ROBUST SLIDING-MODE CONTROL FOR NONLINEAR FLEXIBLE ARM USING NEURAL NETWORK

R. J. WAI, AND M. C. LEE
Dept. of Electrical Engineering, Yuan Ze University, Chung Li 320, Taiwan, R.O.C.
Email: rjwai@saturn.yzu.edu.tw

Abstract – This study addresses the design and properties of a sliding-mode neural-network control (SMNNC) system for a nonlinear flexible arm that is driven by a permanent magnet (PM) synchronous servo motor. First, the dynamic model of a flexible arm system with a tip mass is introduced. When the tip mass of the flexible arm is a rigid body, not only bending vibration but also torsional vibration occur. In this study, the vibration states of the nonlinear system are assumed to be unmeasurable, i.e., only the actuator position can be acquired to feed into a suitable control system for stabilizing the vibration states indirectly. Then, a SMNNC scheme without the feedback of the vibration measure is proposed to control the motor-mechanism coupling system for periodic motion. All adaptive learning algorithms in the SMNNC system are derived in the sense of Lyapunov stability analysis, so that the system-tracking stability can be guaranteed in the closed-loop system. The effectiveness of the proposed control scheme is verified by both numerical simulation and experimental results.

Keywords – Sliding mode control, neural network, robust control, flexible arm, PM synchronous servo motor

1. INTRODUCTION

In order to achieve high performance requirements such as high-speed operation, increased accuracy in positioning, lower energy consumption, less weight, and safer operation due to reduced inertia, great attention has been paid to the dynamics and control of flexible robot arms in recent years [1-5]. Control of flexible structures involves typical nonlinear interactions including physical modeling errors, parameter uncertainties, high-frequency flexible modes, noise measurement, chaotic behaviors, etc. Moreover, these structures have lightly damped modes and unstable zeros that make control quite difficult. The controller that is to be designed should take into account the uncertain aspect of the plant model, even the unknown system dynamics. In addition, the designed control system of the flexible robot arm must be able to control the motion of the rigid-body mode of the arm and to suppress the vibration modes of the arm due to the flexibility of the flexible robot arm [1]. Therefore, complex modeling procedures and complicated control techniques are usually required [2, 3]. These model-based control systems, originally designed for the demands of high performance, may not be so easy to implement in flexible arm control practice. It is due to uncertainties in design models and large variations of loads at the robot hand, to the ignored high frequency dynamics, and to the high order of the designed control system. Though many modern control techniques [6] have been designed to overcome the mentioned difficulties by using complex control laws, these techniques require an exact knowledge of the nonlinear terms, knowledge of bounds on uncertainties, or knowledge of a nonlinear regression matrix of robot functions. In practice, it is very difficult to have such a priori knowledge of the arm dynamics.

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**Corresponding author
In the past several years, active research has been carried out in neural network control [7-9]. The characteristics of fault-tolerance, parallelism and learning suggest that they may be good candidates for implementing real-time adaptive control for nonlinear dynamical systems. It has been proven that an artificial neural network can approximate a wide range of nonlinear functions to any desired degree of accuracy under certain conditions [7]. It is generally understood that the selection of the neural-network-training algorithm plays an important role for most neural network applications. In the conventional gradient-decent-type approach, the sensitivity of the controlled system should be required in the on-line training process [9]. However, it is difficult to acquire for unknown system dynamics or highly nonlinear dynamics. Besides, the local minimum of the performance index remains to be challenged [7]. In practical control applications, it is desirable to have a systematic method of ensuring the stability, robustness, and performance properties of the overall system. Recently, several neural network control approaches have been proposed based on the Lyapunov stability theorem [10-15]. One main advantage of these control schemes is that the adaptive laws were derived based on the Lyapunov synthesis method, and therefore guarantee the stability of the control system. However, some constrained conditions should be assumed in the control process, e.g., the approximation error, optimal parameter vectors or higher-order terms in the Taylor series are bounded. These requirements are not easy to satisfy in practical control applications.

