Clamped-Free Non Homogeneous Magneto Electro Elastic Plate of Polygonal Cross-Sections with Hydrostatic Stress and Gravity

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ABSTRACT
In this article, the influence of hydrostatic stress and gravity on a clamped-free non-homogeneous magneto electro elastic plate of polygonal cross sections is studied using linear theory of elasticity. The equations of motion based on two-dimensional theory of elasticity are applied under the plane strain assumption of pre-stressed and gravitated magneto electro elastic plate of polygonal cross-sections composed of non-homogeneous isotropic material. The frequency equations are obtained by satisfying the boundary conditions along the irregular surface of the polygonal plate using Fourier expansion collocation method. The complex roots of the frequency equations are obtained by secant method. The numerical computations are carried out for triangular, square, pentagon and hexagon cross sectional plates. Graphical representation is given for the various physical variables via gravity and different edge boundaries and its characteristics are discussed. This result can be applied for optimum design of concrete plates with polygonal cross sections.

Keywords: Stress-strain relation; Non-homogeneous; Pre-stressed and gravitated poly plate; FECM.

1 INTRODUCTION

The hydrostatic concrete solid plates are used in the construction to increase the inner tensile strength. It is widely used in the reduction of structural thickness with the improvement for longer span. The pre-stressed floor slabs consists of bracing wire strings, steel bars are laid over the reinforcement, which are having the longitudinal property. The most advantage of these types of floor slabs is large available span with minimal amount of support. These floor slabs are used in the structural engineering such as multi story car parking, residential, super market etc. Akbar [1] discussed various physical properties of multi layered, mechanical loaded cylinders. Mohammad Arefi et al. [2] presented the thermo elastic analysis of FG cylinder subjected to axially variable thermal and mechanical loads. Khoshgoftar et al. [3] used shear deformation theory to derive the analytical solution of functionally graded

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cylinder with finite length thick cylinder under non-uniform pressure. Mohammed Arefi and Rahimi [4] discussed the thermo elastic behavior of clamped-clamped functionally graded cylinder under internal pressure also calculated the SCF due to clamping. Later, Mohammad Arefi [5] carried out the analytical investigation on the nonlinear behavior of the functionally graded piezoelectric cylinder along the thickness direction. He gave the nonlinear result, which justifies the importance of nonlinear analysis used in the sensors and actuator. Abbas et al. [6] used complex Fourier series to analyze the non-symmetric thermo elasticity of functionally graded cylinder and obtained the result that the considerable effect of loading on the non-symmetric behavior of the FG cylinder. Rahimi [7] introduced the term additional energy to the exact energy functional of a functionally graded piezo electrical rotating cylinder and obtained that the electrical potential is proportional to angular velocity. Nagaya [8, 11] derived a method for finding the solution of vibration problems on an arbitrary cross sectional plate. He also discussed the dispersion of the elastic waves in the cylindrical bars with the polygonal cross sections. Hutchinson [9] discussed the axisymmetric flexural waves over a thick circular plate. The effect of non-homogeneity and boundary conditions of a functionally graded circular plate on the Winkler-Pasternak foundation were investigated by Mohammed Arefi and Allam [10,12]. Also he presented the nonlinear analysis of functionally graded square plate with piezoelectric layers resting on the Winkler-Pasternak foundation and gave the comparison between linear and nonlinear responses of the considered system. Chakraverty et al. [13] studied the flexural vibrations of non-homogeneous elliptic plates. Tanigawa [14] presented some basic thermoelastic problems for non-homogeneous structural materials. Annibale et al.[15] derived the stability of the discrete mechanical system which is controllable by the piezo electric nature. Selvamani [16] analyzed the elasto dynamic wave propagation in a piezo electric plate which is having the effect of fluid. Bin et al. [17] analyzed the wave propagation in non-homogeneous magneto-electro-elastic plates. Chen et al. [18] worked on free vibration of non-homogeneous transversely isotropic magneto-electro-elastic plate. Li [19] discussed the magneto electro elastic multi-inclusion and inhomogeneity problems and their applications in composite materials. Kong et al.[20] presented the thermo-magneto-dynamic stresses and perturbation of magnetic field vector in a non-homogeneous hollow cylinder. Rajneesh [21] discussed various behavior of the porous magneto electro elastic medium. Pan[22] and Pan and Heyliger [23] analyzed the three-dimensional behavior of magneto electro elastic laminates under simple support boundary conditions. Pan and Han [24] studied the exact solution for functionally graded and layered magneto-electro-elastic plates. Feng and Pan[25] discussed the dynamic fracture behavior of an internal interfacial crack between two dissimilar magneto-electro-elastic plate. Rajneesh [26] solved the wave propagation in a slightly stretched elastic solid material. Selim [27] discussed the dissipation of the torsional wave in a prestressed cylinder. Rajneesh [28] studied the problem of propagation of wave and the reflection of plane waves under the initial stress on the generalized transversely isotropic half space which is insulated thermally. Akbarov [29] presented the numerical results on the influence of the initial compression on the three layered hollow cylinders along the direction of torsional wave propagation. Kakar[30] discussed the magneto elastic torsional surface waves in the prestressed fiber reinforced medium. Wave propagation in compressed materials with reinforcement in preferred direction subjected to gravity and initial compression reads from Kakar [31]. Kumari [32] investigated the edge wave propagation on the visco elastic initially stressed plate. Wilson[36] investigated the propagation of elastic waves with strain energy in a pre stressed plate and also derived the dispersion equation. De [33] modified the classical equations of motion using Biot’s theory for the elastic wave propagation in a homogeneous isotropic elastic solid medium with the assumption that the gravitational force creates an initial stress naturally. Dey [34] used the damped equation in the complex form for the effect of torsional waves in a prestressed cylinder. Chen [35] discussed the exact solution of waves in the pre stretched electro active cylinders. Zhang[37] discussed the effects of initial stress on the frequency variables and solved the guided wave propagation under gravity in the single directed plate. Lofty [38] solved the effect of the gravity and analyzed the energy dissipation of fiber reinforced thermo elastic. He analyzed also the magnetic effect on the considered material. He represented the graph with the damping and the initial stress. Ahamed [39] analyzed and gave the determinant form of equation the impact of the gravity on wave propagation in a granular medium. Hou [40] used the general solutions of transversely isotropic electro magneto thermo elastic material and constructed the three dimensional Green’s function for a steady point heat source in the semi infinite transversely isotropic electro magneto thermo elastic material.

