TLM Modeling of Microwave Cooking by Consideration of Temperature Effects on Dielectric Properties

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Abstract

The interaction of microwave with the food is mainly based on its dielectric properties, which can change with temperature. Here dielectric properties of some different kinds of materials are introduced over determined range of temperature. A combined electromagnetic and thermal diffusion model using Transmission line modeling (TLM) is introduced and the power dissipation within the food is computed. Temperature distribution within a food are then generated using this data and then with this amount of temperature, after passing each time step the new amount of dielectric constant and loss factor are determined and considered for the rest of calculations.

Keywords: Microwave Cooking, Transmission Line Modeling, Dielectric Constant, Heat.

1 Introduction

Due to the non-uniform heating nature of microwave cooking, there exists a serious concern over complete elimination of photo gens in the food. There are points where little or no heating effect is experienced there so knowing the positions of these cold spots is important. At a work by Gunasekaran [1] the relationship between dielectric properties, fat level, and temperature at microwave frequency was determined. In another work by Desai [2] the TLM method is used for modelling both electromagnetic fields and thermal diffusion. This numerical measuring instrument is ideal because it does not deform the electromagnetic field or affect the temperature obtained. first the electromagnetic power distribution inside the heated objects is found and then this is used as a source in the thermal simulation. In this study we introduce the relationship between dielectric properties and temperature changes for some different kinds of materials, such as water, Ground Beef with different fat levels, and frozen Broccoli. Then we used these results of the dielectric change patterns in modeling the microwave cooking using TLM.

The paper is divided into nine further sections: section 2 introduces microwave cooking; section 3 reviews the effect of dielectric properties; section 4 describes the effect of temperature on dielectric properties; section 5 introduces the TLM method and gives
electromagnetic model results; section 6 describes the heat diffusion model; section 7 combines model considering the effect of temperature on dielectric properties; section 8 draws the paper to conclusion.

2 Microwave Cooking

In microwave heating, the heat is generated due to the molecular friction between dipole molecules of water and other ingredients. Since water is easy to stimulate and most of the food products contain water, microwave heating is highly suitable for cooking. Thus microwave penetrates food easily and heats the food from inside out. Curnerte [3] has shown that microwave ovens produce an uneven heating effect because of the standing wave patterns inside the cavity, so foods of poor thermal conductance should not be placed continuously in cold spots because, if a temperature sufficient to kill micro-organisms is not achieved throughout the heating food, there is a possibility of food poisoning [2]. There are many factors that determine how an object will heat when subjected to microwave radiation. These include the geometry of the cavity in which heating takes place and the geometry and size of the object and its electromagnetic and thermal parameters [4]. Dielectric properties are one of these parameters that are specially considered here.

3 Effect of Dielectric Properties

A factor that is highly important during microwave processing is dielectric properties of the material. The interaction of microwave with the food is mainly based on its dielectric properties which can change with temperature [5 and 6]. Therefore determination of dielectric properties of food with respect to temperature becomes critical.

Metals are good electrical conductors and good reflectors of microwave. They are not heated by microwave. Materials such as dielectrics are categorized under electrical insulators and are good absorber of microwave energy. Heat is generated in the dielectric primarily through absorption of microwave energy and hence the dielectric properties, depends on microwave frequency, and the current produced temperature [7]. Absorption characteristics of a food can also be changed by some other factors such as food composition or physical state of water in the food.

Microwave heating is mostly involves the rotation of dipolar molecules. The dielectric properties of a food depend on the interaction of electromagnetic energy with the materials. The dielectric properties such as dielectric constant and the dielectric loss factor play a major role in microwave heating. The capability of food to store electric-field energy is dielectric constant. The dielectric loss factor measures the ability of food material to dissipate electrical energy as heat [8]. Therefore, the amount of dipolar molecules significantly affects the heating. When the dipolar food components are not evenly distributed in the food, uneven heating can be expected. Obviously, the difference in dielectric activity is a common problem in foods with more than one ingredient [9].
4 Effect of Temperature on Dielectric Properties

The change of dielectric properties is determined at first for water and then for Ground Beef patties at 4%, 9% and 20% fat levels and at last for frozen broccoli.