The neural-network-based control techniques have represented an alternative method to deal with the uncertainties of the robot control system available in the literature [16-19]. In Vemuri and Polycarpou [17], a learning architecture with sigmoidal neural networks is introduced to monitor the robotic system for any off-nominal behavior due to faults. The disadvantages of this structure are that the modeling uncertainty and the robotic system states after the occurrence of a fault are assumed to be bounded. Gutierrez et al. [18] described the results of the practical implementation of a neural network tracking controller on a single flexible link and compared its performance to that of proportional-derivative (PD) and proportional-integral-derivative (PID) standard controllers. This approach avoids the requirement of the knowledge of friction, gravity, and Coriolis/centripetal, or any regression matrix. However, the major drawback of this control technique is that some constrained conditions of the neural network should be satisfied. Lewis et al. [19] developed a multilayer neural-net controller for a general serial-link rigid robot arm, in which the adaptation laws of all network parameters are derived in the sense of the Lyapunov theorem using the Taylor series expansion. Though the stability of the control system can be guaranteed, the bounds of ideal weights and higher-order terms in the Taylor series are required in the design process. The motivation of this study is to design a neural control scheme that not only guarantees the stability of the controlled system, but also the constrained conditions, and prior knowledge of the controlled system are not necessary in the design process such that it can easily extend to other nonlinear machines. On the other hand, sliding-mode control is one of the most effective nonlinear robust control approaches since it provides system dynamics with an invariance property to uncertainties once the system dynamics are the control in the sliding mode [20]. Therefore, a neural control scheme based on the sliding-mode technique is proposed in this study to accomplish the mentioned motivation.

2. DYNAMIC ANALYSIS

a) Field-oriented PM synchronous motor drive

The configuration of a general field-oriented PM synchronous motor drive system is depicted in Fig. 1a [21], which consists of a PM synchronous motor coupled with a mechanism, a ramp comparison current-controlled PWM voltage source inverter (VSI), a unit vector generator (where $\theta$ is the position of rotor flux), a coordinate translator, a speed control loop and a position control loop. The PM synchronous motor used in this drive system is a three-phase four-pole 750W 3.47A 3000 rpm type. The current-controlled VSI is implemented by IGBT switching components with a switching frequency of 5kHz.
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With the implementation of the field-oriented control, the PM synchronous servo motor drive system can be simplified to a control system block diagram as shown in Fig. 1b [21], in which

\[ \tau_e = K_T i_q^* \]  
\[ K_T = \frac{3}{2} P_l L_{md} I_{fd} \]  
\[ H_p(s) = \frac{1}{(J_m s + B_m)} \]  
\[ \tau_e = \tau_m + B_m \dot{\theta}_r + J_m \ddot{\theta}_r \]

where \( \tau_e \) is the electric torque; \( K_T \) is the torque constant; \( i_q^* \) is the torque current command; \( P_l \) is the number of pole pairs; \( L_{md} \) is the d-axis mutual inductance; \( I_{fd} \) is the equivalent d-axis magnetizing current; \( s \) is the Laplace operator; \( \tau_m \) is the load torque; \( B_m \) is the torsional damping coefficient; \( J_m \) is the inertia of rotor and gear; \( \theta_r \) is the rotor position. Moreover, in Fig. 1b, \( \theta_r^* \) and \( \omega_r^* \) are the rotor position and speed commands; \( \omega_r \) is the rotor speed.

b) Mathematical model of motor-mechanism coupling system

Figure 2 shows a PM synchronous servo motor system which is applied to a flexible arm, including a geared speed-reducer with a gear ratio

\[ g_r = \frac{n_a}{n_b} = \frac{\tau}{\tau_m} = \frac{\dot{\theta}_r}{\dot{\theta}} \]

where \( g_r \) is the gear ratio; \( \theta \) is the rotation angle of the flexible arm, and \( n_a \) and \( n_b \) are the gear numbers. Substituting (1) and (5) into (4), the following applied torque can be obtained.
\[ \tau = g_r (\tau_c - J_m \ddot{\theta}_r - B_m \dot{\theta}_r) = g_r (K_r i^*_q - g_r J_m \ddot{\theta} - g_r B_m \dot{\theta}) \]  

\hspace{1cm} (6)

where \( \tau \) is the applying torque.

\[ \theta \]

\[ \tau \]

\[ m \]

\[ r \]

\[ \theta \]

\[ \phi \]

\[ \psi \]

\[ \tau \]

\[ J \]

\[ i \]

\[ B \]

\[ \theta \]

