Application of Chaos Theory in Hazardous Material Transportation

Abbas Mahmoudabadi¹, Seyed Mohammad Seyedhosseini²

Received: 2012.04.03   Accepted: 2012.10.22

Abstract
Risk factors are generally defined and assigned to road networks, as constant measures in hazmat routing problems. In fact, they may be dynamic variables depending on traffic volume, weather and road condition, and drivers’ behavior. In this research work, risk factors are defined as dynamic variables using the concept of chaos theory. The largest Lyapunov exponent is utilized to determine the presence of chaos for road accident rates. Risk factors with the property of chaotic behavior are considered to solve hazmat routing problem using a developed mathematical model. Evaluation process has been done based on travel distance which mainly represents travel cost, as well as results show that the application of chaos to define dynamic risk factor is an appropriate method to solve hazmat routing problem, comparing to constant measures of risks.

Keywords: Risk analysis, chaos theory, hazmat routing problem, transport distance and cost.
Application of Chaos Theory in Hazardous Material Transportation

1. Introduction

Hazardous materials (hazmat) are classified into nine classes of explosives, gases, flammable liquid, flammable solids, oxidizing substances, toxic substances, radioactive materials, corrosive substances and miscellaneous dangerous goods [Environmental Health & Safety, 2011]. Determining the safest path in hazmat transportation is an important issue because of their accident impacts and also wide range of carrying goods [Erkut and Gzara, 2008]. Although hazmat transportation covers a large part of economic activities in industrialized countries [Zografos and Androutsopoulos, 2004], management of hazardous materials is an extremely complex issue involving a multitude of environmental, engineering, economic, social and political concerns [Diaz-Banez Gomez and Toussain, 2005]. Determining the route for hazardous materials transportation, known as hazmat routing problem, is usually known as a two-sided problem in which local authorities are primarily interested in minimizing public risk and the carriers are concerned about minimizing transport costs [Erkuta and Alpb, 2007].

Based on the above concern, bi-objective function is observed in the literature while a number of methods are utilized to solve hazmat routing problem. A dynamic programming model was extended to solve bi-objective formulation of routing problem in large scale by Serafini [Serafini, 2006]. Erkut and Gzara considered passing trucks along cities as the main concern to solve hazmat transportation problem using a heuristic method [Erkut and Gzara, 2008]. Shariat and colleagues developed a model to assign the optimal truck flow within transportation network that minimizes the weighted combination of objective [Shariat Mohaymany and Khodadadian, 2008].

Societal risk and out-of-pocket costs were also studied as the main objectives to solve risk-based routing optimization problem by Bonvicini and Spadoni [Bonvicini and Spadoni, 2008]. A two-stage model was developed using both routing and scheduling daily departure times for hazmat trucks by Carotenuto et al [Carotenuto et al, 2007]. Nielsen and Colleagues [Nielsen, Pretolani and Andersen, 2005] developed K shortest path algorithms in stochastic and dynamic networks and focused on identifying the exact solution which leads to computational challenges in large network. Travel time and the consequence measures were considered uncertain and stochastic by Dadkar and colleagues because of their depending on characteristics such as visibility, traffic volume and activity patterns [Dadkar, Jones and Nozick, 2008]. Several defining methods can be observed for hazmat transport risks in the literature. For example, they are quantified with path evaluation function [Erkut and Ingolfsson, 2005] and probability of terrorist attacks is considered as stochastic variable by Dadkar and colleagues [Dadkar, Nozick and Jones, 2010]. Population risk and travel time were defined by probability distribution function to solve the routing of hazmat transportation by Shariat and colleagues, due to the existing variation of traffic condition [Shariat Mohaymany and Khodadadian, 2008]. Weather conditions were considered as a main parameter in hazmat transport problem routing by Akgun and colleagues [Akgun, Parekh, Batta and Rump, 2007]. Qiao and colleagues used risk factors in hazmat transportation based on the frequency of accidents and developed a fuzzy model to estimate frequency of hazmat transport accidents using a qualitative approach where driver, road, and truck characteristics are presented as membership function regarding experts’ experiences. [Qiao, Keren and Mannan, 2010]. The main objective of this paper is to consider the concept of chaos theory to define dynamic risk factors in intercity road network, as well as utilizing them in hazmat transportation. An iterative procedure and corresponding mathematical model have been proposed to obtain the safest path regarding the most frequent outlined paths where dynamic risk factors are updated using one-dimensional logistic map equation. In other words, this paper attempts to use chaotic property of risk factors in hazmat routing problem.