We have formulated the problem on the non-homogeneous wave propagation in a prestressed magneto piezoelectric polygonal plate. The plate possesses the gravitational force and the plate is kept in the magnetic field. The solution of the problem is derived using the Bessel function by considering the convenient form to uncouple the terms. The double Fourier series and the collocation methods are used to obtain the frequency equations for the polygonal shape of the plate. The numerical results and graphical representations are presented for the impact of different edge conditions and the gravitational force on physical variables.
2 FORMULATION OF THE PROBLEM

The considered non homogeneous hydrostatic magneto electro poly plate is having the stress displacement and the constitutive stress-strain relations for linear elastic medium in polar cylindrical coordinates \((r, \theta, t)\) in the absence of body force from Kakar [30] as follows:

\[
\Pi_{r,r} + r^{-1} \Pi_{r,\theta,\theta} + r^{-1} \left( \Pi_{r,r} - \Pi_{\theta,\theta} \right) - p_s \left[ u_{r,r} + \frac{u_{r,r}}{r} - \frac{u_{\theta,\theta}}{r^2} \right] + \rho g v_r = \rho u_{r,t} \tag{1}
\]

\[
\Pi_{r,\theta,\theta} + r^{-1} \Pi_{\theta,\theta,\theta} + 2r^{-1} \Pi_{r,\theta} - p_s \left[ u_{\theta,\theta} + \frac{u_{\theta,\theta}}{r} - \frac{u_{\theta,\theta}}{r^2} \right] - \rho g v_\theta = \rho u_{\theta,t} \tag{2}
\]

where \(\Pi_{r,r}, \Pi_{\theta,\theta}, \Pi_{\theta,\theta}\) the stress components and \(\rho\) are is the mass density of the material, \(p_s\) denotes the prestressed factor (when \(P_s < 0\) is hydrostatic tension and \(P_s > 0\) represents hydrostatic compression) and \(g\) is the gravitational force. \(u_r, u_\theta\) are the displacement components.

The electric conduction equation is given by [41],

\[
C_{r,r} + r^{-1} C_{r,\theta} + r^{-1} C_{\theta,\theta} = 0 \tag{3}
\]

The magnetic conduction equation is given by [41],

\[
N_{r,r} + r^{-1} N_{r,\theta} + r^{-1} N_{\theta,\theta} = 0 \tag{4}
\]

where \(C_r, C_\theta\) and \(N_r, N_\theta\) are electric displacements and magnetic displacement components. The mechanical, electrical and magnetic stress strain relation for the isotropic material is given as:

\[
\Pi_{rr} = \lambda \left( e_{rr} + e_{\theta\theta} \right) + 2\mu e_{rr} \tag{5}
\]

\[
\Pi_{\theta\theta} = \lambda \left( e_{rr} + e_{\theta\theta} \right) + 2\mu e_{\theta\theta} \tag{6}
\]

\[
\Pi_{r\theta} = 2\mu e_{r\theta} \tag{7}
\]

\[
C_r = \varepsilon_{11} E_r + m_1 H_r \tag{8}
\]

\[
C_\theta = \varepsilon_{11} E_\theta + m_1 H_\theta \tag{9}
\]

and

\[
N_r = m_1 E_r + \mu_1 H_r \tag{10}
\]

\[
N_\theta = m_1 E_\theta + \mu_1 H_\theta \tag{11}
\]

