INSTANTANEOUS WAVELET TRANSFORM DECOMPOSITION FILTER FOR ON-LINE APPLICATIONS*

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Abstract— In this paper, a new digital filter in the category of parallel implemented Wavelet Transform Decomposition (WTD) is proposed for on-line applications. The presented method allows a faster filtering process, even if one considers calculation with every newly drawn sample, and the successive procedure of signal processing in basic Discrete Wavelet Transform (DWT) is omitted. The most important feature of this filter is its reduced order form. Although it gives an approximated filter, its output has a good agreement with full order form and deliberately offers less delay and calculation burden to give the outputs.

Keywords— Digital wavelet transform, reduced order filter, on-line application

1. INTRODUCTION

The Wavelet Transform (WT) represents a powerful signal-processing tool with a wide variety of engineering applications. The main reason for this growing attraction is the ability of WT, not only to decompose a signal into its frequency components, but also to provide a non-uniform division of the frequency domain, whereby it focuses on short time intervals for high frequency components and long intervals for low frequencies [1]. This main characteristic provides the extraction of signal features such as trends, breakdown points and discontinuities, which can be very useful in classifying the source of the transient signals. WT is also often used to de-noise a signal without any appreciable degradation [2].

Based on this transformation, the signal is decomposed into wavelet levels, each representing a part of the original signal occurring at a particular time and in a particular frequency band.

Inspecting some applications of WT (31 papers, published between 1997 and 2008, which have applied WT to different cases, are listed in Appendix B) has shown that the feature extraction task is performed by employing decomposed components from different wavelet levels. Most of them have used only one or two components, but there are some others that have employed more than two. Fig. 1 shows the statistical distribution of using various wavelet components in those published papers.

It is well known that generating any of the upper level WT components is traditionally performed through a successive and/or step-by-step calculation from the first level component up to the desired level. Therefore, in order to find a new element of the desired level component, there is a time-consuming calculation process (depending on the number of the desired level) for each new sample of the original signal when it is drawn. This burden of calculation is not desirable, especially when the components of high levels are needed, and moreover, this burden is not acceptable in some on-line applications.

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Fig. 1. Statistical distribution of the number of WT components used in the published papers

Moreover, there are various papers about different applications which have used Wavelet Transform (WT) in general form and/or with an on-line approach [3-18]. The main feature of these methods is in considering a moving data window containing the samples of the original signal in order to apply the WT. However, the authors believe that in on-line applications of WT, it is not enough to consider only a moving data window. In addition, there must be a procedure in which different elements of each component (detail or approximate) at any decomposition level can be derived simultaneously and as soon as a new sample of Partial Discharge (PD) signals is drawn. This feature, in conjunction with a moving data window ending at the latest drawn sample, completes an on-line approach in applying WT.

It is worth mentioning that in the literature, mainly in image processing, there are papers which have proposed WT based methods with the characteristics of fast in calculation and/or parallel in implementation [19-30]. These are generally for multi dimensional applications and parallel computation of WT.

In this paper, a novel implementation of DWT is proposed which instantly delivers the new elements of any components at any decomposition level. This new filter is well suited for on-line 1D applications of WT, as the number of mathematical operations through its application is reduced.

The explanation of the proposed filter is given in Section III where it is preceded by a general description of DWT in Section II. Some discussions about the proposed Instantaneous Wavelet Transform Decomposition (IWTD) are given through some examples in Section IV.

2. WAVELET DECOMPOSITION METHOD IN GENERAL MODE

It is well known that DWT is used for digitized (or sampled) signals to show their time-scale representation. In order to perform this transformation the original signal is passed through a band-pass filter (called by $G$ and is named mother wavelet) to give a detail component for the first level. At the same level, convolving the signal with a low-pass filter (called by $H$) brings another component named approximate due to its low resolution. $G$ and $H$ are orthogonal vectors with $N \times 1$ elements [31-32].

For the second level, the approximate component is down-sampled by two, i.e. its samples are halved, and it is then passed through $G$ and $H$ to give detail and approximate components respectively at this level. Continuing this procedure up to the $j^{th}$ level causes the original signal to be decomposed to $j$ detail components and an approximate one. This procedure is shown in Fig.2 up to three decomposition levels.

Fig. 2. Decomposition of original signal $X$ by DWT up to three levels
Similarly, the components of the first level detail component may be found as (if down sampling by two is used):

\[
D_1 = \begin{bmatrix}
d_{1}(1) \\
d_{1}(2) \\
d_{1}(k)
\end{bmatrix} = \begin{bmatrix}
x(1) & x(2) & \ldots & x(N) \\
x(3) & x(4) & \ldots & x(N+2) \\
\vdots & \vdots & \ddots & \vdots \\
x(2(k-1)+1) & x(2(k-1)+2) & \ldots & x(2(k-1)+N)
\end{bmatrix} \begin{bmatrix}
g(1) \\
g(2) \\
g(N)
\end{bmatrix}
\]