This paper is presented in seven sections. After introduction, a brief description of chaos theory and its applications in road transportation are given. Checking the presence of chaos using experimental data is presented in section 3, followed by methodology definition and developing mathematical model in sections 4 and 5, respectively. Description of case study and the process of running the model using two different methods of constant and dynamic risk factors are discussed in section 5 and comparing results and sensitivity anal-
analysis in section 6. Summary, conclusion and recommendations for future study are presented in the last section.

2. Chaos Theory

For the first time, the concept of chaos theory was introduced by Edvard Lorenz [Gleick, 1987]. He found the chaotic attractions in complex systems of weather forecasting when he entered different seeds of starting points in his computer program. Chaos is one type of behaviour exhibited in nonlinear dynamical systems. Some sudden and dramatic changes in nonlinear systems may give rise to the complex behaviour, called chaos [Lawrence, Lin and Huang, 2003]. A nonlinear system is said to be chaotic, if it exhibits sensitive dependence upon initial conditions. It may happen that small differences in the initial conditions produce great differences in the final outputs. A small error in the former will produce an enormous one in the latter [Yuan, Yuan and Zhang, 2002]. Chaotic systems are known as complicated ones, so in order to represent chaotic systems understandably it is a common way to use term ‘phase space reconstruction’, where chaotic system is transformed into another space [Sugihara and May, 1990]. Chaos theory is commonly applied for short-term prediction because of its property of ‘sensitive dependence upon initial condition’, which hampers the success of long-term prediction [Lawrence, Lin and Huang, 2003]. Further studies show that chaos is useful in many disciplines such as high-performance circuits and devices, collapse prevention of power systems and information processing [Mingjun and Huanwen, 2004], that may have the same behaviour comparing to traffic incidents. It has also been widely applied in various fields of science, particularly in the area of traffic flow theory. Researchers showed that the dynamics of traffic flow time series data exhibit chaotic phenomena [Lawrence, Lin and Huang, 2003]. Frazier and colleague [Frazier and Kockelman, 2004] believe that chaos theory is used to analyse highly complex systems. In our research scope, as transportation systems are complex entities and legal and social constraints may bind behaviour, chaos may be useful for transportation applications particularly in short-time prediction in transportation systems to allow a researcher to more accurately predict human actions enabling them for system evolution.

3. Presence of Chaos

Determining the presence of chaotic behaviour is a very important step. Calculating the largest Lyapunov exponent is a more common technique to determine the presence of chaos which measures the divergence of nearby trajectories. As the system evolves, the sum of a series of values (in each dimension) will converge or diverge. Lyapunov exponents measure the rate of convergence or divergence in each dimension. Thus, if the largest Lyapunov exponent exceeds zero, the system is chaotic [Frazier and Kockelman, 2004]. Equation (1) is used to determine the largest Lyapunov exponent where S(t) is the sum of a series of observed values and S′(t) is its nearest neighbour in time (t). N is the number of available data points and Δt is defined as time step.

\[ \lambda_{\text{max}} = \lim_{N \to \infty} \frac{1}{N \Delta t} \sum_{t=0}^{N-1} \ln \left( \frac{|S(t + \Delta t) - S'(t + \Delta t)|}{|S(t) - S'(t)|} \right) \]  

(1)

In this research work, if Δt=1, means that time step is a day (24 hours) and if Δt=7, means time step is a week (seven days). As Δt increases, the value of the exponent will, theoretically, converge to its true value.

The ratio of daily fatal accidents in Fars province to the whole accidents in all over the country (Iran) is considered as risk factor. As observed in figure 1, primed and non primed data points are distinct but close, so equation (1) is appropriate to be used.

Time step is set to 24 hours for experimental data and risk factors are limited into open interval of (0,1). Accident risk for each edge is calculated by multiplying the number of daily accidents in Fars province (Iran) by risk factor belonging to each edge [Ehsani, Azar and Saffarzadeh, 2010]. While time step is considered as 24 hours, the well-known chaotic equation of logistic map [Loa and Cho, 2005] is applied to generate risk factors in each iteration. Risk generating equation based on logistic map equation is defined as equation (2) where P(t) is risk factor regarding the whole accidents [Mingjun and Huanwen, 2004].

\[ P(t + 1) = K \times P(t) \times (1 - P(t)) \]  

(2)

Figure 1 shows the ratio of accidents during 365 days in the scope of case study that will be presented in section 6. In order to estimate parameter K in equation (2),
Application of Chaos Theory in Hazardous Material Transportation

minimizing mean square errors between real observations and expected values of data, has been used, while logistic map equation is utilized to generate model risk factors. In this case, seed is accident ration of the first day, therefore others have been calculated based on the risk related to their own previous days. Using 365 set of experimental data $\lambda_{\text{max}}$ corresponding to equation (1) is equal to 0.046 when $\Delta t = 1$, and 0.024 when $\Delta t = 7$. K has been estimated to 3.6219 means that there is chaotic behaviour in experimental data because largest Lyapunov exponent in experimental data exceeds 0 [Frazier and Kockelman, 2004].