In which

\[
E_r = -E_r \quad \text{and} \quad E_\theta = r^{-1} E_\theta \quad H_r = -H_r \quad H_\theta = r^{-1} H_\theta \tag{12}
\]
where \( \lambda, \mu \) are Lame’s constants, \( \varepsilon_1 \) is the dielectric constant and \( m_{11}, \mu_{11} \) are the electro-magneto material coefficients and magnetic displacements respectively. \( E_r, E_\theta \) are the electric potentials, \( H_r, H_\theta \) are the magnetic potentials. The strain \( e_{ij} \) can be represented by displacement as:

\[
e_{rr} = u_{r,r}
\]

\[
e_{\theta\theta} = r^{-1} \left( u_r + u_{\theta,\theta} \right)
\]

\[
e_{r\theta} = u_{\theta,r} - r^{-1} \left( u_\theta - u_{r,\theta} \right)
\]

The stress equation of motion is obtained in the following form

\[
\Pi_r = (\lambda + 2\mu)u_{r,r} + \lambda r^{-1} \left( u_r + u_{\theta,\theta} \right)
\]

\[
\Pi_{\theta\theta} = \lambda u_{r,r} + (\lambda + 2\mu)r^{-1} \left( u_r + u_{\theta,\theta} \right)
\]

\[
\Pi_{r\theta} = 2\mu \left( u_{\theta,r} - r^{-1} \left( u_\theta - u_{r,\theta} \right) \right)
\]

Since the material properties vary in the direction of thickness, the material constants can be taken with the rational number ‘\( m \)’ and the non-homogeneous form of the equation of motion is obtained by the following substitution

\[
\lambda = \lambda r^{2m}, \mu = \mu r^{2m}, m_{11} = m_{11} r^{2m}, \varepsilon_{11} = \varepsilon_1 r^{2m}, \rho = \rho r^{2m}
\]

Upon using the Eqs. (16)-(18) together with Eq.(19) in Eqs.(1)-(4), we can get the following displacement equation of motions for non-homogeneous material

\[
(\lambda + 2\mu - p_s) \left( u_{r,r} + r^{-1} u_{r,r} - r^{-2} u_r \right) + (2\mu - p_s) r^{-2} u_{r,\theta,\theta} + r^{-1} \left( \lambda + 2\mu \right) u_{\theta,\theta,\theta} - \rho g v_r = \rho u_{r,tt}
\]

\[
(2\mu - p_s) \left( u_{\theta,r} - r^{-1} u_{\theta,r} - r^{-2} u_\theta \right) + r^{-2} \left( \lambda + 2\mu - p_s \right) u_{\theta,\theta,\theta} + r^{-1} \left( \lambda + 4\mu + 2\mu \right) u_{r,\theta,\theta} + r^{-1} \left( 4\mu + 4\mu - p_s \right) u_{\theta,r,\theta,\theta} + 2\mu r^{-2} u_{r,\theta,\theta} + \rho g v_\theta = \rho u_{\theta,tt}
\]

\[
\varepsilon_{11} \left( E_{rr} + r^{-1} E_r + r^{-2} E_{\theta\theta} \right) + m_{11} \left( H_{rr} + r^{-1} H_r + r^{-2} H_{\theta\theta} \right) + 2mr^{-1} \left( \varepsilon_{11} E_{rr} + m_{11} H_{rr} \right) = 0
\]

\[
m_{11} \left( E_{rr} + r^{-1} E_r + r^{-2} E_{\theta\theta} \right) + \mu_{11} \left( H_{rr} + r^{-1} H_r + r^{-2} H_{\theta\theta} \right) + 2mr^{-1} \left( m_{11} E_{rr} + \mu_{11} H_{rr} \right) = 0
\]

3 SOLUTION OF THE PRESTRESSED AND GRAVITATED POLY PLATE

The recent Eqs.(20)-(23) are coupled partial differential equations and it can be uncoupled by considering the following solution of the form

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\[ u_r (r, \theta) = \sum_{n=0}^{\infty} \mathcal{E}_n \left[ (r^{-1}\psi_{n,\theta} - \phi_{n,r}) + (r^{-1}\overline{\psi}_{n,\theta} - \overline{\phi}_{n,r}) \right] \] (24)

\[ u_\theta (r, \theta) = \sum_{n=0}^{\infty} \mathcal{E}_n \left[ (r^{-1}\phi_{n,\theta} - \psi_{n,r}) + (r^{-1}\overline{\phi}_{n,\theta} - \overline{\psi}_{n,r}) \right] \] (25)

\[ E (r, \theta) = \sum_{n=0}^{\infty} \mathcal{E}_n \left( E_n + \overline{E}_n \right) \] (26)

\[ H (r, \theta) = \sum_{n=0}^{\infty} \mathcal{E}_n \left( H_n + \overline{H}_n \right) \] (27)

where \[ \mathcal{E}_n = 1/2 \quad \text{for} \quad n = 0 \quad \text{and} \quad \mathcal{E}_n = 1 \quad \text{for} \quad n \geq 1. \]