(1)

and the approximate component at the first level is determined by:

\[
A_1 = \begin{bmatrix}
a_{1}(1) \\
a_{1}(2) \\
a_{1}(k)
\end{bmatrix} = \begin{bmatrix}
x(1) & x(2) & \ldots & x(N) \\
x(3) & x(4) & \ldots & x(N+2) \\
\vdots & \vdots & \ddots & \vdots \\
x(2(k-1)+1) & x(2(k-1)+2) & \ldots & x(2(k-1)+N)
\end{bmatrix} \begin{bmatrix}
h(1) \\
h(2) \\
h(N)
\end{bmatrix}
\]

(2)

In sequel, the second level detail component is derived through substituting the samples of the original signal by the elements of \(A_1\) in Eq. (1), and therefore it may be found as:

\[
D_2 = \begin{bmatrix}
d_{2}(1) \\
d_{2}(2) \\
d_{2}(k)
\end{bmatrix} = \begin{bmatrix}
a_{1}(1) & a_{1}(2) & \ldots & a_{1}(N) \\
a_{1}(3) & a_{1}(4) & \ldots & a_{1}(N+2) \\
\vdots & \vdots & \ddots & \vdots \\
a_{1}(2(k-1)+1) & a_{1}(2(k-1)+2) & \ldots & a_{1}(2(k-1)+N)
\end{bmatrix} \begin{bmatrix}
g(1) \\
g(2) \\
g(N)
\end{bmatrix}
\]

(3)

and for the accompanying approximate component, it can be written:

\[
A_2 = \begin{bmatrix}
a_{2}(1) \\
a_{2}(2) \\
a_{2}(k)
\end{bmatrix} = \begin{bmatrix}
a_{1}(1) & a_{1}(2) & \ldots & a_{1}(N) \\
a_{1}(3) & a_{1}(4) & \ldots & a_{1}(N+2) \\
\vdots & \vdots & \ddots & \vdots \\
a_{1}(2(k-1)+1) & a_{1}(2(k-1)+2) & \ldots & a_{1}(2(k-1)+N)
\end{bmatrix} \begin{bmatrix}
h(1) \\
h(2) \\
h(N)
\end{bmatrix}
\]

(4)

Similarly, the components of the \(j^{th}\) level of decomposition are represented as:

\[
D_j = \begin{bmatrix}
d_{j}(1) \\
d_{j}(2) \\
d_{j}(k)
\end{bmatrix} = \begin{bmatrix}
a_{j-1}(1) & a_{j-1}(2) & \ldots & a_{j-1}(N) \\
a_{j-1}(3) & a_{j-1}(4) & \ldots & a_{j-1}(N+2) \\
\vdots & \vdots & \ddots & \vdots \\
a_{j-1}(2(k-1)+1) & a_{j-1}(2(k-1)+2) & \ldots & a_{j-1}(2(k-1)+N)
\end{bmatrix} \begin{bmatrix}
g(1) \\
g(2) \\
g(N)
\end{bmatrix}
\]
3. INSTANTANEOUS WAVELET TRANSFORM DECOMPOSITION

According to the above-mentioned descriptions, it is clear that in order to find the \( j \)-th level detail component, a serial process must be performed through successive convolutions of approximate components with band-pass and low-pass filters up to the desired level of decomposition.

In most on-line applications of DWT, this time consuming successive procedure is not acceptable due to numerous necessary mathematical operations. In these applications, mainly performed for feature extraction from a signal, one or more detail or approximate components at some predefined frequency bands (different levels of decomposition) must be inspected. But, according to Eq. (5) and (6), the elements of the components in the \( j \)-th level cannot be computed unless the approximate component of the \((j-1)\)-th level is fully determined, and the latter cannot be determined unless the computations of the previous level are finalized. Therefore, the monitoring or inspecting mission cannot be provided except through a successive procedure.

In this section, a new digital filter for on-line application of DWT is developed and its mathematical representation is proposed. The first feature of this filter to be mentioned, is that the \( k \)-th element of \( D_1 \) (the first level detail component) or \( A_1 \) (the first level approximate component) may be calculated along with the \( k \)-th element of any of the upper level detail or approximate components.

\[ A_j = \begin{bmatrix}
    a_j(1) \\
    a_j(2) \\
    \vdots \\
    a_j(k) \\
\end{bmatrix} = \begin{bmatrix}
    a_{j-1}(1) & a_{j-1}(2) & \ldots & a_{j-1}(N) \\
    a_{j-1}(3) & a_{j-1}(4) & \ldots & a_{j-1}(N+2) \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{j-1}(2(k-1)+1) & a_{j-1}(2(k-1)+2) & \ldots & a_{j-1}(2(k-1)+N) \\
\end{bmatrix} \begin{bmatrix}
    h(1) \\
    h(2) \\
    \vdots \\
    h(N) \\
\end{bmatrix} \]  

\[ D_2 = \begin{bmatrix}
    x(1) & x(2) & \ldots & x(3N-2) \\
    x(5) & x(6) & \ldots & x(3N+2) \\
    \vdots & \vdots & \ddots & \vdots \\
    x(2^j(k-1)+1) & x(2^j(k-1)+2) & \ldots & x(2^j(k-1)+3N-2) \\
\end{bmatrix} \begin{bmatrix}
    h(1) & 0 & \ldots & 0 \\
    h(2) & 0 & \ldots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    h(N) & 0 & \ldots & 0 \\
\end{bmatrix} \begin{bmatrix}
    g(1) \\
    g(2) \\
    \vdots \\
    g(N) \\
\end{bmatrix} \]  