A Water

The research has done in Londen South Bank University [4] about water dielectric properties is led to obtain the graphs that can show the relationship between dielectric constant and loss factor, at different microwave frequencies. The applied field potential (E, volts) of electromagnetic radiation as could be seen in figure 1 is given by,

\[ E = E_{\text{max}} \cos(\omega t) \]  

(1)

Where \( E_{\text{max}} \) is the amplitude of the potential, \( \omega \) is the angular frequency in radians.second\(^{-1}\) and \( t \) is the time in seconds. If the polarization lags behind the field by the phase (\( \delta \), radians) then the polarization (P, coulombs) varies as,

\[ P = P_{\text{max}} \cos(\omega t - \delta) \]  

(2)

Where \( P_{\text{max}} \) is the maximum value of the polarization. Hence the current (I, amperes) varies as,

\[ I = \frac{dP}{dt} = -\omega P_{\text{max}} \sin(\omega t - \delta) \]  

(3)

The power (P, watts) given out as heat is the average value of (current \( \times \) potential). This is zero if there is no lag (\( i.e. \) if \( \delta = 0 \)), otherwise,

\[ P = 0.5 P_{\text{max}} E_{\text{max}} \omega \sin(\delta) \]  

(4)

It is convenient to express the dielectric constant in terms of a complex number dielectric permittivity (\( \varepsilon_r^* \)) defined as,

\[ \varepsilon_r^* = \varepsilon_r' - i\varepsilon_r'' \]  

(5)

Figure 1 applied field potential (E) and the polarization (P)
The angle $\delta$ as the phase difference (lag) between the electric field and the resultant polarization of the material.

Where $\varepsilon_r'$ is the ability of the material to be polarized by the external electric field, $L_f$ (the loss factor) quantifies the efficiency with which the electromagnetic energy is converted to heat and $i = \sqrt{-1}$. This equation is visualized in figure 2 by considering the total current is the vector sum of the charging current and the loss current; the angle $\delta$ as the phase difference (lag) between the electric field and the resultant (orientation) polarization of the material.

$$\tan(\delta) = L_f/\varepsilon_r'$$  \hspace{1cm} (6)

The terms ($\varepsilon_r^*, \varepsilon_r', L_f$) are all affected by the frequency of radiation; the relative permittivity ($\varepsilon_r'$, dielectric constant) at low frequencies ($\varepsilon_S$, static region) and at high (visible) frequencies the ($\varepsilon_\infty$, optical permittivity) are the limiting values. The relative permittivity changes with the wavelength (and hence frequency) as,

$$\varepsilon_r' = \frac{\varepsilon_S - \varepsilon_\infty}{1 + (\omega \eta/\lambda_S)^2} + \varepsilon_\infty$$  \hspace{1cm} (7)

Where $\varepsilon_S$ is the relative permittivity at low frequencies (static region), and $\lambda_S$ is the critical wavelength (maximum dielectric loss).

$$\varepsilon_r^* = \frac{\varepsilon_S - \varepsilon_\infty}{1 + (\omega \eta/\lambda_S)^2} + \varepsilon_\infty$$  \hspace{1cm} (8)

$$\tau = \frac{4\pi r^3}{kT}$$

Where $\tau$ is the relaxation time, a measure of the time required for water to rotate also considered as the delay for the particles to respond to the field change or for reversion after disorientation, and $r$ is the molecular radius, $k$ is the Boltzmann constant and $\eta$ is the viscosity. The maximum loss occurs when $\omega = 1/\tau$. For water at 25°C, $\tau$ is 8.27 ps and $r$ is half the (diffraction-determined) inter-oxygen distance (1.4 Å).
The dielectric loss factor (Lf) increases to a maximum at the critical frequency. The dielectric loss factor of water is plotted at figure 3. Note that the shifts in the dielectric properties with temperature rises to **maxima in the temperature behavior at constant frequency**. Plotted opposite are equations derived for pure water over the range for -20°C ~ +40°C, extrapolated (dashed lines) to indicate trends. The equations that generate these curves involve 15 optimized parameters.