By introducing the non dimensional quantity \[ \Omega^2 = \frac{\rho^2 \omega^2 (a+b)}{2\lambda} \]

where \[ a \quad \text{and} \quad b \] are the inner and outer radius of the polygonal plate \( \omega \) is the angular velocity and Substituting Eqs.(24)-(27) in Eqs.(20)-(23), we get the following decoupled equations in the following form

\[ (\lambda + 2\mu - p_s) \nabla^2 \phi_n + m r^{-1} (2\lambda + \mu - p_s) \phi_{n,r} - 2mr^{-2} \lambda \phi_n - \rho(gv_r - \phi_{n,\mu}) = 0 \] (28)

\[ (2\mu - p_s) \nabla^2 \psi_n + (4\mu - p_s)m \left( r^{-1}\psi_{n,\theta} - r^{-1}\psi_n \right) + \rho g \psi_\theta = 0 \] (29)

\[ \varepsilon_{11} \nabla^2 E_n + m_{11} \nabla^2 H_n + 2mr^{-1} \left( \varepsilon_{11} E_{n,\theta} + m_{11} H_{n,r} \right) = 0 \] (30)

\[ m_{11} \nabla^2 E_n + \mu_{11} r^{-2m} \nabla^2 H_n + 2mr^{-1} \left( m_{11} E_{n,\theta} + 2\mu_{11} r^{-2m} H_{n,r} \right) = 0 \] (31)

where \[ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}. \]

We consider the free vibration of non-homogeneous polygonal cross-sectional plate and we seek the displacement function, electric and magnetic displacement function as:

\[ \phi_n (r, \theta, t) = r^{-m} \phi_n (r) \cos n \theta e^{i\omega t} \] (32)

\[ \psi_n (r, \theta, t) = r^{-m} \psi_n (r) \cos n \theta e^{i\omega t}. \] (33)

\[ E_n (r, \theta, t) = r^{-m} E_n (r) \cos n \theta e^{i\omega t} \] (34)

\[ H_n (r, \theta, t) = r^{-m} H_n (r) \cos n \theta e^{i\omega t} \] (35)

Using the Eqs. (32)-(33) in the Eqs. (28)-(29), we get

\[ \phi_n (r) + r^{-1} \phi_n (r) \left( 1 - \frac{3m \mu + mp_s}{\lambda + 2\mu - p_s} \right) + r^{-2} \phi_n (r) \left[ \frac{m^2 (\mu - \lambda) + 2m \lambda - \rho(gv_r - \phi_{n,\mu})}{\lambda + 2\mu - p_s} - n^2 \right] = 0 \] (36)

\[ \psi_n (r) + r^{-1} \psi_n (r) \left( \frac{m - 1}{2\mu - p_s} \right) + r^{-2} \psi_n (r) \left[ \frac{\rho (gv_\theta + m^2 - 1)}{2\mu - p_s} - (m^2 - n^2) \right] = 0 \] (37)
The above equation can be reduced to the following forms of Bessel equations:

\[
\phi_n(r) + r^{-1}\phi'_n(r) + \phi_n(r)r^{-2}(n^2r^2 - \beta^2) = 0
\]

\[
\psi_n(r) + r^{-1}\psi'_n(r) + r^{-2}\psi_n(r)((m^2 - n^2)r^2 - \delta^2) = 0
\]

and the corresponding solution is obtained by

\[
\phi_n(r) = (P_{1n}J_\beta(Nr) + P_{2n}Y_\beta(Nr))\cos n\theta
\]

\[
\psi_n(r) = (P_{1n}J_\delta(Mr) + P_{2n}Y_\delta(Mr))\cos n\theta
\]

Here, \(P_{1n}, P_{2n}\) are arbitrary constants and \(J_\beta(Nr), J_\delta(Mr)\) and \(Y_\beta(Nr), Y_\delta(Mr)\) are the Bessel functions of first and second kind of order \(\beta\) and \(\delta\) respectively.

Again, substituting Eqs.(33)-(35) in to Eqs.(30)-(31) and reducing to the simplest form as follows,

\[
\varepsilon_{11} \left( E_n''(r) + r^{-1}E_n'(r) - c^2r^2E_n(r) \right) + m_{11} \left( H_n''(r) + r^{-1}H_n'(r) - c^2r^2H_n(r) \right) = 0
\]

\[
\varepsilon_{11} \left( H_n''(r) + r^{-1}H_n'(r) - c^2r^2H_n(r) \right) + m_{11} \left( E_n''(r) + r^{-1}E_n'(r) - c^2r^2E_n(r) \right) = 0
\]

where \(c^2 = m^2 + n^2\).

Solving Eq.(40) and Eq.(41), we can get,

\[
E_n''(r) + r^{-1}E_n'(r) - c^2r^2E_n(r) = 0
\]

\[
H_n''(r) + r^{-1}H_n'(r) - c^2r^2H_n(r) = 0
\]

The general solution of Eqs.(42) and (43) are as follows:

\[
E_n(r, \theta, t) = (P_{3n}r^p + P_{3n}r^{-p})\cos n\theta e^{iat}
\]

\[
H_n(r, \theta, t) = (P_{4n}r^p + P_{4n}r^{-p})\cos n\theta e^{iat}
\]

where \(P_{3n}, P_{3n}', P_{4n}, P_{4n}'\) are the arbitrary constants.

