

ON m -GENERALIZED INVERTIBLE OPERATORS ON BANACH SPACES

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ABSTRACT. A bounded linear operator S on a Banach space X is called an m -left generalized inverse of an operator T for a positive integer m if

$$T \sum_{j=0}^m (-1)^j \binom{m}{j} S^{m-j} T^{m-j} = 0,$$

and it is called an m -right generalized inverse of T if

$$S \sum_{j=0}^m (-1)^j \binom{m}{j} T^{m-j} S^{m-j} = 0.$$

If T is both an m -left and an m -right generalized inverse of T , then it is said to be an m -generalized inverse of T .

This paper has two purposes. The first is to extend the notion of generalized inverse to m -generalized inverse of an operator on Banach spaces and to give some structure results. The second is to generalize some properties of m -partial isometries on Hilbert spaces to the class of m -left generalized invertible operators on Banach spaces. In particular, we study some cases in which a power of an m -left generalized invertible operator is again m -left generalized invertible.

1. INTRODUCTION AND PRELIMINARIES

Throughout this paper, X shall denote a complex Banach space, and $\mathcal{L}(X)$ shall denote the algebra of all bounded linear operators on X . We denote X by

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