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where \( v_a(x,t) = \frac{\partial v(x,t)}{\partial x} \); \( I_b \) is the moment inertia of the flexible arm; \( m \) is the attached rigid tip mass. Rearrange (7), the dynamic equation can be rewritten as

\[
M(X;t)\ddot{\theta} + D(X;t)\dot{\theta} + F(X;t) = BU(t)
\]

where \( X = [v \quad \phi]^T \); \( B = g_rK_T > 0 \); \( U(t) = i_a^* \) is the control input, and

\[
M(X;t) = I_b + \rho A_1^L v^2 dx + \rho A_1^L \phi^2 dx + g_r^2J_m + m \{ v^2(L,t) + c^2 v_a^2(L,t) + e^2 \phi^2(L,t) \} + 2 c v_a(L,t)v(L,t) + 2 c c v_a(L,t)\phi(L,t) - 2 c \phi(L,t) + (L+c)^2 \}
\]

\[
D(X;t) = g_r^2B_m + 2 \rho A_1^L v \dot{v} dx + 2 \rho A_1^L \phi \dot{\phi} dx + 2 m \{ v(L,t)v(L,t) + c^2 v_a(L,t)v_a(L,t) \} + e^2 \phi(L,t)\phi(L,t) + c v_a(L,t)v_a(L,t) + c v_a(L,t)\phi(L,t) - e \phi(L,t)v(L,t) - e \phi(L,t)\phi(L,t) \}
\]

\[
F(X;t) = \rho A_1^L x \ddot{v} dx + m(L+c) \{ \dot{v}(L,t) + c \dot{v}_a(L,t) - e \dot{\phi}(L,t) \}
\]

Since all elements of \( M(X;t) \) shown in (9) are positive values, \( M(X;t) > 0 \) can be obtained and it’s invertible exists. In this study, the vibration state \( X \) of the flexible arm is assumed to be unmeasurable. When only the rotation angle of the flexible arm is measurable, a SMNNC system is proposed to control the nonlinear motor-mechanism coupling system in the following section.

3. SLIDING-MODE NEURAL-NETWORK CONTROL SYSTEM

In order to control the position of the flexible arm effectively, a SMNN system is proposed in this section. The configuration of the proposed SMNN system is depicted in Fig. 4, in which the reference model is chosen according to the prescribed time-domain control specifications. The control problem is to find a control law so that the state \( \theta(t) \) can track the desired command \( \theta_d(t) \). To achieve the control objective, define the tracking error \( e_\theta(t) = \theta(t) - \theta_d(t) \), in which \( \theta_m(t) \) represents the reference trajectory. Reformulate (8), the dynamic equation of the motor-mechanism coupling system can be represented as follows:

\[
\ddot{\theta} = f(X;t)\dot{\theta} + G(X;t)U(t) + d(X;t)
\]

where \( f(X;t) = -M^{-1}D \); \( G(X;t) = M^{-1}B > 0 \) and \( d(X;t) = -M^{-1}F \). Now, a sliding surface is defined as

\[
r(t) = \left( \frac{d}{dt} + \lambda \right)^2 \int_0^t e_\theta(\tau)d\tau
\]

where \( \lambda \) is a positive constant. Note that, since the function \( r(t) = 0 \) when \( t = 0 \), there is no reaching phase as in the traditional sliding-mode control [20]. Differentiating \( r(t) \) with respect to time and using (12), it can be seen that

\[
\dot{r}(t) = \ddot{e}_\theta(t) + 2\lambda \dot{e}_\theta(t) + \lambda^2 e_\theta(t) = \ddot{\theta}_m(t) - f(X;t)\dot{\theta} - G(X;t)U(t) - d(X;t) + 2\lambda \dot{e}_\theta(t) + \lambda^2 e_\theta(t)
\]

The tracking problem mentioned above is to find a control law, \( U(t) \), so that the state remaining on the surface \( r(t) = 0 \) for all \( t > 0 \). In the design of the sliding-mode control system, first the equivalent control law \( U_{eq}(t) \), which will determine the dynamics of the system on the sliding surface, is derived. The equivalent control law is derived by recognizing

\[
\dot{r}(t)|_{U=U_{eq}} = 0
\]