### 4 BOUNDARY CONDITIONS AND FREQUENCY EQUATIONS

Here, the problem of vibration of a magneto electro elastic polygonal plate is considered via hydrostatic stress and gravity. Since the boundary is irregular it is difficult to satisfy the boundary conditions directly. Hence, from the formulation of Nagaya [8], the Fourier expansion collocation method is applied to satisfy the boundary conditions. Thus the boundary conditions are obtained as,

\[
(\Pi_{xx})_j = (\Pi_{xy})_j = (C_x)_j = (N_y)_j = 0
\]

where \(x\) is the coordinate normal to the boundary and \(y\) is the tangential to the boundary. \(\Pi_{xx}\) and \(\Pi_{xy}\) are the normal and shear stress. \(\{\}_j\) is the value at the \(j^{th}\) segment of the cross section. Since the angle \(\Gamma_j\) between the
reference axis and normal to the \( j^{th} \) straight line boundary has a constant value in the segment as shown in the following figure.

![Fig.1](image)

In the above Fig. 1 \( 1 = \theta, 2 = \theta, 3 = (\theta + \theta_{i-1}) / 2, 4 = \theta_{i-1}, 5 = \Gamma \). Transforming the vibration displacements into the Cartesian coordinates \( x_i \) and \( y_i \), the relation between the displacements for the \( i-th \) segment of straight line boundaries is

\[
\begin{align*}
    u_x &= u_x \cos(\theta - \gamma_i) - u_\theta \sin(\theta - \gamma_i), \\
    u_\theta &= u_\theta \cos(\theta - \gamma_i) + u_x \sin(\theta - \gamma_i)
\end{align*}
\]  

(47)

and

\[
\begin{align*}
    \frac{\partial r}{\partial x_i} &= \cos(\theta - \gamma_i), \quad \frac{\partial \theta}{\partial x_i} = -r^{-1} \sin(\theta - \gamma_i) \\
    \frac{\partial r}{\partial y_i} &= \sin(\theta - \gamma_i), \quad \frac{\partial \theta}{\partial y_i} = r^{-1} \cos(\theta - \gamma_i)
\end{align*}
\]  

(48)

Using the above Eqs. (47) and (48), the stress equations for the non homogeneity becomes,

\[
\begin{align*}
    \Pi_{xx} &= ((\lambda + 2\mu)\cos^2(\theta - \gamma_i) + \lambda \sin^2(\theta - \gamma_i))u_{r,x} + r^{-1}((\lambda + 2\mu)\sin^2(\theta - \gamma_i) + \lambda \cos^2(\theta - \gamma_i))(u_x + u_{\theta,\theta}) \\
    &+ \frac{u_x}{2}(r^{-1}(u_\theta - u_{r,\theta}) - u_{r,\theta}) \sin 2(\theta - \gamma_i) = 0 \\
    \Pi_{xy} &= \mu(u_{r,x} - r^{-1}u_{\theta,\theta} - r^{-1}u_x) \sin 2(\theta - \gamma_i) + (r^{-1}u_{r,\theta} + u_{\theta,\theta} - r^{-1}u_\theta) \cos 2(\theta - \gamma_i) = 0 \\
    E_x &= -\epsilon_{11}E_x - m_{11}H_x = 0 \\
    M_x &= -m_{11}E_{\theta} - \mu_{11}H_{\theta} = 0
\end{align*}
\]

Applying the solution in the boundary condition (51) we arrive at the transformed form,

\[
\begin{align*}
    \left[ (T_{xx})_j + (\overline{T}_{xx})_j \right] e^{i\alpha} &= 0 \\
    \left[ (T_{xy})_j + (\overline{T}_{xy})_j \right] e^{i\alpha} &= 0 \\
    \left[ (E_x)_j + (\overline{E}_x)_j \right] e^{i\alpha} &= 0 \\
    \left[ (M_x)_j + (\overline{M}_x)_j \right] e^{i\alpha} &= 0
\end{align*}
\]  

(49)

where
\[ T_{xx} = 0.5\left( P_{10}e^{0} + P_{20}e^{2} + P_{30}e^{3}\right) + \sum_{n=1}^{\infty} \left( P_{1n}e^{1} + P_{2n}e^{2} + P_{3n}e^{3} + P_{4n}e^{4}\right) \]
\[ T_{xy} = 0.5\left( P_{10}f^{1} + P_{20}f^{2} + P_{30}f^{3}\right) + \sum_{n=1}^{\infty} \left( P_{1n}f^{1} + P_{2n}f^{2} + P_{3n}f^{3} + P_{4n}f^{4}\right) \]
\[ E_{x} = 0.5\left( P_{10}g^{0} + P_{20}g^{2} + P_{30}g^{3}\right) + \sum_{n=1}^{\infty} \left( P_{1n}g^{1} + P_{2n}g^{2} + P_{3n}g^{3} + P_{4n}g^{4}\right) \]
\[ M_{x} = 0.5\left( P_{10}h^{0} + P_{20}h^{2} + P_{30}h^{3}\right) + \sum_{n=1}^{\infty} \left( P_{1n}h^{1} + P_{2n}h^{2} + P_{3n}h^{3} + P_{4n}h^{4}\right) \]

