Rheological Modelling of Caspian Pony Blood

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ABSTRACT

To study the rheological behaviour of Caspian pony blood which is a suspension of macromolecules in an aqueous medium, steady shear and small amplitude oscillatory shear experiments were conducted on thirty Caspian pony blood samples. The seven different viscoelastic constitutive equations viz., Giesekus, Thurston, four-constant Oldroyd, Williams-Bird, Spriggs, modified Spriggs and Bogue-White were examined. The viscosity data obtained from steady shear experiments and the dynamic viscosity and imaginary part of complex viscosity data obtained from oscillatory shear experiments were used to determine the model parameters. The performance of the models was studied by comparing their prediction for viscosity, dynamic viscosity and imaginary part of complex viscosity with those measured experimentally. As a result, Giesekus and Thurston models were found to be the best models and the Caspian pony blood was characterized by these models. The same experiments mentioned above were carried out on four blood samples of one Caspian pony, different only in hematocrit, in order to find its effect on rheological properties of Caspian pony blood. It was concluded that as the hematocrit of Caspian pony blood increased, the zero shear rate viscosity, the relaxation time, the retardation time and the very high shear rate viscosity increased and the mobility factor decreased.

Key Words: Caspian pony blood, rheology, viscoelastic, model, constitutive equation

INTRODUCTION

A thorough knowledge of the rheological properties of blood and its constituents is essential for understanding and modelling circulatory system dynamics and capillary transport phenomena in the body [1].

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Similarly in the design and development of extracorporeal devices such as artificial kidneys, blood oxygenators and blood pumps, the same knowledge is critical. Many of the major practical problems involved in artificial organ applications result in fact from the sensitivity of blood to the unfamiliar shear
stress imposed by such devices, that give rise to blood cell rupture and clotting problems.

From the rheological point of view, whole blood is a shear thinning non-Newtonian fluid that exhibits both viscous and elastic properties [2-4]. Blood is a suspension of red blood cells, white blood cells and platelets in a continuous aqueous medium called plasma. Since red blood cells constitute about 97% of the total particle volume of blood, they together with the plasma proteins especially fibrinogen are thought to be responsible for the rheological behaviour of blood [2].

Red blood cell consists of a large number of macromolecules, i.e., phospholipids, proteins, enzymes with different ranges of molecular weight. The central feature of the structure of red blood cell membrane is matrix formed by a double layer of phospholipids. This membrane surrounds a solution of proteins and electrolytes. It has the ability to maintain the membrane integrity while exhibiting extreme deformability under normal physiological circumstances [5].

The flow properties of blood depend primarily on the elastic behaviour of red blood cells [6]. The red blood cell changes its shape from biconcave disc at rest to ellipsoid under shear and the elliptical cell is oriented in a flow field with long axis parallel to the flow direction [5-6].

Since the abundance of red blood cells at normal hematocrit (volumetric percent of red blood cells), leaves little space for free cell motion or deformation without direct interaction with neighbouring cells, the tendency for interacting macromolecular cells to aggregate and undergo elastic deformation is high and is an important factor in blood viscoelastic behaviour [6].

Most of the rheological models used for blood have been in the form of power law [7-8] or Bingham plastic equations [9-12] such as Casson model [9] and Walburn-Schneck model [7]. However, blood is a viscoelastic fluid and its flow in arteries is pulsatile in which the elastic response becomes important. Therefore, the Casson [9] and Walburn-Schneck [7] equations, which do not account for the elastic nature, are inappropriate for the description of blood flow. In this work steady shear and small amplitude oscillatory shear experiments were conducted on CP blood samples.

The seven viscoelastic constitutive equations viz., four-constant Oldroyd [13], Williams-Bird [14], Spriggs [14], modified Spriggs [14], Giesekus [13, 15], Thurston [16] and Bogue-White [17-18] were examined and model parameters were estimated. Then blood samples of one CP with different hematocrit were prepared and used to study the effects of changing the hematocrit on the observed rheological behaviour.