Substituting (14) into (15), and assuming all system parameters are well known, then
\[ \ddot{\theta}_m - f(X(t))\dot{\theta} - G(X(t))U(t) - d(X(t)) + 2\dot{\lambda} \dot{e}_\theta(t) + \lambda^2 e_\theta(t) = 0 \] (16)

Sliding-Mode Neural-Network Control System

Solving (16), one can obtain
\[ U_{eq}(t) = G(X(t))^{-1} [\ddot{\theta}_m(t) - f(X(t))\dot{\theta} - d(X(t)) + 2\dot{\lambda} \dot{e}_\theta(t) + \lambda^2 e_\theta(t)] \] (17)
Thus, given \( \dot{r}(t) = 0 \), the dynamics of the system on the sliding surface for \( t \geq 0 \) is given by
\[ \dot{e}_\theta(t) + 2\dot{\lambda} \dot{e}_\theta(t) + \lambda^2 e_\theta(t) = 0 \] (18)

Properly choosing the value of \( \dot{\lambda} \), the desired system dynamics such as rise time, overshoot, and settling time can be easily designed by the second-order system. Since the vibration state (\( X \)) of the flexible arm is unmeasurable, the equivalent control law (17) in the traditional sliding-mode control can not be practically implemented due to the unknown system dynamics (\( f(X(t)) \), \( G(X(t)) \) and \( d(X(t)) \)). Thus, a SMNNC system including a NN controller to learn the equivalent control law and a robust controller to ensure the stable control performance is designed without the feedback of vibration measure in this study to control the motor-mechanism coupling system for periodic motion.

A general function of a three-layer NN can be represented in the following form [7]:
\[ y = U_{NN}(r, V, W, m, S) = WQ(VX) \] (19)
where \( X_i = [r \quad r(1-z^{-1})]^T \in \mathbb{R}^{2 \times 1} \) is the input state of the NN, in which \( z^{-1} \) is a time delay; \( V \in \mathbb{R}^{n \times 2} \) is the input-to-hidden layer interconnection weights, in which \( n \) is the hidden layer nodes; \( W \in \mathbb{R}^{1 \times n} \) is the hidden-to-output layer interconnection weights; the active function used in the NN is chosen as \( Q(VX_i) = \exp[-(VX_i - m)^2]/S \in \mathbb{R}^{n \times 1} \), in which \( m \in \mathbb{R}^{n \times 1} \) and \( S \in \mathbb{R}^{n \times 1} \) are the adjustable parameters of the radial basis functions (RBF); \( y \) is the output of NN. Thus, an optimal NN controller \( U_{NN}^* \) will be designed to learn the equivalent control law, such that
\[ U_{eq}(t) = U_{NN}^*(r, V^*, W^*, m^*, S^*) + \varepsilon = W^*Q^*(V^*X_i) + \varepsilon \] (20)
where \( \varepsilon \) is a minimum reconstructed error; \( V^*, W^*, m^* \) and \( S^* \) are optimal parameters of \( V, W, m \) and \( S \) in the NN. The control law for the SMNNC system is assumed to take the following form:
\[ U(t) = \hat{U}_{NN}(r, \hat{V}, \hat{W}, \hat{m}, \hat{S}) + U_s = \hat{W}Q(\hat{V}X_i) + U_s \] (21)
\[\dot{\theta} + 2\lambda \dot{\theta} + \dot{\lambda} \theta = G(X,t)[U_{eq}(t) - U(t)] = r(t)\]  

Moreover, \( \bar{U} \) is defined as  

\[\bar{U} = U_{eq} - U = \mathbf{W}^* \mathbf{Q}^* + \varepsilon - \dot{\mathbf{W}} \dot{\mathbf{Q}} - U_x = \dot{\mathbf{W}} \dot{\mathbf{Q}}^* + \mathbf{W}^* \dot{\mathbf{Q}} + \varepsilon - U_x,\]  

where \( \dot{\mathbf{W}} = \dot{\mathbf{W}}^* - \dot{\mathbf{W}} \) and \( \dot{\mathbf{Q}} = \dot{\mathbf{Q}}^* - \dot{\mathbf{Q}} \). In this study, a control methodology is proposed to guarantee the closed-loop stability and the tracking performance. For achieving the goal, the linearization technique is employed to transform the nonlinear active functions into partially linear form so that the expansion of \( \dot{\mathbf{Q}} \) in a Taylor series to obtain [19]