For anti-symmetric mode
\[ \bar{T}_{xx} = 0.5\left( \bar{P}_{40}e^{4}\right) + \sum_{n=1}^{\infty} \left( \bar{A}_{1n}e_{n} + \bar{A}_{2n}e_{n} + \bar{A}_{3n}e_{n} + \bar{A}_{4n}e_{n} \right) \]
\[ \bar{T}_{xy} = 0.5\left( \bar{P}_{40}f^{4}\right) + \sum_{n=1}^{\infty} \left( \bar{P}_{4n}f^{1} + \bar{P}_{2n}f^{2} + \bar{P}_{3n}f^{3}\right) \]
\[ \bar{E}_{x} = 0.5\left( \bar{P}_{40}g^{4}\right) + \sum_{n=1}^{\infty} \left( \bar{P}_{4n}g^{1} + \bar{P}_{2n}g^{2} + \bar{P}_{3n}g^{3} + \bar{P}_{4n}g^{4}\right) \]
\[ \bar{M}_{x} = 0.5\left( \bar{P}_{40}h^{4}\right) + \sum_{n=1}^{\infty} \left( \bar{P}_{4n}h^{1} + \bar{P}_{2n}h^{2} + \bar{P}_{3n}h^{3} + \bar{P}_{4n}h^{4}\right) \]

Using the Fourier series expansion to the boundary Eq. (46) along the boundary surfaces are expanded in the form of double Fourier series. The boundary conditions are obtained as, for symmetric mode,
\[ \sum_{m=0}^{\infty} \left[ E_{m}^{1}P_{10} + E_{m}^{2}P_{20} + \sum_{n=1}^{\infty} \left( E_{mn}^{1}P_{1n} + E_{mn}^{2}P_{2n} + E_{mn}^{1}P_{3n} \right) \right] \cos m\theta = 0 \]
\[ \sum_{m=0}^{\infty} \left[ F_{m}^{1}P_{10} + F_{m}^{2}P_{20} + \sum_{n=1}^{\infty} \left( F_{mn}^{1}P_{1n} + F_{mn}^{2}P_{2n} + F_{mn}^{1}P_{3n} \right) \right] \sin m\theta = 0 \]
\[ \sum_{m=0}^{\infty} \left[ G_{m}^{0}P_{10} + G_{m}^{2}P_{20} + \sum_{n=1}^{\infty} \left( G_{mn}^{1}P_{1n} + G_{mn}^{2}P_{2n} + G_{mn}^{3}P_{3n} \right) \right] \cos m\theta = 0 \]
\[ \sum_{m=0}^{\infty} \left[ H_{m}^{0}P_{10} + H_{m}^{2}P_{20} + \sum_{n=1}^{\infty} \left( H_{mn}^{1}P_{1n} + H_{mn}^{2}P_{2n} + H_{mn}^{3}P_{3n} \right) \right] \cos m\theta = 0 \]

For anti-symmetric mode,
\[ \sum_{m=0}^{\infty} \left[ \bar{E}_{m}^{3}P_{30} + \sum_{n=1}^{\infty} \left( \bar{E}_{mn}^{1}P_{1n} + \bar{E}_{mn}^{2}P_{2n} + \bar{E}_{mn}^{3}P_{3n} \right) \right] \sin m\theta = 0 \]
\[ \sum_{m=0}^{\infty} \left[ \bar{F}_{m}^{3}P_{30} + \sum_{n=1}^{\infty} \left( \bar{F}_{mn}^{1}P_{1n} + \bar{F}_{mn}^{2}P_{2n} + \bar{F}_{mn}^{3}P_{3n} \right) \right] \cos m\theta = 0 \]
\[ \sum_{m=0}^{\infty} \left[ \bar{G}_{m}^{3}P_{30} + \sum_{n=1}^{\infty} \left( \bar{G}_{mn}^{1}P_{1n} + \bar{G}_{mn}^{2}P_{2n} + \bar{G}_{mn}^{3}P_{3n} \right) \right] \sin m\theta = 0 \]
\[ \sum_{m=0}^{\infty} \left[ \bar{H}_{m}^{3}P_{30} + \sum_{n=1}^{\infty} \left( \bar{H}_{mn}^{1}P_{1n} + \bar{H}_{mn}^{2}P_{2n} + \bar{H}_{mn}^{3}P_{3n} \right) \right] \sin m\theta = 0 \]
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NUMERICAL RESULTS AND DISCUSSIONS