4. Methodology

Due to the existing chaotic behaviour, risk factors assigned to the edges will be different. Risk factors are also dynamically updated in the process of solving problem, so an iterative procedure has been designed where total risk is considered as objective function while distance (may be represented by cost) is defined as criteria for evaluation and sensitivity analysis. The proposed procedure including the following steps is also shown in figure 2.

1- Initialize the parameters including network structure, risk factors, number of iterations and starting point (Different starting points are used for sensitivity analysis).

2- Set risk factors for the next day (iteration) using logistic map equation.

3- Run mathematical model and keep the selected path and its measures.

4- If the number of iterations is met, stop and go to step 5, otherwise go to step 2.

5- Choose the most frequent path as the best path.

Figure 2. Overall view of defined iterative procedure

5. Developing the Mathematical Model

There are two kinds of risk factors associated with accident probabilities. One factor corresponds to the rate of road accidents in Fars province, that can be estimated using some other estimation techniques such as artificial neural network [Mahmoudabadi, 2010], and the other is...
Abbas Mahmoudabadi, Seyed Mohammad Seyedhosseini

the accident risk factor associated with each edge, based on the local expert experiences that are the results of converting linguistic variables to crisp values [Ehsani, Azar and Saffarzadeh, 2010]. Eventually, final risk factors are obtained by multiplying the ratio of accidents in Fars province and accident probability in each edge. It is assumed that graph G is predefined road network and risk factor which is assigned to each edge on graph G, is calculated by equation (3):

\[ PA_{ij}(t) = RF(t) \times EO_{ij}(t) \] (3)

where:

- \( PA_{ij}(t) \) is the probability of accidents and consequent impacts for edge \((i,j)\), \( RF(t) \) is risk factor identified as the ratio of accidents in Fars, and \( EO_{ij}(t) \) is hazmat accident risk in edge \((i,j)\), corresponding to the experts experiences and outlined by linguistic variables converted to crisp value [Ehsani, Azar and Saffarzadeh, 2010].
- If \( D_{ij} \) is the length of edge \((i,j)\), objective function identified by equation (4) is to minimize total risk in road network, where \( X_{ij} \) = 1 if edge \((i, j)\) is in the selected path and 0 otherwise.

\[ \text{Min } Z(t) = \sum_{(i,j) \in G} PA_{ij}(t) \cdot D_{ij} \cdot X_{ij} \] (4)

Subject to:

\[ (i,j),(j,i) \in G \] (5)

\[ \sum_{(i,j) \in G} X_{ij} - \sum_{(j,i) \in G} X_{ji} = \begin{cases} -1 & \text{if } j = S \\ 1 & \text{if } j = D \\ 0 & \text{otherwise} \end{cases} \quad \forall (i,j) \in G \] (6)

\[ PA_{ij}(t) = RF(t) \times EO_{ij}(t) \] (7)

\[ RF(t) = K \ast RF(t-1) \left(1 - RF(t-1)\right) \] (8)

\[ EO_{ij}(t) = K \ast EO_{ij}(t-1) \left(1 - EO_{ij}(t-1)\right) \] (9)

Equation (5) guarantees that two opposite directions for each edge are available. Equation (6) corresponds to the nodes located in the path. This equation guarantees that there is no loop or break out in the path unless nodes are defined as origin or destination. Equation (7) is extracted by equation (3) which calculates risk factors of each edge in graph G in iteration t. Iteration (t) represents current day used in the process of finding the best solution. In each iteration, risk factors are updated based on parameters and previous iteration, while one-dimensional logistic map [Mingjun and Huawen, 2004] is used for updating the risk factors, equations (8) and (9) guarantee that risk factors belonging to the edges will be updated during iterations.