or as the following form:
The right coefficient matrix in Eq. (8) can be replaced by:

\[
G_2 = H_{22} G_1
\]  

where \( G_1 = G \), and

\[
A_2 = \begin{bmatrix}
  x(1) & x(2) & \ldots & x(3N-2) \\
  x(5) & x(6) & \ldots & x(3N+2) \\
  \vdots & \vdots & \ddots & \vdots \\
  x(2^2(k-1)+1) & x(2^2(k-1)+2) & \ldots & x(2^2(k-1)+3N-2)
\end{bmatrix}
\]

and its right coefficient matrix is defined by:

\[
H_2 = H_{22} H_1
\]  

where \( H_1 = H \) and has \( N \times 1 \) elements.
This method of defining coefficient matrices for the second level of decomposition makes it possible to give a closed form definition for these coefficients in any upper level decomposition. In general, the right coefficient matrix for the $j^{th}$ level detail component can be defined as:

$$G_j = H_j G_{j-1}$$

(13)

and similarly, the coefficient matrix for the $j^{th}$ level approximate component is:

$$H_j = H_j H_{j-1}$$

(14)

where $H_j$ has $[\alpha_j(N-1)+1] \times [\beta_j(N-1)+1]$ elements, and:

$$\begin{cases}
\alpha_j = 2^j - 1 \\
\beta_j = \frac{\alpha_j - 1}{2}
\end{cases}$$

(15)

$H_j$ is constructed in a similar procedure as $H_{22}$ in Eq. (10), i.e. the first column is filled with $H$ in the first elements and zeros in the remaining ones; then any other column is built in the same way except that the position of vector $H$ is shifted down by two with respect to its position in the previous column.

According to Eq. (13), $D_j$ in Eq. (5) can be converted to:

$$D_j = \begin{bmatrix}
x(1) & x(2) & \cdots & x(\alpha_j(N-1)+1) \\
x(2^j+1) & x(2^j+2) & \cdots & x(2^j+\alpha_j(N-1)+1) \\
\vdots & \vdots & \ddots & \vdots \\
x(2^j(k-1)+1) & x(2^j(k-1)+2) & \cdots & x(2^j(k-1)+\alpha_j(N-1)+1)
\end{bmatrix} \left[ \prod_{i=2}^{k} H_{ji} \right] G_j$$

(16)

In a similar manner for $A_j$ in Eq. (6), the following relation can be derived:

$$A_j = \begin{bmatrix}
x(1) & x(2) & \cdots & x(\alpha_j(N-1)+1) \\
x(2^j+1) & x(2^j+2) & \cdots & x(2^j+\alpha_j(N-1)+1) \\
\vdots & \vdots & \ddots & \vdots \\
x(2^j(k-1)+1) & x(2^j(k-1)+2) & \cdots & x(2^j(k-1)+\alpha_j(N-1)+1)
\end{bmatrix} \left[ \prod_{i=2}^{k} H_{ji} \right] H_j$$

(17)

b) New formulation for instant calculation of elements at different levels

In the case of on-line applications of wavelet transformation there is a need to determine, simultaneously, the new element of any of the upper level detail or approximate components. The proposed filter and its formulation are described in this section.

Obviously, while the new sample of the considered signal (e.g. the $n^{th}$ sample) is obtained/captured, the previous samples have been saved as an array of digitized values. According to Eqs. (16) and (17), the new element (the $n^{th}$ one) of detail or approximate component in the $j^{th}$ decomposition level, $d_j(n)$ and $a_j(n)$ respectively, can be found from the following equation:
Instantaneous wavelet transform decomposition filter…

\[
\begin{bmatrix}
\vdots & \vdots \\
d_j(n-1) & a_j(n-1) \\
\vdots & \vdots \\
d_j(n) & a_j(n)
\end{bmatrix}
= 
\begin{bmatrix}
\vdots & \vdots & \vdots \\
x(n-\alpha_j(N-1)-2^j) & \cdots & x(n-2^j-1) & x(n-2^j) \\
x(n-\alpha_j(N-1)) & \cdots & x(n-1) & x(n)
\end{bmatrix}
\prod_{j=2}^{j=J} H_n
\begin{bmatrix}
G & H
\end{bmatrix}
\]

where all of the parameters in R.H.S. have been introduced before. Eq. (18) can be expanded to the following equation:

\[
\begin{bmatrix}
\vdots & \vdots \\
d_j(n-1) & a_j(n-1) \\
\vdots & \vdots \\
d_j(n) & a_j(n)
\end{bmatrix}
= 
\begin{bmatrix}
g_j(l) & h_j(l) \\
g_j(2) & h_j(2) \\
g_j(\alpha_j(N-1)+1) & h_j(\alpha_j(N-1)+1)
\end{bmatrix}
\begin{bmatrix}
x(n-\alpha_j(N-1)-2^j) & \cdots & x(n-2^j-1) & x(n-2^j) \\
x(n-\alpha_j(N-1)) & \cdots & x(n-1) & x(n)
\end{bmatrix}
\]

