The following interaction formula for this combined loading is proposed that shows satisfactory correlation with the test data.

\[
\left( \frac{V_{de}}{V_u} \right)^2 + \left( \frac{P_{de}}{P_u} \right) \leq 1.0 \tag{12}
\]

where \( V_{de} \) and \( P_{de} \) are the design shear and patch loads for combined loading, and \( V_u \) and \( P_u \) are the ultimate shear resistance in the absence of patch loading, and \( P_u \) is the ultimate patch resistance in the absence of shear.

The presence of the patch loading requires two important additional factors to be considered:
(a) The reduction in the shear buckling resistance
(b) The influence of the patch stresses upon the magnitude of the membrane stresses required producing yield in the web.

Each of these factors will now be considered individually.

**Evaluation of the Shear Buckling Resistance** When the web panel is subjected to combined patch loading and shear, the buckling coefficients are determined by the author from the following interaction formula [8]

\[
\left( \frac{K_{sp}}{K_s} \right)^\alpha + \left( \frac{K_{ps}}{K_p} \right)^\alpha = 1.0
\]  

(12)

where \(K_{sp}\) and \(K_s\) are the buckling shear coefficients in the presence and absence of patch loading, respectively, and \(K_{ps}\) and \(K_p\) are the...
Note: For all patch tests, \( c = 50 \text{ mm} \)

buckling patch coefficients in the presence and absence of shear, respectively. \( \alpha \) is given by

\[
\alpha = 2.1 - 0.77\beta + 0.77\beta^2 - 0.05\beta^3 \quad (13)
\]

in which

\[
\beta = \frac{b_w}{d_w} \quad (14)
\]

In each case, the buckling load \( P_{cr} \) can be expressed as

\[
P_{cr} = K_p \frac{\pi^2 E t_w^3}{12(1 - \nu^2)d_w} \quad (15)
\]

Thus, for any patch load \( P \), by substituting \( \frac{K_p}{K_{ps}} = \frac{P}{P_{cr}} \) in Equation 12, the value of the modified shear buckling stress \( \tau_{erm} \) is determined.

\[
\tau_{erm} = \tau_{cr} \frac{K_{ps}}{K_s} \quad (16)
\]

(b) Evaluation of the Membrane Stresses
The state of stress in the web plate for combined shear and patch loading is shown in Figure 3. The total state of stress in the web plate may be obtained by superimposing all the stresses as shown in Figure 4 and may be defined as follows

\[
\begin{align*}
\sigma_0 &= \tau_{erm} \sin 2\theta - \sigma_{ym} - \sigma_p \sin^2 \theta \\
\sigma_{\theta-90} &= -\tau_{erm} \sin 2\theta - \sigma_p \cos^2 \theta \\
\tau_\theta &= \tau_{erm} \cos 2\theta - \frac{\sigma_p}{2} \sin 2\theta
\end{align*}
\quad (17)
\]

Upon further increase of the applied loading, the tensile membrane stress (\( \sigma_{erm} \)) developed in the web increases. Eventually, the membrane stress reaches such a value that, when combined with the buckling stress as in Equations 17, the resulting
stress \((\sigma_0)\) reaches the yield value \((\sigma_{yw})\) for the web material. This value of the membrane stress will be denoted as \(\sigma_{mn}^2\) and it may be determined by applying the Von Mises-Hencky yield criterion:

\[
\sigma_0^2 + \sigma_{0+90}^2 - \sigma_{0+90} \sigma_0 + 3 \tau_0^2 = \sigma_{yw}^2
\]  

By substituting the stresses from Equations 17, the following modified expression for the membrane stress is obtained.