\[
\dot{\mathbf{Q}} = \mathbf{Q}_V \dot{\mathbf{V}}_X + \mathbf{Q}_m \dot{\mathbf{m}} + \mathbf{Q}_s \dot{\mathbf{s}} + \mathbf{O}_n
\]  

where

\[
\mathbf{Q}_V = \left[ \frac{\partial q_1}{\partial (\mathbf{V}X_1)} \frac{\partial q_2}{\partial (\mathbf{V}X_2)} \ldots \frac{\partial q_n}{\partial (\mathbf{V}X_n)} \right]_{\mathbf{V}X_1, \mathbf{V}X_2, \ldots, \mathbf{V}X_n} \in \mathbb{R}^{n \times n}, \quad \mathbf{Q}_m = \left[ \frac{\partial q_1}{\partial \mathbf{m}} \frac{\partial q_2}{\partial \mathbf{m}} \ldots \frac{\partial q_n}{\partial \mathbf{m}} \right]_{\mathbf{m} = \mathbf{m}^* - \hat{\mathbf{m}}} \in \mathbb{R}^{n \times m},
\]

\[
\mathbf{Q}_s = \left[ \frac{\partial q_1}{\partial \mathbf{s}} \frac{\partial q_2}{\partial \mathbf{s}} \ldots \frac{\partial q_n}{\partial \mathbf{s}} \right]_{\mathbf{s} = \mathbf{s}^* - \hat{\mathbf{s}}} \in \mathbb{R}^{n \times s}, \quad \mathbf{O}_n \in \mathbb{R}^{n \times 1}
\]

is a vector of higher-order terms. Rewrite (25), it can obtain

\[
\dot{\mathbf{Q}}^* = \dot{\mathbf{Q}} + \mathbf{Q}_V \dot{\mathbf{V}}_X + \mathbf{Q}_m \dot{\mathbf{m}} + \mathbf{Q}_s \dot{\mathbf{s}} + \mathbf{O}_n
\]  

Substituting (26) into (23), it is revealed that

\[
\bar{U} = \mathbf{W}^* \dot{\mathbf{Q}}^* + \varepsilon - \dot{\mathbf{W}} \dot{\mathbf{Q}} - U_x
\]

\[
= \mathbf{W}^* \dot{\mathbf{Q}} + \mathbf{W}^* \dot{\mathbf{Q}}_V \dot{\mathbf{V}}_X + \mathbf{W}^* \dot{\mathbf{Q}}_m \dot{\mathbf{m}} + \mathbf{W}^* \dot{\mathbf{Q}}_s \dot{\mathbf{s}} + \mathbf{O}_n + \varepsilon - \dot{\mathbf{W}} \dot{\mathbf{Q}} - U_x
\]

\[
= (\mathbf{W}^* - \dot{\mathbf{W}}) \dot{\mathbf{Q}} + (\mathbf{W}^* + \dot{\mathbf{W}}) \dot{\mathbf{Q}}_V \dot{\mathbf{V}}_X + (\mathbf{W}^* + \dot{\mathbf{W}}) \dot{\mathbf{Q}}_m \dot{\mathbf{m}} + (\mathbf{W}^* + \dot{\mathbf{W}}) \dot{\mathbf{Q}}_s \dot{\mathbf{s}} + \varepsilon - U_x + \mathbf{W}^* \mathbf{O}_n
\]

\[
= \dot{\mathbf{W}} \dot{\mathbf{Q}} + \mathbf{W}^* \dot{\mathbf{Q}}_V \dot{\mathbf{V}}_X + \mathbf{W}^* \dot{\mathbf{Q}}_m \dot{\mathbf{m}} + \mathbf{W}^* \dot{\mathbf{Q}}_s \dot{\mathbf{s}} - U_x + \mathbf{W}^* \mathbf{O}_n + \varepsilon
\]

\[
= \dot{\mathbf{W}} \dot{\mathbf{Q}} + \mathbf{W}^* \dot{\mathbf{Q}}_V \dot{\mathbf{V}}_X + \mathbf{W}^* \dot{\mathbf{Q}}_m \dot{\mathbf{m}} + \mathbf{W}^* \dot{\mathbf{Q}}_s \dot{\mathbf{s}} - U_x + \mathbf{W}^* \mathbf{O}_n + \varepsilon
\]  

where the uncertain term \( E = \mathbf{W}^* \dot{\mathbf{Q}} + \mathbf{W}^* \dot{\mathbf{Q}}_V \dot{\mathbf{V}}_X + \mathbf{W}^* \dot{\mathbf{Q}}_m \dot{\mathbf{m}} + \mathbf{W}^* \dot{\mathbf{Q}}_s \dot{\mathbf{s}} + \mathbf{W}^* \mathbf{O}_n + \varepsilon \) is assumed to be bounded by \( |E| < \psi \).