\[
Lf = \frac{(\varepsilon_r - \varepsilon_\infty)(\varepsilon_r/\varepsilon_\infty)}{1 + (\varepsilon_r/\varepsilon_\infty)^2}
\]

\[
\varepsilon_r = \varepsilon_\infty - 2H_NC + \varepsilon_m + \varepsilon_a
\]

\[
Lf = \frac{(\varepsilon_0 - \varepsilon_\infty)(\varepsilon_0/\varepsilon_\infty)}{1 + (\varepsilon_0/\varepsilon_\infty)^2}
\]

Dissolved salt depresses the dielectric constant dependent on its concentration (C) and the average hydration number of the individual ions (H_N). The dielectric loss is increased by a factor that depends on the conductivity (\(\Lambda S \text{ cm}^2 \text{ mol}^{-1}\); \(S = \text{ siemens} = \text{ mho}\)), concentration and frequency. It increases with rise in temperature and decreasing frequency.

\[
Lf = \frac{(\varepsilon_0 - 2H_NC) - \varepsilon_\infty(\varepsilon_0/\varepsilon_\infty)}{1 + (\varepsilon_0/\varepsilon_\infty)^2} + \frac{AC}{1000\varepsilon_\infty}
\]

Figure. 4
Dielectric constant at different fat levels at 2450 MHz
Figure 5
Dielectric loss factor at different fat levels at 2450 MHz

B Ground Beef

N.Gunasekaran [1] determined the dielectric properties of Ground Beef by measurements on an open-ended, coaxial line with copper conductors, connected to a Network analyzer. The relationship among dielectric properties, fat level, and temperature was obtained.

As can be seen in figure 4 and 5, in 2450 MHz dielectric loss factor and dielectric constant boost with increases in temperature, at temperature below freezing point. Above freezing point, dielectric constant decreases with increase in temperature. Dielectric loss factor remains almost constant with varying temperature.

C Frozen Broccoli

A Network Analyzer is used to help in understanding effect of plant parts and temperature on dielectric properties of frozen broccoli [1]. As shown in figure 6 and 7, at 2450 MHz dielectric loss factor and dielectric constant increase with increase in temperature below 0°C. Above 0°C, dielectric constant decreases with increase in temperature. Dielectric loss factor increases with temperature at 2450 MHz; it decreased as the temperature increases and became almost constant after 45°C. The dielectric constant and the dielectric loss factor of florets were lower than that of stems.

Figure 6
Dielectric properties of Broccoli florets at 2450 MHz
5 TLM Method and Electromagnetic Field Model

TLM was first described by Johns and Beurle as a numerical method of finding EM field patterns within waveguides [10]. Electrical engineers have an initiative understanding of how electrical circuits work and the significance of each circuit component. They are therefore more comfortable with circuit models than with the more abstracts mathematical models normally used. This is the basis of Transmission-Line Modeling (TLM) method [5]. TLM is a method of analysis represents a true computer simulation of wave propagation in the time domain [3].

It can be used as a tool to understand the effect of various factors influencing the microwave cooking. Here we need an electromagnetic (EM) field model which produces a map of the power dissipated within the food and a thermal diffusion model which takes the power from the EM model and uses it to compute the temperature distribution within the food as the cooking progresses [2]. Desai [2] introduced the electromagnetic field model and heat diffusion model in a loaded microwave cooking, using TLM. We use the results equations here within a typical UK domestic oven operating at 2.45 GHz.

Maxwell’s equations, which describe how EM radiation propagates through space, have been modeled using TLM for a number of years [3, 10-12]. By comparing the two-dimensional scalar wave equation, derived from Maxwell’s equation, with the approximate equation for a lumped component equivalent of a two-dimensional transmission line network. It has been pointed out at figure 8.

\[
V = EU_{VE} \quad L = \mu_0 \mu_r \Delta \quad 2G = \omega \varepsilon_r \varepsilon_0 \Delta \quad 2C = \omega \varepsilon_r \varepsilon_0 \Delta \quad Z = \sqrt{2}\mu_0/\varepsilon_0
\]

(11)

Capacitance is provided by open-circuited transmission-line stubs with a round trip time equal to the time step. These effectively store pulses reflected into them by the scattering process and return the pulses for the next iteration. The required capacitance that is calculated [2] is,

\[
C_S = \varepsilon_0(\varepsilon_r-1) \Delta L
\]