\[
\sigma_{mn}^2 = \\
\frac{3}{2} \tau_{em} \sin 2\theta + \sigma_p \sin \theta \left( \cos \theta - \frac{1}{2} \sigma_p \cos^2 \theta \right) \\
+ \frac{1}{2} \left( -3 \tau_{em} \sin 2\theta - 6 \tau_{em} \sin 2\theta \sigma_p \cos^2 \theta \right) \\
- 3 \sigma_p^2 \cos^2 \theta - 12 \tau_{em} \cos^2 2\theta \\
+ 12 \tau_{em} \cos 2\theta \sigma_p \sin 2\theta - 3 \sigma_p \sin 2\theta + 4 \sigma_{yw}^2 \right)^{0.5}
\]  

where \(\sigma_p\) is the value of the patch stress is given by

\[
\sigma_p = \gamma \left( P - P_{at} \right) \sigma_{yw}
\]

\[
\gamma = 0 \quad P \leq P_{at}
\]

\[
\gamma = 1 \quad P_{at} \leq P \leq 0.9 P_u
\]

\[
\gamma = \frac{P}{P - P_{at}} \quad 0.9 P_u \leq P \leq P_u
\]  

6. COMPARISON OF THE PROPOSED METHOD WITH EXPERIMENTAL AND THEORETICAL RESULTS

A total of 32 specimens were constructed and tests (two tests per girder) have been conducted on slender-plate girders, details of which are shown in Figure 2. The dimensions of the test girders denoted PG2-1 to PG4-3, are given in Table 1.

The modified shear resistance \((V_{um})\) of the plate girders determined in accordance with Equation 21 is presented in Table 2. To evaluate \(P_u\) used in Table 2, the numerical factor \(F\) in Equation 8 was assumed equal to 1.0 to provide a mean prediction. The theoretical ultimate shear resistances \((V_u)\) in the absence of patch loading, determined in accordance with Equation 1, are compared with experimental results \((V_{exe})\) and the proposed formula \((V_{um})\) in Table 3 and in Figure 5.

Also, as presented in Table 3 and Figure 6, values of \(\lambda\) are determined as follows

\[
\lambda = \frac{V_{exe}}{V_{um}} \geq 1
\]  

All the values of \(\lambda\) are greater than unity, indicating a safe and conservative design procedure.

7. DISCUSSION AND CONCLUSIONS

Experimental investigation of the ultimate resistance of slender, steel-plate girder web panels to combined shear-and-patch loading indicate significant interaction between shear loading and patch loading. To date the only solution that is available for predicting the resistance of plate girders to combined shear-and-patch loading is in the form of an interaction formula and not as a modified shear resistance in accordance with Equations 1 and 21, shows the values of \(\tau_{cr}\) and \(\sigma_{cr}^2\) have been replaced with the \(\tau_{em}\) and \(\sigma_{mn}^2\), respectively.
TABLE 2. Modified Shear Resistance in Accordance with the Proposed Formula.

<table>
<thead>
<tr>
<th>Girder/Test reference</th>
<th>$P_{cr}$ [Eq. (15)] (kN)</th>
<th>$P_u$ [Eq. (8)] (kN)</th>
<th>$P_{exc}$ [reference 5] (kN)</th>
<th>$\tau_{em}$ [Eq. (16)] (N/mm$^2$)</th>
<th>$\sigma_p$ [Eq. (20)] (N/mm$^2$)</th>
<th>$\sigma_{em}$ [Eq. (19)] (N/mm$^2$)</th>
<th>$V_{um}$ [Eq. (21)] (N/mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PG1-2SP1</td>
<td>74</td>
<td>213</td>
<td>205</td>
<td>0</td>
<td>326</td>
<td>61</td>
<td>71</td>
</tr>
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<td>74</td>
<td>213</td>
<td>208</td>
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<td>215</td>
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<td>0</td>
<td>186</td>
<td>260</td>
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<td>PG2-2SP1</td>
<td>21</td>
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<td>0</td>
<td>76</td>
<td>263</td>
<td>215</td>
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<tr>
<td>PG2-2SP2</td>
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<td>16</td>
<td>11</td>
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<td>249</td>
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<td>119</td>
<td>106</td>
<td>0</td>
<td>180</td>
<td>163</td>
<td>83</td>
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<tr>
<td>PG4-2SP1</td>
<td>6</td>
<td>44</td>
<td>38</td>
<td>0</td>
<td>152</td>
<td>149</td>
<td>40</td>
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<tr>
<td>PG4-2SP2</td>
<td>6</td>
<td>44</td>
<td>20</td>
<td>0</td>
<td>67</td>
<td>215</td>
<td>52</td>
</tr>
<tr>
<td>PG4-3SP1</td>
<td>6</td>
<td>44</td>
<td>32</td>
<td>0</td>
<td>118</td>
<td>166</td>
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<td>44</td>
<td>44</td>
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<td>172</td>
<td>112</td>
<td>32</td>
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</table>