or,

\[
\begin{bmatrix}
\vdots & d_j(n-1) & d_j(n) \\
\vdots & a_j(n-1) & a_j(n)
\end{bmatrix}
= 
\begin{bmatrix}
g_j(l) & g_j(2) & \cdots & g_j(\alpha_j(N-1)+1) \\
h_j(l) & h_j(2) & \cdots & h_j(\alpha_j(N-1)+1)
\end{bmatrix}
\begin{bmatrix}
x(n-\alpha_j(N-1)-2^j) & \cdots & x(n-2^j-1) & x(n-2^j) \\
x(n-\alpha_j(N-1)) & \cdots & x(n-1) & x(n)
\end{bmatrix}
\]

where \( g_j(.) \) and \( h_j(.) \) are the elements of \( G_j \) and \( H_j \), respectively.

Each of the Eqs. (19) or (20) is the desired filter that gives the new elements of detail and approximate components at the \( j \)th level according to a mother wavelet with \( N \) elements, i.e. the new element of the desired component is derived as soon as the new sample of the signal is drawn. This filter is named “Instantaneous Wavelet Transform Decomposer” and is presented as \( IWT_{(N)} \).

c) Improvement in calculation burden

The calculation burden and/or the number of mathematical operations in applying a digital filter, which is an indication of the order of the filter, is an important feature for on-line applications. In this section, the order of the proposed filter and the procedure for further order reduction is explained and analyzed.

i. Initial comparison: In order to compare the calculation burden of \( IWT \) and \( DWT \), a schematic diagram from \( DWT \) for three levels of decomposition with \( db_2 \) (where \( N = 4 \)) as the mother wavelet is shown in Fig. 3.
As shown, for each new element of the third detail component, seven new convolution operations are needed, where any of them contains four multiplication and three summation operations, i.e. as a whole 28 multiplications and 21 summations. In general, using a mother wavelet with $N$ elements and for computing any new element of the $j^{th}$ decomposition level by $DWT$, the calculation burden contains $(2^j - 1)N$ new multiplications and $(2^j - 1)(N - 1)$ summations. Applying $IWTD$, the calculation burden while having the same number of summations, exhibits only $(2^j - 1)(N - 1) + 1$ multiplications, i.e. $(2^j - 2)$ fewer multiplying operations.

Figure 3. The DWT algorithm for three levels decomposition by using db2.

Figure 4 shows the relative reduction of calculation burden in computing a new element of any decomposition level by applying $IWTD$ with respect to $DWT$, with db mother wavelet of various lengths. This comparison is made whenever only one detail or approximate component at different levels is going to be calculated.

ii. Further reduction: In order to perform the reduced order filter (named $Gr$), the elements of $G_j$ with small magnitudes in the beginning and end parts of the filter are discarded. The discarded elements are chosen according to their magnitude being smaller than an acceptable threshold, e.g. a percentage of the largest magnitude of the filter elements.

Fig. 4. Percentage of relative reduction in calculation burden for computing components of each decomposition level, independently, using different db mother wavelets.
Figure 5 shows the relative reduction of calculation burden by applying the reduced $IWTD$ with a threshold of 1%, in comparison with $DWT$, whenever only one detail or approximate component at the mentioned level is desired.

Figures 6 and 7 show similar results where the above-mentioned threshold is chosen as 2% and 5%, respectively. It can be seen that the calculation burden (and the order of the filters) has a considerable reduction.
The numerical presentation of $G$ and $H$ and associated reduced order (5% threshold) are shown in Appendix-A.

iii. Characteristics of the reduced order IWTD: The impulse response of $G_8$ and $H_8$, full order filters according to the basic structure of IWTD, and also the impulse response of $G_r8$ and $H_r8$, the reduced order filters (according to 5% threshold), are shown and compared in Figure 8. It is clear that the reduced order filters provide the main features of the former with reduced number of elements, i.e. reduced delay time and calculation burden.

Furthermore, in order to investigate more precisely, the frequency characteristics of the full and reduced order filters are shown in Fig. 9, where the similarity of the characteristics has been proven, even for the case of 5% threshold.

Fig. 8. The full and reduced order of IWTD filter using db5 in eighth level with Threshold=5%

Fig. 9. Frequency characteristics of the full and reduced order filters in eighth level using db5 as the mother wavelet (Sampling rate: 40 MS/sec) for different threshold values
d) Further improvement in applying IWTD by increasing resolution

Equations (16) and (17), as the basic equations, are based on down sampling by two at each level, similar to DWT.

The main reason for employing down-sampling in DWT is to obtain a lower (halved) frequency spectrum output in each level using constant $G$ and $H$, applied to the approximate component of the previous level. But, in the IWTD, any output will be calculated using the original signal samples and by applying pre-determined filters, where there is no need for down sampling. In addition, the resolution of the filter output at any level will be increased if down sampling is not put in use.