(12)

The stub's impedance, required for the scattering matrix is given by [2],

\[
Z_S = \Delta t/2C_S
\]

(13)
Figure. 8
Lumped circuit equivalent of 2D electromagnetic field node and associated transmission lines

And the Scattering matrix is calculated [2] for a loaded microwave cooking in network, using thevenin equations equivalent circuit for each transmission line including lumped component equivalent. The results is described by,

\[
\begin{bmatrix}
V_1' \\
V_2' \\
V_3' \\
V_4'
\end{bmatrix} = \frac{1}{Y_s + 4Y + G'} \begin{bmatrix}
a & b & b & c \\
b & a & b & c \\
b & b & a & c \\
b & b & b & d
\end{bmatrix} \begin{bmatrix}
V_1' \\
V_2' \\
V_3' \\
V_4'
\end{bmatrix}
\]

(14)

Where,
\(Y = 1/Z\), \(Y_s = 1/Z_s\), \(G' = 2G\Delta L\), \(a = (Y + G' + Y_s)\), \(b = 2Y\), \(c = 2Y_s\), \(d = 4Y + G' - Y_s\) and \(C\) (capacitance), \(L\) (inductance) and \(G\) (conductance) are the distributed (per unit line length) parameters. \(E\) is the transverse electric field component at position \((x, y)\), \(\omega\) is the angular frequency of the wave, \(\mu_0\) is the permeability of a vacuum, \(\mu_r\) is the relative permeability of the medium, \(\varepsilon_0\) is the permittivity of a vacuum, \(\varepsilon_r\) is the relative permittivity of the medium and \(\varepsilon_{ir}\) is the imaginary component of permittivity.

6 Heat Diffusion Model

By TLM method, explicit and unconditional stability has been applied in many areas. These have ranged from modeling water diffusion in white rice [13 and 14] to the modeling of semiconductor fuses [14]. The approximate two-dimensional transmission-line equation derived from the lumped-component equivalent of the mesh as figure 8 is,

\[
V = TU_{VT} \\
(RAL) = U_{RK}/2K_T\Delta L \\
(CAL) = \rho C_P\Delta L^3 U_{CC}/2 \\
I = P \Delta L^3 U_{IP}
\]

(15)

\(T\) is the temperature at a point \((x,y)\), \(p\) is the density of the material, \(C_P\) is the specific heat capacity, \(K_T\) is the thermal conductivity and \(P\) is the power dissipated per unit volume within the material that is obtained from EM model. \(U_{VT}\) has units of degrees per volt, \(U_{RK}\), \(U_{CC}\) and \(U_{IP}\) are the appropriate unity constants used to ensure dimensionally correct equations. It can be shown that the power dissipated in the electromagnetic model becomes the current source value in the heat model shown in Figure 9.
The scattering matrix is calculated [2] using components values of the network and thevenin equivalent is described by,

\[
\begin{bmatrix}
V_1' \\
V_2' \\
V_3' \\
V_4'
\end{bmatrix} = \frac{1}{2R + Z} \begin{bmatrix}
2R - Z & Z & Z & Z \\
Z & 2R - Z & Z & Z \\
Z & Z & 2R - Z & Z \\
Z & Z & Z & 2R - Z
\end{bmatrix} \begin{bmatrix}
1 \\
1 \\
1 \\
1
\end{bmatrix} + \frac{1}{4} \begin{bmatrix}
1 \\
1 \\
1 \\
1
\end{bmatrix}
\]  

The algorithm that is used [1] for simulating combined model is modified for considering dielectric properties of foods and shown in Figure 10.
7 Conclusion

Dielectric properties of some ingredients which are mainly used in the food industries and microwave oven cooking is studied and measured. These parameters are then employed as a base to introduce the interaction of microwave field and the heat diffusion in foods by utilizing the TLM method. It has been shown that, dielectric parameters and electromagnetic energy dissipation are both vary with the temperature distribution all over the material. The combined electromagnetic and thermal diffusion model using transmission line modeling (TLM) is then described with more mathematical details and by a versatile computer algorithm. The power dissipated within the food is also computed.

References