THEORETICAL

The following seven viscoelastic constitutive equations were applied in this work:

**Four-constant Oldroyd Model (1)**

Four-constant Oldroyd model is a generalization of convected Jeffreys model to describe non-linear viscoelastic phenomena [13]. The material functions for this model are written as:

\[ \eta = \eta_0 (1 + \lambda_2 \lambda_3 \gamma^2) / (1 + \lambda_1 \lambda_3 \gamma^2) \]  
\[ \eta' = \eta_0 (1 + \lambda_1 \lambda_2 \omega^2) / (1 + \lambda_1^2 \omega^2) \]  
\[ \eta'' = \eta_0 \omega (\lambda_1 - \lambda_2) / (1 + \lambda_1^2 \omega^2) \]

where, \( \eta \) is the steady shear viscosity, \( \eta' \) the dynamic viscosity (or real part of complex viscosity), \( \eta'' \) the imaginary part of complex viscosity, \( \eta_0 \) the zero shear rate viscosity, \( \omega \) the angular velocity, \( \gamma \) the shear rate, \( \lambda_1 \) the relaxation time and \( \lambda_2 \) and \( \lambda_3 \) are time constants.

**Williams-Bird Model (2)**

Williams and Bird [14] simplified the Oldroyd model by assuming that normal stresses perpendicular to the flow direction are equal and the stress tensor is traceless. The material functions for this model are written as:

\[ \eta = \eta_0 (1 + 2/3 \lambda_1 \lambda_2 \gamma^2) / (1 + 2/3 \lambda_1^2 \gamma^2) \]
\[ \eta' = \eta_0 (1 + \lambda_1 \lambda_2 \omega^2 / (1 + \lambda_2^2 \omega^2)) \] (0 < \lambda_2 < \lambda_1) \tag{5} \\
\eta'' = \eta_0 \omega (\lambda_1 - \lambda_2 / (1 + \lambda_2^2 \omega^2)) \tag{6}

**Spriggs Model (3)**

Spriggs [14] developed a flexible model containing six constants, by non-linear modification of generalized Jeffrey's model. The material functions for this model are written as:

\[ \eta = \frac{\eta_0}{Z(\alpha_1)} \sum_{n=1}^{\infty} n^{-\alpha_1} \frac{1 + A \lambda_1 \lambda_2 n^{-(\alpha_1+\alpha_2)} \gamma \omega^2}{1 + B \lambda_1^2 n^{-2\alpha_1} \gamma^2} \] (7) \\
\[ \eta' = \frac{\eta_0}{Z(\alpha_1)} \sum_{n=1}^{\infty} n^{-\alpha_1} \frac{1 + \lambda_1 \lambda_2 n^{-(\alpha_1+\alpha_2)} \omega^2}{1 + \lambda_1^2 n^{-2\alpha_1} \omega^2} \] (8) \[
\eta'' = \frac{\eta_0}{Z(\alpha_1)} \sum_{n=1}^{\infty} n^{-\alpha_1} \frac{(n^{-\alpha_1} \lambda_1 - n^{-\alpha_2} \lambda_2) \omega}{1 + \lambda_1^2 n^{-2\alpha_1} \omega^2} \] (9)

where, \( A, B, \alpha_1 \) and \( \alpha_2 \) are adjustable parameters, and 

\[ Z(\alpha_1) = \sum_{j=1}^{\infty} j^{-\alpha_1} \]

**Modified Spriggs Model (4)**

Spriggs model was modified by changing the power of \( \gamma \) and \( \omega \) in the denominator of the expressions for \( \eta, \eta' \) and \( \eta'' \) from 2 to \( \beta \), a variable power [14].

**Giesekus Model (5)**

The above mentioned models are some extensions to the Jeffrey's model [13] formulated upon combining spring-dashpot elements to simulate the behaviour of material. However, Giesekus model is originated from the kinetic theory and is written as a superposition of solvent and polymer contributions \( \tau_s \) and \( \tau_p \) to the stress tensor [13, 15, 19]. The material functions for this model are written as:

\[ \eta = \eta_{\infty} + \sum_{n=1}^{\infty} \eta_{0,n} / [1 + (\gamma \lambda_n)^2]^{b} \] (15) \\
\[ \eta' = \eta_{\infty} + \sum_{n=1}^{\infty} \eta_{0,n} / (1 + \omega \lambda_n^2) \] (16) \[
\eta'' = \sum_{n=1}^{\infty} \omega \eta_{0,n} \lambda_n / (1 + \omega \lambda_n^2) \] (17)

where, \( \eta_{\infty} \) is the viscosity of terminating dashpot, \( \eta_{0,n} \) the ground state value of viscosity of the \( n \)th element, \( \lambda_n(=\eta / \lambda_0) \) the relaxation time of the \( n \)th element and \( b \) is a constant.