6. Running Model and Discussion

6-1 Case Study

Fars, the second largest province in Iran, shown in figure 3, is nominated as case study. Risk factors for each edge were provided as crisp values using linguistic data forms filled out by local experts. It consists of 57 nodes and 80 two-way edges. Each edge contains length and initial risk factors outlined based on local experts’ experiences which are consisting of accident, population, environmental and infrastructure risks obtained by AHP process [Ehsani, Azar and Saffarzadeh, 2010]. Some of nodes are borders that connect the case study location to other provinces. The number of vehicles entering and departing Fars province are considered as supplies and demands, respectively and internal supplies and demands are defined as domestic nodes. Low risk factor represents low probability of accident and consequent impacts independent from the edge length and vice versa [Ehsani, Azar and Saffarzadeh, 2010]. Proposed procedure has been applied in the above predefined Fars road network and five pairs of origin – destination commonly used in rural transportation fuel in Fars province.

6-2 Comparison Results

As mentioned before, different risk factors may cause to find different paths, so considering the most frequent path as the best one is quite understandable. In other words, the most frequent path can represent general situation and different risk factors during the time study of a year. The other objective function is to minimize total risk, so the selected best path would be the safest path that can satisfy local authorities to choose the safest paths for routing and scheduling transportation of hazardous materials. If the most frequent path satisfies transport companies, it would be the best path which also would satisfy decision makers’ requirements. Travel cost is one of the main concerns in the process of finding hazmat transport path, which is usually computed by transport companies; therefore travel distance which represents travel cost is the best criterion for evaluation process.

Five origin-destination routes are selected for evaluation. Travel distances are calculated in both methods of traditional and proposed procedure using constant and
Application of Chaos Theory in Hazardous Material Transportation

dynamic risk factors, respectively. Results are shown in table 1, where travel distance is the main criterion. Table 1 also shows statistical measures of distance in evaluation process. It can be revealed that applying chaos factors cause to achieve shorter paths of hazmat transport with lower cost of transportation rather than using constant risk factors. It is concluded that because of the existing chaotic behaviour of road accidents [Loo and Cho, 2005], application of chaos theory is more capable to find the safest path in hazmat routing problem, with lower cost. Comparing the maximum and minimum lengths for defined routes in table 1 shows that the edges with the higher norm of expected risk factors may be selected, because risk factors during the year are not necessarily considered as higher and vice versa. Results also revealed that there are some differences between selected paths in short and long origin-destination paths using the concept of chaos theory. When long trips such as (33-57) are considered, more satisfactory results will be outlined, but in short trips such as (14-45), chaos theory cannot play a significant role to find the safest path. The main reason is existing limitation of routes in short origin-destination routes.

6-3 Sensitivity Analysis
It is believed that chaos theory is commonly applied for short-term prediction [Lawrence, Lin and Huang, 2003] so initial condition may change results. It is important to check this concern in this proposed method. Two origin-destination pairs of 33-57 and 6-48 have been selected for checking procedure sensitivity upon initial condition. Outlined paths and distances, using dynamic risk factors by proposed procedure, corresponding to O-D (33-57) and (6-48) are shown in tables 2 and 3. Three different initial risk factors named “seed” have been used to check dependency on initial conditions. For each seed, results for three different iterations (5, 10 and 15) have been discussed, and outlined paths are shown. For example

<table>
<thead>
<tr>
<th>Method</th>
<th>Measure</th>
<th>(4-51)</th>
<th>(14-45)</th>
<th>(52-58)</th>
<th>(33-57)</th>
<th>(6-48)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td></td>
<td>250</td>
<td>126</td>
<td>606</td>
<td>656</td>
<td>507</td>
</tr>
<tr>
<td>Proposed Method</td>
<td>Minimum</td>
<td>237</td>
<td>126</td>
<td>529</td>
<td>530</td>
<td>436</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>260</td>
<td>126</td>
<td>616</td>
<td>700</td>
<td>517</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>246</td>
<td>126</td>
<td>567</td>
<td>631</td>
<td>479</td>
</tr>
<tr>
<td></td>
<td>Most Frequent</td>
<td>237</td>
<td>126</td>
<td>567</td>
<td>563</td>
<td>456</td>
</tr>
</tbody>
</table>

*One - dimensional logistic map equation (This is general for of Equation)

Table 1. Path length using the mathematical model after solving problem (KM)
the first row in table 2 shows the outlined path for origin-destination (33-57) in iteration 5, while the ratio of Fars Province accidents to the whole accidents is considered as 0.05, therefore seed is entered as 0.05. Results in tables 2 and 3 reveal that the model is dependent upon initial condition, so it is recommended that proposed procedure should be used for solving hazmat routing problem in short time prediction when updated data are available. But in the state of solving hazmat routing problem in long-term prediction, the most frequent path should be selected as the safest path. It is not necessary to check dependency upon initial condition in short-length paths because there are no different alternatives for routes, and consequently not significantly different from outlined paths on constant and dynamic risk factors.