Therefore, the new formulation to apply full order IWTD (without down sampling) for higher resolution in the output should be given as:

$$\begin{bmatrix} d_j(n) \\ d_j(n-1) \end{bmatrix} = \begin{bmatrix} x(n - \alpha_j(N - 1) - 1) & x(n - 2) & x(n - 1) \\ x(n - \alpha_j(N - 1)) & x(n - 1) & x(n) \end{bmatrix} \cdot \frac{1}{G} \prod_{j=2}^{l} H_{i_j} G_j$$  \hspace{1cm} (21)

and

$$\begin{bmatrix} a_j(n) \\ a_j(n-1) \end{bmatrix} = \begin{bmatrix} x(n - \alpha_j(N - 1) - 1) & x(n - 2) & x(n - 1) \\ x(n - \alpha_j(N - 1)) & x(n - 1) & x(n) \end{bmatrix} \cdot \frac{1}{H} \prod_{j=2}^{l} H_{i_j} H_r$$  \hspace{1cm} (22)

These equations can be substituted for Eqs. (18) or (19).

Also, the formulation to apply the reduced order IWTD (without down sampling) can be written as:

$$\begin{bmatrix} d_j(n) \\ d_j(n-1) \end{bmatrix} = \begin{bmatrix} x(n - \ell_{Gr_j}) & x(n - 2) & x(n - 1) \\ x(n - \ell_{Gr_j} + 1) & x(n - 1) & x(n) \end{bmatrix} \cdot G_{r_j}$$  \hspace{1cm} (23)

and

$$\begin{bmatrix} a_j(n) \\ a_j(n-1) \end{bmatrix} = \begin{bmatrix} x(n - \ell_{Hr_j}) & x(n - 2) & x(n - 1) \\ x(n - \ell_{Hr_j} + 1) & x(n - 1) & x(n) \end{bmatrix} \cdot H_{r_j}$$  \hspace{1cm} (24)

where $\ell_{Gr_j}$ and $\ell_{Hr_j}$ are the length of filters $G_{r_j}$ and $H_{r_j}$, respectively.

It should be noted that there is no change in full and/or reduced order IWTD, and it is only the involved samples of the original signal that are changed to provide higher resolution in the output.

Based on the above descriptions, on-line decomposition process for obtaining any arbitrary detail component with the best resolution is as shown schematically in Fig. 10. This figure depicts both features of IWTD, appropriate for on-line application, i.e. the new element of any desired decomposition level component is derived as soon as the new sample of the original signal is drawn, where the output is calculated according to the samples contained in a moving data window.

Moreover, deriving the new elements of, for example, 7th and 8th detail components, simultaneously, using full and reduced order IWTD is shown in Figs. 11a and 11b, respectively. These figures show the advantages of the proposed reduced order against full order new filters. Whenever the reduced order filters...
are used, the size of the data window decreases, resulting in the reduction of the calculation burden and the delay time (between the last sample of the original signal and the related filter output).

Fig. 10. Proposed full order IWTD in on-line condition and without down-sampling for obtaining jth detail

Fig. 11. Proposed IWTD in on-line condition and without down-sampling for obtaining 7th and 8th details, simultaneously, using db5 as mother wavelet and (a) full order G, (b) reduced order Gr with 5% threshold level

Similarly, using $H_j$ or $H_{rj}$, approximate component(s) can be obtained in on-line applications. It is worth mentioning that by applying the final proposed filter, i.e. the reduced order IWTD without down sampling, not only is a considerable saving in computation burden achieved, but also the complete set of the elements of any desired detail and/or approximate components will be derived. Moreover, the outputs of IWTD (without down sampling) in each detail component act as the output of a band-pass filter for the original signal being applied, and therefore, reveals the characteristics of the relevant frequency spectrum from the original signal.

e) Extracting successive components using reduced order IWTD

There are some applications where more than one component in the output of any WT-based filter is needed, e.g. $d_1$ to $d_7$. Even in these cases, the calculation burden of the proposed reduced order filter (with 5% threshold) is less than applying DWT and also full order IWTD for calculating only one component in the upper most level.

Table 1, in columns 2 to 5, depicts the number of multiplications and summations for DWT, full order IWTD and reduced order IWTD with 5% threshold filters in calculating detail or approximate components of the highest level (mentioned in the first column). In agreement with figures 4 to 7, it is seen that there is a comprehensive reduction in the calculation burden for the proposed filters to be applied. However, the
last two columns show the number of operations for calculating successive detail or approximate components up to the mentioned level, i.e. for the eight level it is the number of operations in calculating \( d_1 \) up to \( d_8 \) or \( a_1 \) up to \( a_8 \). It is clear that there is still a considerable reduction in the calculation burden, for example in extracting \( d_1 \) to \( d_8 \) in comparison with \( DWT \) and full order \( IWTD \) for the calculation of only \( d_8 \).