**Theorem 1:** Consider the nonlinear motor-mechanism coupling system represented by (12), if the SMNNCC law is designed as (21), in which the adaptation laws of the NN controller are designed as (28)–(31) and the robust controller is designed as (32) with the adaptive bound estimation algorithm shown in (33), then the tracking error dynamic trajectory can be always kept on the sliding surface, and the asymptotically stability can be guaranteed.

\[
\dot{\mathbf{W}} = \eta_t r(t) G(X,t) \dot{\mathbf{Q}}^T
\]  

where \( \eta_t \) is a positive constant.
where $\eta_1$, $\eta_2$, $\eta_3$, $\eta_4$ and $\eta_5$ are positive constants; $\text{sgn}(\cdot)$ is a sign function, and $\hat{\psi}(t)$ is the estimated value of $\psi$.

**Proof:** Define the following Lyapunov function candidate:

$$L_a(r(t), \hat{\psi}(t), \hat{W}, \hat{V}, \hat{m}, \hat{S}) = \frac{1}{2} r^2(t) + \frac{1}{2 \eta_1} \hat{W}^T \hat{W} + \frac{1}{2 \eta_2} \hat{V}^T \hat{V} + \frac{1}{2 \eta_3} \hat{m}^T \hat{m} + \frac{1}{2 \eta_4} \hat{S}^T \hat{S} + \frac{1}{2 \eta_5} \hat{\psi}^2(t)$$

where $\hat{\psi}(t) = \psi - \hat{\psi}(t)$ and $\text{tr}(\cdot)$ is the trace operator. Differentiating (34) with respect to time, and using (22) and (27), one can obtain

$$\dot{L}_a = r(t) \dot{r}(t) + \frac{1}{2} \hat{W}^T \dot{W} + \frac{1}{2} \hat{V}^T \dot{V} + \frac{1}{2} \hat{m}^T \dot{m} + \frac{1}{2} \hat{S}^T \dot{S} + \frac{1}{2} \hat{\psi} \dot{\psi} = r(t) G(X; t)[\dot{W} \dot{Q} + \dot{W} \dot{V} \dot{X}]$$

If the adaptation laws of the NN controller are chosen as (28)–(31) and the robust controller are designed as (32) with the adaptive bound estimation algorithm shown in (33), (35) can be rewritten as follows:

$$L_a = \frac{1}{2} r(t) E - \frac{1}{2} \hat{W}^T \hat{W} + \frac{1}{2} \hat{V}^T \hat{V} + \frac{1}{2} \hat{m}^T \hat{m} + \frac{1}{2} \hat{S}^T \hat{S} + \frac{1}{2} \hat{\psi} \dot{\psi} = \frac{1}{2} r(t) G(X; t)[\hat{W} \dot{Q} + \dot{W} \dot{Q} \dot{X}]$$

Since $L_a(r(t), \hat{\psi}(t), \hat{W}, \hat{V}, \hat{m}, \hat{S}) \leq 0$, $L_a(r(t), \hat{\psi}(t), \hat{W}, \hat{V}, \hat{m}, \hat{S}) \leq L_a(r(0), \hat{\psi}(0), \hat{W}, \hat{V}, \hat{m}, \hat{S})$, which implies $r(t)$, $\dot{r}(t)$, $\hat{W}$, $\hat{V}$, $\hat{m}$ and $\hat{S}$ are bounded. By using Barbalat’s lemma [20, 24], it can be shown that $r(t) \to 0$ as $t \to \infty$. As a result, the SMNNC system is asymptotically stable. Moreover, the tracking error of the control system, $e_{\theta}$, will converge to zero according to $\frac{1}{2} r(t) \to 0$.