In general, it is far from local authorities’ points of view to determine the safest path regularly or daily, so regarding the concept of depending upon the initial condition, the most frequent path is the much better path instead of one path being nominated as the best solution.

7. Summary and Conclusion

In this research work, the basic principle of hazmat routing problem and attributes, chaos theory and its applications on transportation are briefly presented. Checking the presence of chaos on intercity accidents reveals that risk factors in road safety can be chaotic variables. Because of existing chaotic behaviour on road accidents, risk factors are dynamically being updated according to the one-dimensional logistic map equation. An iterative procedure combined with a developed mathematical model has been proposed to solve hazmat routing problem considering dynamic risk factors. Fars, the second largest province in Iran, including 57 nodes and 80 edges, has been used for running model and checking evaluation process.

While paths that are outlined based on minimizing total risk and travel distance, are considered as criteria, results reveal that utilizing the concept of chaos theory is capable to find better routes or paths rather than using constant risk factors. Sensitivity analysis approved that outlined paths are dependent upon the initial condition of risk factors and iteration, so it is concluded that applying chaos theory will be useful to find the best paths in road hazmat transportation in short-time prediction.

For solving hazmat routing problem in long-term prediction, it is suggested to select the most frequent path as the safest or best path for hazmat transportation.

For researchers who are interested in further studies, it is recommended to focus on estimating risk factors and combining them with other effective variables and opening an interesting idea in emergency situation.

Table 2. Outlined paths considering different initial conditions for O-D (33-57)

<table>
<thead>
<tr>
<th>Seed</th>
<th>Iteration</th>
<th>Path</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>5</td>
<td>33 - 33 - 31 - 30 - 40 - 41 - 14 - 13 - 51 - 56 - 57</td>
<td>563</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Seed</th>
<th>Iteration</th>
<th>Path</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>5</td>
<td>33 - 33 - 31 - 30 - 40 - 41 - 14 - 13 - 51 - 56 - 57</td>
<td>563</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Seed</th>
<th>Iteration</th>
<th>Path</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>5</td>
<td>33 - 32 - 34 - 38 - 42 - 45 - 46 - 13 - 51 - 56 - 57</td>
<td>563</td>
</tr>
</tbody>
</table>

Table 3. Outlined paths considering different initial conditions for O-D (6-48)

<table>
<thead>
<tr>
<th>Seed</th>
<th>Iteration</th>
<th>Path</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>5</td>
<td>6 - 5 - 4 - 55 - 54 - 53 - 51 - 50 - 49 - 47 - 48</td>
<td>517</td>
</tr>
<tr>
<td>10</td>
<td>6 - 5 - 4 - 7 - 9 - 11 - 13 - 46 - 45 - 47 - 48</td>
<td>436</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>6 - 5 - 4 - 55 - 54 - 53 - 51 - 50 - 49 - 47 - 48</td>
<td>517</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Seed</th>
<th>Iteration</th>
<th>Path</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>5</td>
<td>6 - 5 - 4 - 55 - 54 - 53 - 51 - 50 - 49 - 47 - 48</td>
<td>507</td>
</tr>
<tr>
<td>10</td>
<td>6 - 5 - 4 - 55 - 54 - 53 - 51 - 50 - 49 - 47 - 48</td>
<td>517</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>6 - 5 - 4 - 7 - 9 - 11 - 13 - 46 - 45 - 47 - 48</td>
<td>436</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Seed</th>
<th>Iteration</th>
<th>Path</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>5</td>
<td>6 - 5 - 4 - 55 - 54 - 53 - 51 - 50 - 49 - 47 - 48</td>
<td>507</td>
</tr>
<tr>
<td>10</td>
<td>6 - 5 - 4 - 7 - 9 - 11 - 13 - 46 - 45 - 47 - 48</td>
<td>436</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>6 - 5 - 4 - 7 - 9 - 11 - 13 - 46 - 45 - 47 - 48</td>
<td>436</td>
<td></td>
</tr>
</tbody>
</table>
8. References


Abbas Mahmoudabadi, Seyed Mohammad Seyedhosseini