Table 1. Number of mathematical operations for \( DWT \), the full and reduced order (5% threshold) of \( IWTD \) filters in various levels based on \( db5 \) as the mother wavelet

<table>
<thead>
<tr>
<th>Level</th>
<th>( DWT ) (G or H)</th>
<th>( IWTD ) (full) (G or H)</th>
<th>( IWTD ) (reduced) (Gr)</th>
<th>( IWTD ) (reduced) (Hr)</th>
<th>Successive (Gr) (1-Level)</th>
<th>Successive (Hr) (1-Level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10*</td>
<td>10*</td>
<td>7*</td>
<td>7*</td>
<td>7*</td>
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<td>30*</td>
<td>27*</td>
<td>16*</td>
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<td>69*</td>
<td>66*</td>
<td>127*</td>
<td>120*</td>
</tr>
<tr>
<td>5</td>
<td>310*</td>
<td>280*</td>
<td>140*</td>
<td>132*</td>
<td>267*</td>
<td>252*</td>
</tr>
<tr>
<td>6</td>
<td>630*</td>
<td>568*</td>
<td>264*</td>
<td>263*</td>
<td>546*</td>
<td>516*</td>
</tr>
<tr>
<td>7</td>
<td>1270*</td>
<td>1144*</td>
<td>1103*</td>
<td>1044*</td>
<td>1103*</td>
<td>1044*</td>
</tr>
<tr>
<td>8</td>
<td>2550*</td>
<td>2295*</td>
<td>2219*</td>
<td>2100*</td>
<td>2219*</td>
<td>2100*</td>
</tr>
</tbody>
</table>

4. DISCUSSION

In order to explain more explicitly the features of this new digital filter and also to compare its results with the results obtained from \( DWT \), the following examples were studied.

Example 1- Comparison between the full order \( IWTD \) (without down sampling) and \( DWT \): The following input signal with a changing frequency characteristic is considered (its waveform is shown in Fig. 12):

\[
Signal(k) = \begin{cases} 
\sin(2\pi k \Delta t) & f = 850 KHz \\
-\sin(2\pi k \Delta t) & f = 1.75 MHz 
\end{cases} \quad k = 1 : 250
\]

Using \( db_2 \) as the mother wavelet and \( 10 \) MS/sec as the sampling rate, the detail component at the third decomposition level through \( DWT \) and \( IWTD \) are shown in Fig. 12.

The following remarks may be emphasized:

a. The marked points in parts (b) and (c) show that similar elements are derived by \( DWT \) and \( IWTD \) (extra elements in part (c) between each two marked points are due to not using down sampling).

b. As soon as the data window is filled with \([2^j - 1)(N - 1) + 1\] samples of the original signal (e.g. 22 samples in this example, where \( j = 3 \) and \( N = 4 \)), the new element of the desired detail (or approximate) component is calculated through \( IWTD \), while any calculation through \( DWT \) for elements of components in different levels requires a sequential procedure of calculating all elements of approximate components of lower levels). In on-line application as soon as the moving data window includes a new sample of the signal (while withdrawing the oldest sample), the desired output is found. For instance, by using Eq. (21):

\[
d^{IWTD}_3(t_{200}) = [x(179) \quad x(180) \quad ... \quad x(200)] \cdot G_3
\]

and as the data window goes on:

\[
d^{IWTD}_3(t_{201}) = [x(180) \quad x(181) \quad ... \quad x(201)] \cdot G_3
\]

\[
d^{IWTD}_3(t_{202}) = [x(181) \quad x(182) \quad ... \quad x(202)] \cdot G_3
\]

\[
d^{IWTD}_3(t_n) = [x(n-21) \quad x(n-20) \quad ... \quad x(n)] \cdot G_3
\]
C. Based on IWTD and by using Eqs. (21) and (22) every element of different components up to the \( j \)th level can be obtained, simultaneously. The numerical results, as the data window is moving on, are shown in Table 2 and graphically in Fig. 13.

Fig. 12. A window of a) Original Signal(k), b) The 3\textsuperscript{rd} detail component by DWT algorithm, and c) The 3\textsuperscript{rd} detail component, by the full order IWTD (without down sampling).

Table 2. Numerical output of IWTD as data window moves in on-line applications

<table>
<thead>
<tr>
<th>At ( t=t_n )</th>
<th>By entering the sample ( x(n) )</th>
<th>( d_1^{\text{IWTD}}(t_n) )</th>
<th>( d_2^{\text{IWTD}}(t_n) )</th>
<th>( d_3^{\text{IWTD}}(t_n) )</th>
<th>( a_3^{\text{IWTD}}(t_n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{240} )</td>
<td>( x(240) )</td>
<td>0.1126</td>
<td>-0.0014</td>
<td>0.3147</td>
<td>0.2121</td>
</tr>
<tr>
<td>( t_{241} )</td>
<td>( x(241) )</td>
<td>0.0793</td>
<td>0.2051</td>
<td>-0.1268</td>
<td>0.3146</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( t_{249} )</td>
<td>( x(249) )</td>
<td>0.0450</td>
<td>-0.4037</td>
<td>0.8075</td>
<td>-0.3146</td>
</tr>
<tr>
<td>( t_{250} )</td>
<td>( x(250) )</td>
<td>0.0941</td>
<td>-0.3288</td>
<td>0.8171</td>
<td>0.0852</td>
</tr>
<tr>
<td>( t_{251} )</td>
<td>( x(251) )</td>
<td>0.0695</td>
<td>-0.1579</td>
<td>0.5988</td>
<td>0.0915</td>
</tr>
<tr>
<td>( t_{252} )</td>
<td>( x(252) )</td>
<td>0.1992</td>
<td>0.0410</td>
<td>0.2151</td>
<td>0.2431</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( t_{270} )</td>
<td>( x(270) )</td>
<td>0.3179</td>
<td>-0.8223</td>
<td>-0.2699</td>
<td>0.1797</td>
</tr>
<tr>
<td>( t_{271} )</td>
<td>( x(271) )</td>
<td>-0.1106</td>
<td>-0.1720</td>
<td>-0.0514</td>
<td>0.1784</td>
</tr>
</tbody>
</table>