**Bogue-White Model (7)**

Savarmand et al. [18] modified the Bogue-White model and applied it to predict the rheological behaviour of solutions of polyacrylamide in a 50% mixture by weight of water and glycerine. The material functions for the model are written as:

\[ \eta = \sum_{n=1}^{\infty} G_n \lambda_{on} / (1 + b \lambda_n \gamma^2) \] (18) \[
\eta' = \sum_{n=1}^{\infty} G_n \lambda_{on}^2 / (1 + \lambda_n^2 \gamma^2) \] (19) \[
\eta'' = \sum_{n=1}^{\infty} G_n \lambda_{on}^2 \omega / (1 + \lambda_n^2 \omega^2) \] (20)

\( G_n \) is a parameter having dimension of stress, \( \lambda_{on} \) a
time constant, β a constant and β is an adjustable parameter.

EXPERIMENTAL

All the experiments were conducted under the ICSH guidelines on blood rheology [20]. The guidelines include, standardization of the blood collection procedure, laboratory processing of blood samples and measurement of hematological indices on rheological samples.

Fresh blood was drawn from the jugular vein of 45 young male CPs. Shear damage to red blood cells was avoided during blood collection by using a large-bore needle syringe. NaEDTA was used as an anticoagulant at a concentration of 1.8 mg/mL. The blood samples should have a hematocrit and erythrocyte (red blood cell) indices, particularly the mean cell hemoglobin concentration (MCHC) and the mean cell volume (MCV) within the normal reference range. Therefore, the following hematological and biochemical measurements were done on the blood samples in order to find the healthy CPs.

Hematocrit was obtained by centrifugation in a Hawksley microhematocrit centrifuge at 12,000 g for 5 min. Red blood cell count (RBC) and white blood cell count (WBC) were measured by Coulter counter method. Differential cell count (diff.) was measured by manual method. Hemoglobin (Hb) was measured by spectrophotometry after reaction with Drabkin. Total protein (TP) was determined in plasma by refractometry. Fibrinogen was determined by four steps: (1) heating the plasma at 56 °C for 3 min., (2) centrifuging it to remove the fibrin clot, (3) reading the refractive index of the remaining solution by refractometer and (4) subtracting this value from the refractive index of total protein in plasma. Albumin was measured by Bromocresol green dye binding method. MCHC and MCV were obtained from the following relations:

\[
\text{MCHC} = \frac{\text{Hemoglobin}}{\text{Hematocrit}} \times 100 \\
\text{MCV} = \frac{\text{Hematocrit}}{\text{RBC}} \times 10
\]

Among the studied CPs, 30 were chosen for rheological experiments. To study the effect of hematocrit on the blood rheology, just one CP was considered. Then blood samples of various hematocrit levels (14, 37, 52, 78%) were prepared by centrifuge separating the red blood cells from plasma and recombining them at desired concentrations. All of the above measurements were made at the Laboratory of Clinical Pathology of Veterinary College, Tehran University.

To perform steady shear experiments and small amplitude oscillatory shear experiments, Haake couette rheometer (RV100+CV100 with diameter of inner cylinder 13.91 mm, diameter of beaker 15 mm, cap size 0.545 mm and length of inner cylinder 32.3 mm) at the Laboratory of Rheology in the Faculty of Chemical Engineering, Amir Kabir University of Technology was used. The rheological experiments were divided in two parts: 1-Experiments were conducted on blood samples of 30 CPs. 2-Experiments were conducted on four samples of one CP with different hematocrit to find its effect on the rheological properties of CP blood. The steady shear experiments were done over a shear rate range of 0.3–1000 s⁻¹ [21]. The small amplitude oscillatory shear experiments were done over a frequency range of 0.24–20 Hz. The measurement temperature was 29± 0.1 °C. The precision of measured values was ±5%. All experiments were completed within 4 h of collecting the blood sample.