According to the unavailable system states, the $G(X; t)$ in the tuning algorithms is reorganized as $G(X; t) \text{sgn}(G(X; t))$ in practical applications. Therefore, the adaptation laws of the NN controller shown in (28)–(31) and the adaptive algorithm for the upper bound of $|E|$ shown in (33) can be reorganized as follows:

$$\dot{\hat{W}} = \alpha_1 r(t) \text{sgn}(G(X; t)) \dot{Q}^T, \quad \dot{\hat{V}} = \alpha_2 r(t) \text{sgn}(G(X; t)) [\dot{X}, \dot{W} \dot{Q} \dot{X}]^T, \quad \dot{\hat{m}} = \alpha_3 r(t) \text{sgn}(G(X; t)) [\dot{W} \dot{Q} \dot{m}]^T$$

where the term $\eta |G(X; t)|$ is absorbed by the positive tuning constant $\alpha$. Consequently, only the sign of $G(X; t)$ is required in the design procedure, and it can be easily obtained from (8), (9) and (12). Moreover,
the adaptive algorithm for the upper bound of $|E|$ designed to adjust the upper bound on the uncertain term, which has the superiority of simple structure, can guarantee the tracking error to be zero. However, the guaranteed convergence of tracking error to be zero does not imply convergence of the estimated value of the upper bound to its optimal value. The persistent excitation condition [20, 24] should be satisfied for the estimated value to converge to its optimal value.

4. NUMERICAL SIMULATION AND EXPERIMENTAL RESULTS

For numerical simulations, the parameters of the flexible arm are given as

\[ L = 0.3 \text{ m}, \ b = 2.228 \times 10^{-2} \text{ m}, \ w = 2.36 \times 10^{-3} \text{ m}, \ A = 5.2581 \times 10^{-5} \text{ m}^2, \ c = 4.02 \times 10^{-3} \text{ m}, \ e = 1.498 \times 10^{-2} \text{ m}, \ \rho = 2700 \text{ kg/m}^3, \ I_b = 9.7618 \times 10^{-11} \text{ m}^4, \ I = 9.7618 \times 10^{-11} \text{ m}^4, \ g_r = 1, \ w_f = 4.26 \times 10^{-2} \text{ kg} \]

in which $b$, $w$ and $w_f$ are the width, high and nominal mass of the flexible arm. Moreover, the parameters of the motor system are

\[ K_T = 0.6732 N \cdot m / A, \ J_m = 1.32 \times 10^{-3} N \cdot m \cdot s^2, \ B_m = 5.78 \times 10^{-3} N \cdot m \cdot s / \text{rad} \]

In addition, the gains of the proposed control system are selected as

\[ \lambda = 0.6, \ \alpha_1 = 35, \ \alpha_2 = 35, \ \alpha_3 = 1, \ \alpha_4 = 1, \ \alpha_5 = 0.01 \]

All the control gains are chosen to achieve the superior vibration-free positioning and accurate tracking control performance in both simulation and experimentation considering the requirement of stability, limitation of control effort, and possible occurrence of uncertainties. Furthermore, a second-order transfer function of the following form with rise time of 0.63s is chosen as the reference model for the periodic step command

\[ \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2} = \frac{36}{s^2 + 12s + 36} \]

where $\xi$ and $w_n$ are the damping ratio (set at one for critical damping) and undamped natural frequency. On the other hand, when the command is a periodic sinusoidal reference trajectory, the reference model is set to be one. Two simulation cases due to periodic commands are addressed as follows:

\[ \ \text{Case 1: nominal case (} m = 0 \text{ kg)} \]
\[ \ \text{Case 2: parameter variation case (} m = 2.5 \times 10^{-2} \text{ kg)} \]