Example 2– Comparison between the full & reduced order IWTD (in disclosing the frequency spectrum of a signal): An input signal with a non-stationary frequency characteristic is considered as the following:

\[
\text{Signal}(t) = \begin{cases} 
\sin(2\pi f_1 t) & f_1 = 0.5 \text{ MHz} \\
-\sin(2\pi f_2 (t - 13 \times 10^{-6})) & f_2 = 1 \text{ MHz} \\
\sin(2\pi f_3 (t - 25.5 \times 10^{-6})) & f_3 = 2 \text{ MHz} \\
\sin(2\pi f_4 (t - 38 \times 10^{-6})) & f_4 = 4 \text{ MHz} 
\end{cases} \\
0 \leq t \leq 13 \mu \text{sec}
\]

According to the chosen sampling frequency (40 MS/sec), the components with frequencies \( f_2, f_3 \) and \( f_4 \) must be disclosed by \( G_5, G_4 \) and \( G_3 \) respectively. Table 3 compares the performance of the reduced filter and the full one, in revealing different components of the original signal at different levels of decomposition, where it can be seen that the largest error (in spite of the order of the filter which is halved) is less than 1\%.
Table 3. Output amplitudes of the full and reduced order IWTD filters using db5 as the mother wavelet

<table>
<thead>
<tr>
<th>Threshold Percentage</th>
<th>$f_1$ amp. in $D_3$</th>
<th>$f_2$ amp. in $D_4$</th>
<th>$f_3$ amp. in $D_5$</th>
<th>$f_4$ amp. in $D_6$</th>
<th>$f_5$ amp. in $D_7$</th>
<th>$f_6$ amp. in $D_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.9133</td>
<td>-----</td>
<td>.9100</td>
<td>-----</td>
<td>.9129</td>
<td>-----</td>
</tr>
<tr>
<td>2</td>
<td>.9084</td>
<td>.54%</td>
<td>.9057</td>
<td>.47%</td>
<td>.9080</td>
<td>.54%</td>
</tr>
<tr>
<td>5</td>
<td>.9058</td>
<td>.82%</td>
<td>.9024</td>
<td>.84%</td>
<td>.9044</td>
<td>.93%</td>
</tr>
</tbody>
</table>

Fig. 13. The simultaneity performance of the IWTD algorithm at different times ($t = t_{230}$ (Left), $t = t_{265}$ (Middle) and $t = t_{300}$ (Right)) a) Original signal, b) First detail component, c) Second detail component, d) Third detail component, and e) Third approximate component

Example 3– Comparison between the full & reduced order IWTD (in disturbance detection): This example gives another comparison between the time delay in the full and reduced order IWTD (with 2% threshold).

As is shown in Fig. 14, a mono-sample disturbance has been inserted in the original signal at $t = 20 \mu\text{sec}$. This disturbance has been detected with the reduced order filter at only $0.025 \mu\text{sec}$ later, where by the full order filter this delay equals $0.375 \mu\text{sec}$, i.e. fifteen times later (for the reduced order filter with 5% threshold, this delay is not recognizable, and the filter output starts as soon as the disturbance occurs in the original signal). This capability of the reduced order IWTD (which may become even less according to the chosen threshold) makes it very suitable for any on-line disturbance detection.

Fig. 14. Disturbance detection by the full (left figures) and reduced (right figures) order filters based on db5 as the mother wavelet (Threshold=2% & Sampling rate=40MS/sec)
5. CONCLUSION

The reduced order IWTD (without down sampling) as a new digital filter for on-line applications has been proposed and its mathematical implementation has been derived. The proposed filter can produce, simultaneously, the elements of detail and/or approximate components at different levels. In addition, the overall time delay of the new filter is much reduced in comparison with DWT. These advantages make this new filter a desired tool for on-line applications of Wavelet Transform in feature extraction and/or classification tasks in different signal processing procedures.

