“Technical Note”

AN ANALYSIS OF SHEAR CORRECTION FACTORS IN A THERMOPLASTIC COMPOSITE CANTILEVER BEAM

F. OKUMUŞ

1inci Mknz.P.Tüm.K.İlg, Ankara, Turkey
Email: fokumus1953@hotmail.com

Abstract—In the present study, an analysis of shear correction factors is carried out for a homogeneous polyethylene thermoplastic cantilever beam which is reinforced by steel fibers with a rectangular cross-section. An applied shear force produces extension-shear coupling which has not been considered in previous out-of-plane shear deformation studies of beams. The shear correction factor is defined taking into account the coupling effect. Normal and shear stress distributions, as well as shear strain cross sections of the rectangular solid beam are obtained analytically. The variation of shear correction factors versus ply angle for the composite beam is also presented.

Keywords – Thermoplastic composite, shear correction factor, stress and strain analysis

1. INTRODUCTION

Fiber-reinforced composite materials and in particular polymer-matrix composites, offer considerable possibilities of application in aircraft, aerospace, and automotive sectors on account of their high specific stiffness, strength-to-weight ratio, and the possibility of changing its stiffness characteristics by manipulating the laminate sequences with keeping its weight constant. Thermoplastic-matrix composites are presently of interest because of the improved interlaminar fracture toughness, increased impact resistance, and higher solvent resistance in relation to thermoset composites systems. Moreover, thermoplastic composites possess the unique characteristic that enables them to be remelted, reprocessed, and reformed. They are easily repaired and they can be remelted for repairing local cracks and delaminations. As a result of their potential for high production rates and low material costs, glass mat and woven thermoplastic composites are of interest in a wide range of sectors as stated previously. Experimental investigations on the forming of thermoplastic composites can be found in Refs. [1-3]. The shear correction factors of beams and plates have been studied by researchers during the past decades [4-8]. Chow [4] yielded a method to calculate the shear correction factor of symmetrically laminated plate by assuming normal stress distribution across the cross-section. Whitney [5] suggested methods to calculate the shear correction factors for general orthotropic laminated composite beams and plates, respectively. Vlachoutsis [6] suggested a procedure for the shear correction factor and applied it to shell analysis. Cowper [7] presented analytical solutions for various beam cross-sections. Bert [8] presented methods to find the shear correction factors for composite plates. In the present study, the distribution of stresses are investigated for a polyethylene thermoplastic composite beam having a rectangular cross-section reinforced with woven steel fibers. In this study the shear correction factor is based on Timoshenko’s beam theory which imposes the equality of shear deformation energies and includes the influences of extension-shear and bending shear coupling. Finally, the stress distributions of the polyethylene thermoplastic composite beam are identified and the shear correction factors are calculated for rectangular shaped beams with a change in their composite layer angles.
2. MATHEMATICAL FORMULATION

In this study we consider a composite cantilever beam whose free end is loaded by transverse shear force in the y direction as seen in Fig. 1.

In Fig. 1, the positive fiber angle $\theta$ is defined as the angle between the fiber direction and the x-axis $u, v$ is defined as axial displacement and $\psi$ is the bending angle. Under these conditions, the beam deflects in x-y plane by the applied force and extension-shear couplings exist due to lamination [9]. The equality of shear deformation energies is given as

$$\frac{1}{2} \int N_{xy} \gamma_{xy} dy = \frac{1}{2} [Q(v' - \psi)]$$

where $N_{xy}$ is stress resultant on the cross section, $\gamma_{xy}$ is the shear strain component and $Q$ is shear force.

The governing equation of a composite beam can be written as

$$\begin{bmatrix} \quad N \\ \quad Q \\ \quad M \end{bmatrix} = \begin{bmatrix} \quad bA_{11} \\ \quad bA_{16} \\ \quad nbA_{66} \end{bmatrix} \begin{bmatrix} \quad u' \\ \quad v' - \psi \\ \quad -\psi \end{bmatrix}$$

where $\{N, Q, M\} = (N_x, N_y, N_{xy}) dy$ and

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} \\ A_{16} \\ A_{66} \end{bmatrix} \begin{bmatrix} \quad \varepsilon_x \\ \quad \varepsilon_y \\ \quad \gamma_{xy} \end{bmatrix}$$

Stress resultants $(N_x, N_y, N_{xy})$ on the cross section are defined through the integration of normal stress $\sigma_x$ and shear stress $\tau_{xy}$ along the thickness of the beam as $\int \left\{ \sigma_x, \tau_{xy} \right\} dz$. In order to find the shear correction factor from (4), it is necessary to calculate the distributions of actual shear strain and stress resultants across the cross-section. From (2), the first one should be modified by considering the actual shear strain as follows:

$$(v' - \psi) = \frac{1}{kbA_{66}} (Q - bA_{16}u')$$

Inserting Eq. (3) into (1), we can obtain the shear correction factor as follows:

$$k = \frac{\int N_{xy} \gamma_{xy} dy \left[ \int N_{xy} dy - bA_{16}u' \right]}{bA_{66} \int N_{xy} \gamma_{xy} dy}$$