The NN in the proposed SMNNC system has two, twenty and one neuron at the input, hidden and output layers, respectively. Moreover, to test the learning ability of the SMNNC system, all the connecting weights in the NN controller are set to zero. The simulated results of the SMNNC system due to periodic step command at Case 1 and Case 2 are depicted in Fig. 5. Observing the tracking errors shown in Figs. 5b and 5f, favorable tracking responses can be obtained under all the simulated conditions. The little tracking errors in the first cycle are caused by the zero initial weights, moreover, there is no chattering phenomena existing in the control efforts owing to the on-line adjustment of the upper bound of the uncertain term. To test the effectiveness of the proposed SMNNC system with different reference trajectory, the simulated results, due to periodic sinusoidal command at Case 1 and Case 2, are depicted in Fig. 6. From the simulated results, no chattering phenomena exist in the control efforts owing to the on-line adjustment of the upper bound of the uncertain term, and perfect tracking responses can be obtained under the presence of parameter variation and different reference trajectory. It is worth noting that the transient tracking errors shown in Fig. 6 are larger than those in Fig. 5. According to the control gains of the SMNNC system which are chosen based on the periodic step command to achieve the best transient control performance, the powerful learning ability of the SMNNC system can be verified.
Some experimental results are provided here to further demonstrate the effectiveness of the proposed control system. The control algorithms are realized in a Pentium computer with a 2ms sampling interval. Two conditions are tested here; one is the nominal condition, that is without a tip mass, and the other is the parameter variation condition, that is with a $2.5 \times 10^{-2}$ kg tip mass. Figure 7 depicts the experimental results of the SMNNC system due to periodic step command at the nominal condition and parameter variation.
condition. From the experimental results, good tracking responses can be obtained after one cycle of on-line training mechanism due to zero initial weights. Moreover, the near total sliding motions shown in Figs. 7b and 7f confirm the absence of the reaching phase in the control system. In addition, the chattering phenomena in the control efforts are significantly reduced owing to the on-line adjustment of the upper bound of the uncertain term. To test the effectiveness of the proposed SMNNC system with different reference trajectory, the experimental results of the tracking responses, sliding surfaces, control efforts, and estimated bounds due to periodic sinusoidal command at the nominal condition and parameter variation condition are depicted in Fig. 8. From the experimental results, robust control performance with near total sliding motions can be obtained under the occurrence of parameter variation and different reference trajectory; moreover, the chattering phenomena are greatly reduced in the control efforts according to the on-line adjustment of the bound value in the robust controller.

Fig. 7. Experimental results of SMNNC system due to periodic step command: a) position response at nominal condition; b) sliding surface at nominal condition; c) control effort at nominal condition; d) estimated bound at nominal condition; e) position response at parameter variation condition; f) sliding surface at parameter variation condition; g) control effort at parameter variation condition; h) estimated bound at parameter variation condition
The mathematical derivation of system dynamics in Sec. II is provided to numerical simulation for verifying the effectiveness of the proposed SMNNC system in this study. However, there still exist unpredictable uncertainties in practical applications; therefore, model-based control design is not suitable for good control in the highly nonlinear motor-mechanism coupling system. Besides, the cost of this flexible arm equipped with an expensive vibration measure will be increased broadly such that it may not satisfy the cost requirement in industrial applications. Thus, the vibration states of the nonlinear system are assumed to be unmeasurable, i.e., only the actuator position can be acquired to feed into a suitable control system for stabilizing the vibration states indirectly. Though many modern control techniques [1-6] (e.g. sliding-mode control or adaptive control) have been published for dealing with the positioning problem of the flexible arm, the well-known or partial-known system dynamics and more than one type of sensor equipment are always required in the control process. According to the aforementioned limitations, it is reasonable to adopt
the NN control scheme according to the powerful approximation capability of NN. Moreover, observing the numerical simulation and experimental results, the proposed SMNNC system can, admittedly, yield favorable control performance without detailed system knowledge and vibration sensors.

5. CONCLUSIONS

This study has successfully demonstrated the application of a SMNNC system to the position control of a nonlinear mechanism system. The nonlinear mechanism used in this study is a flexible arm driven by a PM synchronous motor drive. The object of the control scheme is to achieve good tracking performance without the strict constraints and prior knowledge of the controlled system. The design procedure of the proposed SMNNC system was described in detail. Moreover, simulation and experimentation were carried out using periodic reference trajectory to test the effectiveness of the proposed control system. The major contributions of this study are: (i) the successful development of a SMNNC system, in which a NN controller is used to learn an equivalent control law as in the traditional sliding-mode control, and a robust controller is designed to ensure the near total sliding motion through the entire state trajectory without a reaching phase, (ii) an adaptive bound estimation algorithm was proposed to estimate the bound of the uncertain term, avoiding the chattering phenomena, and (iii) the successful application of the SMNNC system to control the motor-mechanism coupling system considering the existence of uncertainties.

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