REFERENCES


Appendix A – Numerical Presentation of Full and Reduced Order IWTD Filters

The numerical presentation of $G$, $G_2$, $G_3$, $H$, $H_2$, $H_3$, and related reduced order (with 5% threshold) for calculation of detail and approximate components, up to the third level of decomposition and based on $db_2$ as the mother wavelet, are as below:

$$G = \begin{bmatrix} -0.915 \\ -1.585 \\ 0.5915 \\ -0.3415 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0.1769 \\ 0.3644 \\ 0.029 \\ 0.2561 \\ 0.051 \\ 0.0312 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 0.1124 \\ 0.1993 \\ 0.0499 \\ 0.0469 \\ 0.0911 \\ 0.1494 \\ 0.0591 \\ 0.0086 \\ 0.0234 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.3415 \\ 0.5915 \\ -0.0915 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0.1166 \\ 0.3186 \\ 0.1479 \\ 0.0061 \\ 0.0145 \\ 0.0084 \\ 0.0061 \\ 0.0023 \\ 0.0063 \\ 0.0006 \\ 0.0006 \\ 0.0013 \\ 0.0008 \end{bmatrix}$$

$$G_{r} = G$$

$$G_{r2} = G_2$$

$$G_{r3} = G_3$$

$$H_{r} = H$$

$$H_{r2} = H_2$$

$$H_{r3} = H_3$$
### Appendix B – Bibliography of the Wavelet Transform Applications

<table>
<thead>
<tr>
<th>Title</th>
<th>Address</th>
<th>Used Comp.</th>
<th>Main Subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>An Approach to Power Transformer Protection Based on Wavelet Transform</td>
<td>L. Jipping, C. Ying, Y. Xue, W. Li The 7th International Power Engineering Conference, 2005. IPEC 2005, 29 Nov.-2 Dec. 2005</td>
<td>D3</td>
<td>Identify an internal fault and distinguish it from an inrush current or an external fault</td>
</tr>
<tr>
<td>A Signal Processing Module for the Analysis of Heart Sounds and Heart Murmurs</td>
<td>J. Faizan, P. A. Venkatachalam, M. H. Ahmad Fadzil Journal of Physics: Conference Series 34, 2006 Page(s):1098-1105</td>
<td>D4,D5</td>
<td>Analysis of heart sounds and heart murmurs</td>
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</table>
### Appendix B – Continued.

<table>
<thead>
<tr>
<th>Applications of Discrete Wavelet Transform for Transformer Inrush Current Detection in Protective Control Scheme</th>
<th>A. Kunakorn</th>
<th>D4,A4</th>
<th>Transformer inrush current detection in protective control scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discrete Wavelet Analysis of Acoustic Emissions During Fatigue Loading of Carbon Fiber Reinforced Composites</td>
<td>G. Kamala J. Hashemi, A. A. Barhorst</td>
<td>D8,D9</td>
<td>Acoustic emission analysis</td>
</tr>
<tr>
<td>Analyzing Power System Waveforms Using Wavelet-Transform Approach</td>
<td>K. P. Wong, V. L. Pham</td>
<td>D2,D3</td>
<td>Harmonic analysis of capacitor bank switching waveforms</td>
</tr>
<tr>
<td>Computation of Continuous Wavelet Transform via A New Wavelet Function for Visualization of Power System Disturbances</td>
<td>S. J. Huang, C. T. Hsieh</td>
<td>D3,D4</td>
<td>Visualization of electric power system disturbances</td>
</tr>
<tr>
<td>Power Transformer Protection Based On Transient Detection Using Discrete Wavelet Transform</td>
<td>F. Jiang, Z. Q. Bo, P. S. M. Chin, M. A. Redfern, Z. Chen</td>
<td>D3,A5</td>
<td>Discrimination between internal and external faults</td>
</tr>
<tr>
<td>Using Wavelet Transforms for ECG Characterization</td>
<td>J. S. Sahambi, S. N. Tandon, R. K. P. Bhatt</td>
<td>A3,A4</td>
<td>Obtaining of timing intervals that define the morphology of the ECG</td>
</tr>
<tr>
<td>Validation of a New Method for the Diagnosis of Rotor Bar Failures via Wavelet Transform in Industrial Induction Machines</td>
<td>J. A. Antonino-Daviu, M. Riera-Guasp, J. R. Folch, M. P. M. Palomares</td>
<td>D7,D8,A8</td>
<td>Diagnosis of bar breakages in induction machines</td>
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<tr>
<td>Online Rotor Bar Breakage Detection of Three Phase Induction Motors by Wavelet Packet Decomposition and Artificial Neural Network</td>
<td>Z. Ye, B. Wu</td>
<td>D8-D10</td>
<td>Detection of induction motor rotor bar breakage</td>
</tr>
<tr>
<td>Anti-distortion method for wavelet transform filter banks and non-stationary power system waveform harmonic analysis</td>
<td>V. L. Pham, K. P. Wong</td>
<td>D5,D6,D8</td>
<td>Capacitor bank tripping</td>
</tr>
<tr>
<td>A Novel Approach to the Classification of the Transient Phenomena in Power Transformers Using Combined Wavelet Transform and Neural Network</td>
<td>P. L. Mao, R. K. Aggarwal</td>
<td>D1–D3</td>
<td>Accurate discrimination between an internal fault and a magnetizing inrush current in the power transformers</td>
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</table>