Stress resultants $(N_x, N_y, N_{xy})$ on the cross section are defined through the integration of normal stress $\sigma_x$ and shear stress $\tau_{xy}$ along the thickness of the beam as $\int \left\{ \sigma_x, \tau_{xy} \right\} dz$. In order to find the shear correction factor from (4), it is necessary to calculate the distributions of actual shear strain and stress resultants across the cross-section. From (2), the first one should be modified by considering the actual shear strain as follows:

$$N = \int N_x dy = bA_{11}u' + A_{16} \int \gamma_{xy} dy$$

From (5) and (2) the strain components and bending moment in the composite beam can be written as

$$U' = \frac{1}{bA_{11}} \left[ N - A_{16} \int \gamma_{xy} dy \right], \quad -\psi' = 12(b^3A_{11})^{-1} M$$
Using (2) and noting that $N=0$, the normal stress resultants can be written as

$$N_x = \frac{1}{b} A_{16} \int_{y} \gamma_{xy} \, dy + \frac{12}{b^3} M_y + A_{16} \gamma_{xy}$$

(7)

According to Timoshenko’s beam theory [10], two-dimensional static equilibrium equations can be written as follows:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \quad \frac{\partial N_{xy}}{\partial x} = 0$$

(8)

Shear stress resultant can be obtained by integrating the first one of (8)

$$N_x = \frac{12}{b^3} Qy(x - L), \quad N_{xy} = \frac{3}{2} \frac{Q}{b^3} (b^2 - 4y^2)$$

(9)

The stress resultants and strain across the cross section of the composite beam can be found by using (9). The strain is represented from the first one of (6)

$$U' = -\frac{A_{16}}{A_{11}} \delta_{66} Q b^{-1}$$

(10)

The shear correction factors of the laminated composite beam can be calculated by using Eqs. (9) and (10) in (4) in terms of known quantities. As can be seen in the above equations, the shear correction factors for a composite beam are the function of material properties, the geometric dimension of a beam and the lay-up sequences of composite layers.

3. PRODUCTION OF THE COMPOSITE BEAM AND ITS MECHANICAL PROPERTIES

The composite beam consists of a polyethylene matrix and woven steel fibers. Thermoplastic is melted by electrical resistance up to 1750°C. The woven steel fibers are put into two polyethylene layers. The beam has a 4.4 mm thickness. Its length ($L$) is 250 mm and width 25 mm. The mechanical properties are given as $E_1$(MPa) = 3040, $E_2$(MPa) = 3040, $\sigma_{12}$(MPa) = 290, $\nu_{12} = 0.30$, X(MPa) = 16.2, Y(MPa) = 16.20, S(MPa) = 5.25.

4. RESULTS AND DISCUSSION

A stress and strain analysis is carried out for a cantilever composite beam. The rectangular composite beam is clamped at one end and loaded by a terminal force $P = 7$ N. in the direction of $y$-axis at the free end. The stress and strain across the cross section are analytically calculated using (9). The cross section shear strain distribution for the polyethylene composite beam is illustrated in Fig. 2. Results have shown that the distribution of shear strain across the beam cross section is a superposition of parabolic and linear expansion, i.e., quadratic. This superposition is more significant in the constitutive relation. It is produced by the coupling effect between normal stress and shear stress. The influence of ply orientation angle $\theta$ on the shear correction factor is displayed in Fig. 3. This analysis is done for two cases. In the first case, the calculation is made taking account of the coupling terms. In the second case this calculation is carried out without coupling terms. When the coupling terms are considered, the results show that the shear correction factor of a rectangular composite beam has the same constant value as that of an isotropic rectangular beam, i.e., $k = 5/6$. In the second case where coupling is ignored, the shear correction factors vary with the change of ply orientation. The main reason for the difference between the two cases is the extensional beam strain. In an isotropic beam the natural axis coincides with the mid-depth of the beam, whereas, an extensional strain is present in a rectangular composite beam by means of extension shear coupling. It should be noted that in the composite laminated beam the neutral axis does not coincide with the beam mid-depth due to the coupling effect. Results showed that the values of the shear correction factor are a function of the ply orientation angle due to extensional shear strain coupling.
5. CONCLUSION

In the present study, the shear correction factors have been determined considering the effect of extension-shear and bending-shear couplings for various ply angles. The shear correction factors for polyethylene cantilever composite beams have the same constant value as an isotropic beam in spite of the change of its orientation angles for the cases in which the effects of extension-shear coupling have been considered. But the shear correction factor results that do not account for the coupling influence displayed values that are dependent on the fiber orientation angle under the same shear loading. Thus, the numerical calculations indicate that the shear correction factors of composite rectangular beams should be calculated, taking into account the coupling effects of the extension-shear and the bending-shear. From the above results, the shear correction factors determined with the coupling effects of the cantilever beam will play important roles in the analyses of some of the practical problems such as the elastic stability analysis of thermoplastic composite beam elements. The current analytic calculation method can be easily extended to the cantilever beams of arbitrary cross-sections in the same manner.

REFERENCES