DETERMINATION OF THE MODEL PARAMETERS

The model parameters in the above mentioned constitutive equations were determined by minimizing the total sum of squares of errors:

\[
\sum_{i=1}^{n} e_i^2 \quad \text{where} \quad e_i = y_i - f_i
\]

in which \(y_i\) and \(f_i\) are the measured and calculated values of a property, i.e., non-Newtonian viscosity, \(\eta\), dynamic viscosity, \(\eta'\) and imaginary part of complex viscosity, \(\eta''\). Statistica and Table-curve softwares were used to estimate the parameters.
Figure 1. Evaluation of various rheological models for the CP blood behaviour with the following specification: H: 37 %, RBC: 9.7 (10^6/μL), WBC: 8.9 (10^3/μL), HB: 11.25 (g/dL), MCV: 38.1 (fL), MCHC: 30.4 %, TP: 8.8 (g/dL), Fibr.: 300 (mg/dL), albumin: 3 (g/dL), diff. (Segmented granule: 59, Lymphocyte: 38, Eosin granule: 1, Monocyte: 2, blood density: 1.047 (g/L)).

Figure 1 shows the sum of squares of errors obtained by using viscosity, dynamic viscosity and imaginary part of complex viscosity functions of the seven rheological models for one CP blood sample. Note that the exact value of sum of squares of errors obtained from the imaginary part of complex viscosity function of Bogue-White model (9.55×10^2) is beyond the scale of Figure 1. The estimated values of parameters of the seven rheological models for the same CP blood sample are given in Table 1. The range of the estimated values of parameters of the best rheological models i.e., Giesekus and Thurston together with the hematological and biochemical indices for 30 CPs is given in Tables 2 and 3. Giesekus and Thurston models were applied to rheological data of four-different hematocrit blood samples. The estimated values of parameters are given in Tables 4 and 5.

RESULTS AND DISCUSSION

It is seen from Figure 1 that the sum of squares of

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Oldroyd</th>
<th>Williams &amp; Bird</th>
<th>Spriggs n=2</th>
<th>Modified Spriggs n=2</th>
<th>Giesekus</th>
<th>Thurston n=1</th>
<th>Bogue-White n=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_0 (\text{Pa.s}) )</td>
<td>0.081</td>
<td>0.082</td>
<td>0.0835</td>
<td>0.0885</td>
<td>0.093</td>
<td>0.1016</td>
<td>0.74</td>
</tr>
<tr>
<td>( \lambda_1 (s) )</td>
<td>0.7825</td>
<td>0.417</td>
<td>0.493</td>
<td>0.515</td>
<td>0.632</td>
<td>0.83</td>
<td>9.9</td>
</tr>
<tr>
<td>( \lambda_2 (s) )</td>
<td>0.09</td>
<td>0.0345</td>
<td>0.016</td>
<td>0.0159</td>
<td>0.0291</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda_3 (s) )</td>
<td>0.172</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>( \lambda_{ad} (s) )</td>
<td></td>
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<tr>
<td>( \lambda_{aq} (s) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>( \alpha )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>3.355</td>
<td>3.269</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>13.106</td>
<td>46.84</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A )</td>
<td>1.765</td>
<td>1.759</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B )</td>
<td>0.686</td>
<td>0.718</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( G_1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.5</td>
</tr>
<tr>
<td>( G_2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.075</td>
</tr>
<tr>
<td>( b )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.494</td>
<td>1.3257</td>
<td>1.</td>
</tr>
<tr>
<td>( \beta )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.097</td>
<td></td>
<td>0.0046</td>
</tr>
<tr>
<td>( \eta_0 (\text{Pa.s}) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \eta_1 (\text{Pa.s}) )</td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>
Table 2. The range of Thurston and Giesekus model parameters for 30 CPs.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Thurston model</th>
<th>Giesekus model</th>
</tr>
</thead>
<tbody>
<tr>
<td>η₀ (Pa.s)</td>
<td>0.0048–0.0084</td>
<td>0.0590–0.23</td>
</tr>
<tr>
<td>η₀ (Pa.s)</td>
<td>0.0570–0.27</td>
<td>0.1094±0.0415</td>
</tr>
<tr>
<td>γ₀ = γ₀₁ + γ₀₂</td>
<td>0.0625–0.2753</td>
<td>0.0098–0.0899</td>
</tr>
<tr>
<td>λ₀ (s)</td>
<td>0.5200–1.72</td>
<td>0.0469±0.0259</td>
</tr>
<tr>
<td>B</td>
<td>0.4200–0.799</td>
<td>0.5240±0.1545</td>
</tr>
</tbody>
</table>