Herein, the shear resistance of plate girders in the presence of patch loading has been investigated, theoretically. A modified formula for this combined loading is proposed that shows satisfactory correlation with the available test results. The solution presented is acceptable for practical purposes.

8. NOMENCLATURE

- $b_f$: Width of flange
- $b_w$: Clear width of web plate between stiffeners
- $c$: Length of Patch load
- $c_e$: Effective length of patch load
- $d_w$: depth of web panel
- $E$: Young's modulus
TABLE 3. Comparison of Proposed Method with Test Results and Theoretical Predictions.

<table>
<thead>
<tr>
<th>Girder/Test reference</th>
<th>$V_u$ [Eq. (1)] (kN)</th>
<th>$V_{exc}$ [Reference 5] (kN)</th>
<th>$V_{um}$ [Eq. (21)] (kN)</th>
<th>$\lambda = \frac{V_{exc}}{V_{um}}$ [Eq. (22)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PG1-2SP1</td>
<td>386</td>
<td>158</td>
<td>71</td>
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<td>56</td>
<td>1.73</td>
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<td>1.04</td>
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<td>70</td>
<td>43</td>
<td>32</td>
<td>1.34</td>
</tr>
</tbody>
</table>

$F$ Numerical factor  
$K_p$ Buckling coefficient for patch loading  
$K_{ps}$ Buckling coefficient for combined patch loading and shear  
$K_s$ Buckling coefficient for shear  
$K_{sp}$ Buckling coefficient for combined shear and patch loading  
$M_p^*$ Flange strength parameter  
$M_{pf}$ Plastic moment of flange  
P Patch load  
P_{er} Buckling patch load  
P_{dc} Design patch load for combined loading  
P_{exc} Experimental ultimate patch resistance to combined shear-and-patch loading  
P_a Theoretical ultimate patch resistance  
P_{ab} Theoretical ultimate patch resistance to compounded shear and patch loading.
Figure 5. Comparison of proposed method with test results.

Figure 6. Comparison of modified shear resistance with test data.

web crippling

\[ P_{wy} \]

Theoretical ultimate patch resistance to web yielding

\[ R \]

Reaction at support

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\( t_f \)  \( t_w \)  Thickness of flange Thickness of web
\( V \)  \( V_{dc} \) \( V_{exc} \) \( V_u \) \( V_{um} \)  Shear load Design shear load for combined loading Experimental ultimate shear resistance to combined shear-and-patch loading Theoretical ultimate shear resistance Modified ultimate shear resistance to combined shear-and-patch loading

Greek Symbols

\( \alpha \)  Function of \( \beta \)  
\( \beta \)  Aspect ratio (\( b_w/d_w \))  
\( \gamma \)  Numerical factor for patch stress  
\( \theta \)  Inclination of web tension field  
\( \lambda \)  Design factor (defined by equation 21)  
\( \nu \)  Poisson’s ratio  
\( \sigma^y_t \)  Tension field membrane stress at yield  
\( \sigma^y_{tm} \)  Modified tension field membrane stress at yield  
\( \sigma_yf \)  Yield stress of flange  
\( \sigma_{yw} \)  Yield stress of web  
\( \sigma_0 \)  Normal stress  
\( \tau_{cr} \)  Buckling shear stress  
\( \tau_{crm} \)  Modified buckling shear stress  
\( \tau_{yw} \)  Yield shear stress of web  
\( \tau_0 \)  Shear stress

9. REFERENCES