To illustrate the analytical result presented earlier, now we consider the numerical example for which computational results are presented. The physical constants for the numerical computation is taken from copper at $42^\circ K$, Poisson ratio $\nu = 0.3$, density $\rho = 8.96 \times 10^3 \text{ kg} / \text{m}^3$, the Young’s modulus $E = 2.139 \times 10^{11} \text{ N} / \text{m}^2$, $\lambda = 8.20 \times 10^{11} \text{ kg} / \text{ms}^2$, $\mu = 4.20 \times 10^{10} \text{ kg} / \text{ms}^2$, $c_\epsilon = 9.1 \times 10^2 \text{ m}^2/\text{ks}^2$ and $K = 113 \times 10^2 \text{ kglm/ks}^2$. The material properties of the magneto-electro elastic material based on Peng-Fei Hou et al.[40] are $\epsilon_{11} = 8.26 \times 10^{-11} \text{ C}^2 \text{Ns}^{-1} \text{m}^{-2}$, $\mu_i = -5 \times 10^{-6} \text{ Ns}^2 / \text{C}^2$, $m_{11} = -3612 \times 10^{-11} \text{Ns}/\text{V}$/C . The geometric relations for the polygonal cross-sections given by Nagaya [8] as $R_i/a = \left[ \cos (\theta - \gamma_i) \right]^{-1}, R_i/jb = \left[ \cos (\theta - \gamma_i) \right]^{-1}, \gamma_i = \gamma_i$ . To study the convergence and validation of the present study, the frequency equation of a thick polygonal cross-sectional plate without non homogeneous parameter and other interacting field variables is chosen to show the exactness of the result obtained in the present case with the results of Nagaya [2]. The non-dimensional frequencies $[\Omega]$, obtained for a ring shaped polygonal plates using Fourier expansion collocation method, with clamped edges, in both the Author and Nagaya’s [2], have exact coincidence, which is shown in Table 1.

**Table 1**
Comparative display of non-dimensional frequencies $[\Omega]$ of Author results with the results of Nagaya [8] of a ring shaped polygonal plates with clamped (ICOc) edges.

<table>
<thead>
<tr>
<th>$a/b$</th>
<th>Mode</th>
<th>Triangle</th>
<th>Square</th>
<th>Pentagon</th>
<th>Hexagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>$n_1$</td>
<td>4.148</td>
<td>4.146</td>
<td>4.850</td>
<td>4.149</td>
</tr>
<tr>
<td></td>
<td>$n_2$</td>
<td>4.392</td>
<td>4.393</td>
<td>4.997</td>
<td>4.997</td>
</tr>
<tr>
<td></td>
<td>$n_3$</td>
<td>4.547</td>
<td>4.551</td>
<td>5.889</td>
<td>5.886</td>
</tr>
<tr>
<td>0.15</td>
<td>$n_1$</td>
<td>4.269</td>
<td>4.267</td>
<td>5.128</td>
<td>5.127</td>
</tr>
<tr>
<td></td>
<td>$n_2$</td>
<td>4.510</td>
<td>4.511</td>
<td>5.271</td>
<td>5.272</td>
</tr>
<tr>
<td></td>
<td>$n_3$</td>
<td>4.769</td>
<td>4.765</td>
<td>6.069</td>
<td>6.068</td>
</tr>
<tr>
<td>0.2</td>
<td>$n_1$</td>
<td>4.413</td>
<td>4.412</td>
<td>5.431</td>
<td>5.431</td>
</tr>
<tr>
<td></td>
<td>$n_2$</td>
<td>4.622</td>
<td>4.613</td>
<td>5.573</td>
<td>5.572</td>
</tr>
<tr>
<td>0.25</td>
<td>$n_1$</td>
<td>4.474</td>
<td>4.573</td>
<td>5.757</td>
<td>5.756</td>
</tr>
</tbody>
</table>
A graph is drawn for the variation of radial stress $\Pi_{rr}$ with plate radius $r$ for IFOC and IFOF edge boundary conditions via $g = 0.0, 9.8$ for the polygonal plate in Fig.3 and Fig.4. From the Figs. 3 and 4, it is observed that, the radial stress $\Pi_{rr}$ increases with respect to its radius $r$, also it is noted that the radial stress attains the maximum value in IFOF edge boundary than the radial stress of IFOC edge boundary conditions and also in higher gravity. The energy level is higher in hexagonal and pentagonal cross sections as compared to other cross sections. Figs. 5-6 display the distribution of displacement $u$ against the radius $r$ with IFOC and IFOF of polygonal plates for $g = 0.0, 9.8$. From the Figs. 5 and 6, it is observed that the displacement is higher for a plate with inner free and outer free edges. It is shown that, the displacement of the pentagon and hexagon cross section is higher in both the values of the gravity.
Dispersion curves are plotted for the variation of electric displacement with radius $r$ of Triangle, Square, Pentagon and Hexagonal cross-sectional plates with IFOC and ICOF edges via $g = 0.0, 9.8$ in Figs. 7-8. From the Figs. 7 and 8, it is observed that the dispersions behavior of Triangle, Pentagon and Square, Hexagon are similar to each other at $g = 0.0, 9.8$. The crosses over points denote the transfer of energy between the modes of vibrations due to the hydrostatic stress, gravity and clamped- free boundaries. Figs. 9-10 show magnetic displacement with radius $r$ of polygonal plates with IFOC and IFOF boundary conditions via $g = 0.0, 9.8$. The magnetic displacement is higher in the lower values of radius and became linear in the other part of radius in both boundary conditions and higher in the case of $g = 9.8$ and ICOF edge. It is shown that, the magnetic displacement of the pentagon and hexagon plate is more in two cases due to the symmetry of cross section and the influence of other interactive fields.

Fig. 5
Displacement with radius of the polygonal plate with IFOC edges via $g = 0.0$.