Table 3. The range of blood density, hematological and biochemical indices for 30 Cps.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Mean± SD</th>
<th>Parameter</th>
<th>Range</th>
<th>Mean± SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (%)</td>
<td>32–54</td>
<td>40.70±8.24</td>
<td>Fibr. (mg/dL)</td>
<td>200–400</td>
<td>275±76.6</td>
</tr>
<tr>
<td>RBC (10^6/µL)</td>
<td>4–15</td>
<td>9.72±2.87</td>
<td>Albumin (g/dL)</td>
<td>2.3–4.14</td>
<td>3.308±0.44</td>
</tr>
<tr>
<td>WBC (10^3/µL)</td>
<td>4.6–13.95</td>
<td>8.52±2.72</td>
<td>Differential cell count:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hb (g/dL)</td>
<td>11.25–18.5</td>
<td>14.53±2.28</td>
<td>Segmented granule</td>
<td>22–64</td>
<td>44±12.29</td>
</tr>
<tr>
<td>MCV (fl)</td>
<td>25.40–64.5</td>
<td>41.91±9.37</td>
<td>Lymphocyte</td>
<td>34–74</td>
<td>51.8±12.2</td>
</tr>
<tr>
<td>MCHC (%)</td>
<td>30.40–42.5</td>
<td>35.79±3.48</td>
<td>Eosin granule</td>
<td>1–8</td>
<td>2.7±1.95</td>
</tr>
<tr>
<td>TP (g/dL)</td>
<td>4–8.3</td>
<td>6.97±1.11</td>
<td>Monocyte</td>
<td>1–4</td>
<td>1.75±0.94</td>
</tr>
<tr>
<td>Blood density (g/L)</td>
<td>1.0101–1.083</td>
<td>1.0216±0.0264</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SD: Standard deviation

Errors is the highest for Bogue-White (n=2) model and is the lowest for Giesekus and Thurston (n=1) models. Figure 2 shows the observed values of viscosity vs. shear rate for the blood sample mentioned in Figure 1 and the predicted values by the models. It is seen that at low shear rates viscosity approaches a Newtonian region with a high value. At sufficiently high shear rates blood behaviour is similar to a Newtonian fluid with a constant low viscosity. Between these two regions, there is a region with pronounced non-Newtonian behaviour. This type of behaviour has been observed for suspensions of butadiene-styrene copolymer latex [22, 23] and for solutions of macromolecules [24, 25]. It is seen from this figure that Giesekus and Thurston models (n=1) can predict the observed values well, while Bogue-White model (n=2) is the most unsuitable one, showing a large deviation from the experimental data at high and low shear rates.

The above-observed behaviour is interpreted more fully in microscopic level as follows: the high concentration of red blood cells at normal condition of blood, leaves little space for any motion or deformation without direct interactions with neighbouring cells [26]. Therefore, there is a tendency for interacting macromolecular cells to aggregate and undergo elastic deformation. When a volume of normal blood
Table 4. The estimated values of parameters for Giesekus model at different hematocrit.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hematocrit (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14</td>
</tr>
<tr>
<td>$\eta_0$ (Pa.s)</td>
<td>0.0601</td>
</tr>
<tr>
<td>$\lambda_1$ (s)</td>
<td>0.527</td>
</tr>
<tr>
<td>$\lambda_2$ (s)</td>
<td>0.0187</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.4999</td>
</tr>
</tbody>
</table>

is at stasis, the cells tend to aggregate with random orientations [6, 27]. A large portion of blood plasma volume is trapped within the cell aggregates. As a result there is very little free plasma space or “rattling room” for free motion of the aggregates of red blood cells. This kind of cellular microstructure can be sheared by a small amount, slightly deforming the elastic structures and repositioning them within the “rattling room”. The stress-shear rate relation is linear and the viscosity is therefore constant. As the shear rate is increased, the red blood cells are progressively deformed, the larger aggregates will be disrupted, the trapped plasma will be released and as a result the viscosity decreases. At high shear rates disaggregation is complete and the red blood cells are aligned. The cells may be elongated and compacted into layers. At very high shear rates the cell stretching and compaction may be complete and the plasma dominates the rheological behaviour of blood.

Figures 3 and 4 show the observed values of low amplitude oscillatory shear tests (dynamic viscosity and imaginary part of complex viscosity vs. frequency) and the predicted values by the models for the same blood sample mentioned in Figure 1. It is also seen from these figures that Giesekus and Thurston models (n=1) can again predict the observed values well, while Bogue-White model (n=2) fails completely by predicting a very high unrealistic imaginary part of complex viscosity. It is noteworthy that the zero shear rate viscosity predicted by this model is also very different from the predicted values by the other models. Greater values of $n$, did not give rise to better prediction of $\eta$, $\eta'$ and $\eta''$ by Bogue-White model.

Because of the similarities found between the

Table 5. The estimated values of parameters for Thurston model at different hematocrit.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hematocrit (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14</td>
</tr>
<tr>
<td>$\eta_{01}$ (Pa.s)</td>
<td>0.06</td>
</tr>
<tr>
<td>$\eta_0$ (Pa.s)</td>
<td>0.0026</td>
</tr>
<tr>
<td>$\eta_0$ (Pa.s)</td>
<td>0.0626</td>
</tr>
<tr>
<td>$\lambda_1$ (s)</td>
<td>0.616</td>
</tr>
<tr>
<td>$b$</td>
<td>0.529</td>
</tr>
</tbody>
</table>
rheological behaviour of blood and those of some polymeric solutions [22-25, 28, 29], the above non-linear viscoelastic models of value for polymeric solutions were examined in order to explain large deformations in flow of blood, together with a special model developed for human blood by Thurston [16], Oldroyd [13], Williams and Bird [14], Spriggs [14] and modified Spriggs [14] constitutive equations are linear in terms of stress but non-linear in terms of rate of strain tensor. They are unable to describe the material functions especially at high shear rates or high frequencies. There is not a theoretical background for adding second order terms in the rate of strain tensor in Jeffrey’s model or generalized Jeffrey’s model in order to establish the above non-linear viscoelastic models. In the case of Bogue and White model [14, 18], which was formulated upon the second order integral theory of Coleman and Noll, the memory function was empirically modified to be dependent on the strain rate invariants to include the non-linearity. One serious defect of such models that include the strain rate in the memory function is that they are not simplified properly to a linear viscoelastic model in the limit of small displacement gradients.

Giesekus constitutive equation [13, 15] was originated from a clear systematic procedure taken from the kinetic theory and has attracted considerable attention in the recent years. It is non-linear in terms of stress and rate of strain tensors and the inclusion of \( \tau \) term gives material functions that are more realistic than those obtained by Oldroyd or other constitutive equations. Large decreases in viscosity and normal stress coefficients with increasing shear rate are plausible by applying this model. It is seen that at both high and low shear rates and frequencies this model fits very well the experimental blood viscosity and viscoelasticity data. Thurston model [16] is a generalization of Maxwell model in which a shear rate degradation function serves to adjust the model elements to the flow conditions of blood. This model fitted the CP blood rheological data well and can be used to explain small displacement gradients in flow of blood.

To characterize the CP blood from rheological point of view, we repeated the experiments for 30 CPs and obtained the range of Giesekus and Thurston model parameters (Table 2). The hematological and biochemical indices were also measured (Table 3) and found to be in the range reported by Atyabi [30].