Fig. 6
Displacement with radius of the polygonal plate with IFOF edges via $g = 9.8$.

Fig. 7
Dispersion of electric potential with radius of polygonal plate with IFOC edges via $g = 0.0$.

Fig. 8
Dispersion of electric potential with plate radius of polygonal plate with and ICOF edges via $g = 9.8$. 
Fig. 9
Dispersion of magnetic potential with plate radius of polygonal plate with IFOC edges via $g = 0.0$.

Fig. 10
Dispersion of magnetic potential with plate radius of polygonal plate with ICOF edges via $g = 9.8$.

Figs. 11-14 illustrates the frequency distribution $\Omega$ with radius $r$ of Triangle, Square, Pentagon and Hexagonal cross-sectional plates with different boundary conditions and aspect ratio through $g = 9.8$. From the Figs. 11-14, it is noted that the frequency increases as the radius increases in all the boundary conditions.

Fig. 11
Frequency with radius of triangular cross-sectional plate with ICOF edges via $g = 9.8$.

Fig. 12
Frequency with radius of square cross-sectional plate with IFOC edges via $g = 9.8$. 
6 CONCLUSIONS

The clamped free non homogeneous wave propagation on a magneto electro elastic plate of polygonal shape is studied in this paper. The interactive force terms gravity and the hydro static stress on the polygonal plate also have been derived using linear elasticity theory. Fourier expansion collocation method is used to solve the irregular boundary of the polygonal shape of the plate. The numerical result of the stress, mechanical, displacement, electric displacement, magnetic displacement and non-dimensional frequency of the wave in the various shapes such as triangle, square, pentagon and hexagon cross section plates with IFOC, ICOC, ICOF edges and fixed gravity are computed and the graphical representations are presented. We concluded that

- The hydrostatic stress, gravity and non-homogeneity of the poly plate have important role on the distribution of physical variables.
- The boundary edge IFOC and IFOF are noted to be more significant in all the field variables.
- Pentagon and Hexagon cross section plate attain higher values in all the physical quantities.
- The Fourier expansion collocation method gave faster convergence in irregular boundaries.
- This type of study is more important in the construction of foundation of footing, turbine disks, floor and roof with irregular boundaries.

APPENDIX

\[
e_{_{n}}^{1} = 2\{ \beta(\beta - 1)J_{\beta}(Nr) + (Nr)J_{\beta+1}(Nr) \} \cos 2(\theta - \gamma) \cos n\theta
\]

\[
-r^{2} \{ N^2 (\Lambda + 2 \cos^2(\theta - \gamma)) \} J_{\beta}(Nr) \cos n\theta
\]

\[
e_{_{n}}^{1} = 2\{ \beta(\beta - 1)J_{\beta}(Nr) + (Nr)J_{\beta+1}(Nr) \} \sin 2(\theta - \gamma) \sin n\theta
\]

\[
-r^{2} \{ N^2 (\Lambda + 2 \cos^2(\theta - \gamma)) \} J_{\beta}(N) \sin n\theta
\]

\[
e_{_{n}}^{2} = \{ n(\delta - 1)J_{\delta}(Mr) - (Mr)J_{\delta+1}(Mr) \} \cos 2(\theta - \gamma) \cos n\theta
\]

\[
-n^{2} \{ \delta(\delta + 2) + n^{2} - (Mr)^2 \} J_{\delta}(Mr) - (Mr)J_{\delta+1}(Mr) \cos n\theta \sin 2(\theta - \gamma)
\]
\[ e_x^2 = \left\{ n (\delta - 1) J_\delta (Mr) - (Mr) J_\delta (Mr) \right\} \sin 2(\theta - \gamma_i) \sin n \theta \\
- \left( (\delta (\delta + 2) + n^2 - (Mr)^2 \right) J_\delta (Mr)\frac{2}{2} - (Mr) J_\delta (Mr) \right\} \cos n \theta \cos 2(\theta - \gamma_i) \]
\[ e_x^2 = 0, e_y^2 = 0 \]
\[ f_1^2 = 2 \left\{ \beta J_\delta (Nr) - (Nr) J_\delta (Nr) \right\} + \left( (Nr)^2 - \beta^2 - n^2 \right) J_\delta (Nr) \right\} \cos n \theta \sin 2(\theta - \gamma_i) \]
\[ + 2n \left\{ (\beta - 1) J_\delta (Nr) - (Nr) J_\delta (Nr) \right\} \sin n \theta \cos 2(\theta - \gamma_i) \]
\[ f_1^2 = 2n \left[ \delta J_\delta (Mr) - (Mr) J_\delta (Mr) \right] \cos n \theta \sin 2(\theta - \gamma_i) + \\
2 \left[ \delta J_\delta (Mr) - (Mr) J_\delta (Mr) \right] \sin n \theta \cos 2(\theta - \gamma_i) + \\
2 \left[ \delta J_\delta (Mr) - (Mr) J_\delta (Mr) \right] \cos n \theta \sin 2(\theta - \gamma_i) \]
\[ f_2^2 = 0, f_3^2 = 0 \]

REFERENCES