The plot of viscosity vs. shear rate for different hematocrit is shown in Figure 5 along with the predicted values by the best models, i.e., Thurston and Giesekus. As the hematocrit increases from 14 to 78% the viscosity also increases at a constant shear rate. The trend of the curves also indicates an increase in the zero shear rate viscosity \( (\eta_0) \) and the very high shear rate viscosity \( (\eta_\infty) \) with an increase in the hematocrit. The plots of dynamic viscosity \( (\eta') \) and imaginary part of complex viscosity \( (\eta'') \) vs. frequency at four different hematocrits are shown in Figures 6 and 7. The plots of these three figures indicate strong dependency of \( \eta, \eta' \) and \( \eta'' \) on the hematocrit. The estimated values of parameters given in Tables 4 and 5 for Giesekus and Thurston models show that as the hematocrit increases, the relaxation time \( (\lambda_1) \) and the retardation time \( (\lambda_2) \) increase and the mobility factor \( (\alpha) \) decreases.
These results may be explained more fully as followed. The relationship between the relative motion and generating force may be quite generally described by a tensorial drag coefficient. The tensorial drag coefficient depends on the entanglement density, itself being a function of the configuration tensor. Giesekus [19] introduced a nonisotropic tensorial drag coefficient or the reciprocal of it (tensorial mobility) which governs the motions of the structural elements of the molecules relative to their surroundings under the influence of elastic tractions. Giesekus stated that when entanglement density increases, the hydrodynamic drag coefficient increases and therefore the mobility factor decreases. The relaxation time has a reciprocal relation to the mobility factor, α. It is known that as the hematocrit of blood increases the interaction of red blood cells and therefore the entanglement density increases [27]. In view of occurrence of orientation and alignment of compacted cells under shear, the idea of non-isotropic tensorial drag coefficient may be justified and applicable.

Because of the similarity existing between the hematological and biochemical indices of horse and human [31], one might use the proposed rheological models to simulate their blood flow behaviour as well.

CONCLUSION

From the steady shear viscosity and small amplitude oscillatory shear data it is clear that CP blood which is a suspension of macromolecules in an aqueous medium, behaves as a viscoelastic fluid. Among the seven viscoelastic models studied in this work, Giesekus and Thurston models can predict the rheological behaviour of CP blood satisfactorily. The viscosity, dynamic viscosity and imaginary part of complex viscosity show strong dependence on the hematocrit. Furthermore, as the hematocrit is increased, the zero shear rate viscosity, the relaxation time, the retardation time and the very high shear rate viscosity are all increased, while the mobility factor is decreased.

The two successful rheological models along with Navier-Stokes equation can simulate the blood flow in vivo and in vitro, and predict the pressure gradient-flow rate relationship, the shear stress distribution and the velocity profile inside the arteries.
SYMBOLS AND ABBREVIATIONS

A: Parameter defined in Spriggs model  
B: Parameter defined in Spriggs model  
CP: Caspian pony  
Diff: Differential cell count  
Fibr: Fibrinogen (mg/dL)  
Gn: Parameters defined in Bogue-White model  
H: Hematocrit (%)  
Hb: Hemoglobin (g/dL)  
Ks: Spring constant  
MCHC: Mean cell hemoglobin concentration (%)  
MCV: Mean cell volume (fl)  
RBC: Red blood cell count (x 10^6 /µL)  
TP: Total protein (g/dL)  
WBC: White blood cell count (x 10^3 /µL)  
b: Constant defined in Bogue-White model and Thurston model  
f: Function defined in eqn (11)  
j: An index  
n: An index

GREEK SYMBOLS

α: Mobility factor  
α1: Parameter defined in Spriggs model  
α2: Parameter defined in Spriggs model  
β: Parameter defined in Bogue-White and modified Spriggs model  
ε: Error  
η0: Zero shear rate viscosity (Pa.s)  
η0,n: Ground state value of viscosity of the nth element (Pa.s)  
η∞: Viscosity of the terminating dashpot (or the very high shear rate viscosity) (Pa.s)  
η: Steady shear rate viscosity (Pa.s)  
η': Dynamic viscosity (or real part of complex viscosity) (Pa.s)  
η'': Imaginary part of complex viscosity (Pa.s)  
γ: Shear rate (1/s)  
λ1: Relaxation time (s)  
λ2: Retardation time in Giesekus model and time constant in the other models (s)  
λ∞: Time constant (s)  
λ3: Time constant (s)  
τs: Solvent contribution to the stress tensor (Pa)  
τp: Polymer contribution to the stress tensor (Pa)  
ω: Angular velocity (1/s)  
χ: Function defined in eqn (12)

REFERENCES


