Wakefield excitation by an intense femtosecond laser pulse in homogeneous underdense plasma

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Abstract

An intense, short-pulse laser interacting with underdense plasma can generate huge amplitude plasma wave wake field. This wakefield can be described in tree-dimensional. A differential equation for wake potential corresponding to this wakefield is derived analytically and is solved numerically for various laser-plasma conditions. It is shown by numerical studies that the amplitude of the wakefield is increased for the large frequency and higher laser intensity, but is decreased with the spot size of laser. The study conducted for various pulse durations and background plasma density reveals that there is an optimum value on excited plasma wake.

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Keywords: LWFA: Plasma wakefield; Laser-plasma interaction.

1. Introduction

Laser-plasma interaction is an area of significant scientific activities since the past several decades due to wide-ranging applications in laser-induced fusion [1,2], high-harmonic generation [3,4], soft x-ray lasers [5,6], laser wake field accelerators [7,8,9], etc.

In the laser wake field accelerators, Plasma wake waves excited by intense laser pulses may produce extraordinarily strong accelerating fields. It can be expected when an electron bunch is properly injected in to the accelerating region of the wakefield, the bunch will co-propagate with the laser wakefield and be accelerated to ultra-relativistic energies within a short distance. The theoretical and experimental studies have shown that the acceleration rate in a wake wave may be as high as tens of GeV/m, i.e., three orders of magnitude higher than the rates achieved in conventional accelerators [10].

The main parameter in the laser plasma interaction is that the amplitude of the wake field as well as interaction length be maintained at higher magnitudes in order to achieve higher acceleration. In the past several years, extensive theoretical and experimental work has been done to enhance the wake field by nonlinearities in the response of the plasma to the pondermotive force of a laser pulse of relativistic intensity [11-13]. Attempts to excite the wake field have been made via the ionization processes of the gases [14], capillary density as a plasma waveguide [15], and tapered plasma channels [16].

In this paper we use a numerical fluid approach of the plasma together with the laser field equations to derive a second order differential equation for wake potential \( \Phi_W \) in nonlinear conditions. The amplitude of wake fields generated by the finite-width laser pulse propagating in an underdense plasma are examined by the effects of laser and plasma parameters.

2. Analytical Investigation

We consider that a wakefield (corresponding potential \( \Phi_W \)) is generated by a linearly \( y \) polarized laser pulse at frequency \( \nu_L \), pulse duration \( \tau_L \) and minimum spot size \( \omega_0 \) in a homogeneous plasma with background density \( n_0 \).

We use fluid approach and write basic fluid equations together with the Maxwell equations describing the propagation of laser pulse in plasma.

\[
\begin{align*}
\nabla \times \vec{E}_L &= -\frac{\partial \vec{B}_L}{\partial t} & (Maxwell) \\
\nabla \times \vec{B}_L &= \frac{1}{c^2} \frac{\partial \vec{E}_L}{\partial t} + \mu_0 \vec{j} & (Maxwell) \\
\varepsilon_0 \nabla^2 \Phi_W &= -e(n_0 - n_e) & (Poisson) \\
\frac{dn_e}{dt} + \nabla \cdot (n_e \vec{v}) &= 0 & (Continuity) \\
\frac{\partial \vec{p}}{\partial t} + \vec{v} \cdot \nabla (\vec{p}) &= -e(\vec{E}_L + \vec{E}_W + \vec{v} \\
&\quad \times \vec{B}_L) & (Relativistic momentum)
\end{align*}
\]

Where \( e \) and \( m_e \) are the electron charge and rest mass,
... the electric and magnetic fields of laser pulse, $E_L$, $B_L$ are the longitudinal excited electric field and corresponding potential that $E_L = E_0 = \gamma v_0^2 M(x, \eta) / n_0 e^2$ and $B_L = \gamma v_0^2 M(x, \eta) / n_0 e^2$. Where $n_0$ is the electron density where $\gamma$ is the density perturbation.

We consider now the frame of reference, $\eta = z - v_g t$, co-moving with the laser pulse at the group velocity $v_g = c(1 - \omega_p^2(1 + \alpha^2)^{-1/2} / \omega)^{1/2}$ [17], where $\alpha$ is the normalized amplitude of the laser vector potential and it is an important parameter that characterizes how strong is the laser effect on the plasma electrons. $\omega_p = (n_0 e^2/m_e \omega_0)^{1/2}$ and $\omega_L$ are the plasma and laser frequency respectively. With above transformation, equations (1-5) yield:

\begin{align}
B_{xL} &= -E_L / v_g \\
B_{zL} &= 1 / v_g \int \frac{\partial E_L}{\partial \eta} d\eta \\
v_y &= \frac{H(x, \eta)}{n_0 + \tilde{n}} \\
v_z &= v_g \left( \frac{\tilde{n}}{n_0 + \tilde{n}} \right) \\
\tilde{n} &= \frac{\varepsilon_0 d^2 \Phi_w}{\varepsilon_0 \beta_0^2} \\
\gamma v_z &= \left( \frac{\varepsilon_0 v_z}{\varepsilon_0 \beta_0^2} \right) M(x, \eta) \\
M(x, \eta) &= \frac{1}{v_g^2} \frac{\partial H}{\partial \eta} - \frac{\partial}{\partial \eta} \frac{\partial H}{\partial \eta} - \frac{\varepsilon_0 \beta_0^2}{\varepsilon_0 \beta_0^2} \\
&= \frac{1}{v_g^2} \frac{\partial H}{\partial \eta} - \frac{\partial}{\partial \eta} \frac{\partial H}{\partial \eta} - \frac{\varepsilon_0 \beta_0^2}{\varepsilon_0 \beta_0^2}.
\end{align}

\begin{equation}
\gamma v_z = \left( \frac{\varepsilon_0 v_z}{\varepsilon_0 \beta_0^2} \right) M(x, \eta) \tag{12}
\end{equation}

Where the function $M(x, \eta)$ is express by

\begin{equation}
M(x, \eta) = \frac{-\omega_p^2}{v_g^2} \Phi_w - \frac{e^2}{2m_e v_g^2} \left( \frac{\partial \Phi_w}{\partial \eta} \right)^2 - \frac{e^2}{m_e v_0^2} \int B_{xL} H(x, \eta) d\eta \tag{13}
\end{equation}

From equation (12) it follows that the function $(\varepsilon_0 v_g m_e / \varepsilon_0 \beta_0^2) M(x, \eta)$ can be treated as longitudinal momentum $P_z$ of plasma electrons.

Using the relativistic factor $\gamma = \sqrt{(1 + \alpha^2)/(1 - \beta_0^2)}$ [12] where $\beta_0 = v_z / c$ and substituting into equation (12), (10), (9), laser wake potential equation can be obtained as follow

\begin{equation}
\frac{d^2 \Phi_w}{d\eta^2} = \frac{M(x, \eta)}{1 + \alpha^2 \frac{\varepsilon_0 M(x, \eta)}{\varepsilon_0 \beta_0^2} - \frac{\varepsilon_0 M(x, \eta)}{\varepsilon_0 \beta_0^2} \varepsilon_0 \beta_0^2} \tag{14}
\end{equation}

The relativistic factor $\gamma$ can now be expressed as:

\begin{equation}
\gamma = \sqrt{1 + \alpha^2 \frac{\varepsilon_0 M(x, \eta)}{\varepsilon_0 \beta_0^2} - \frac{\varepsilon_0 M(x, \eta)}{\varepsilon_0 \beta_0^2} \varepsilon_0 \beta_0^2} \tag{15}
\end{equation}

Equation (14) shows the coupling between the transverse and longitudinal spatial derivatives. It could be solved numerically based on the fourth-order Runge–Kutta scheme for a Gaussian linearly polarized laser pulse, $E_L = E_0 \exp \left( \frac{-x^2}{2L^2} \right)$ with pulse length $L$. Note that $E_0$ is the electric field amplitude which is related to normalized vector potential as:

\begin{equation}
\frac{E_0}{m_e c \omega_L} = \left( \frac{\varepsilon_0 v_0^2}{2m_e \varepsilon_0 \beta_0^2} \right) M(x, \eta) \tag{16}
\end{equation}

Figure 1: Expanded view of plasma oscillations behind the laser pulse: (a) density perturbation; (b) longitudinal electric field; (c) 2-D electron density perturbation in x-z coordinate. The main parameters are, $\alpha_0 = 0.2$, $w_0 = 20 \mu m$, $\tau_L = 10 fs$ and $v_L = 3.7 \times 10^{14} Hz$, $n_0 = 5 \times 10^{14} m^{-3}$. 

112

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3. Results and discussion

Two typical examples of numerical results of the propagation of Gaussian laser beam in the primarily homogeneous plasma with density \( n_0 = 5 \times 10^{24} \text{ m}^{-3} \) are shown in the figure 1 and Fig. 2.

The corresponding intensity are \( I_0 = 8.6 \times 10^{20} \text{ W} \cdot \text{m}^{-2} \) \( (a_0 = 0.2) \) and \( I_0 = 8.6 \times 10^{22} \text{ W} \cdot \text{m}^{-2} \) \( (a_0 = 2) \) respectively. The laser pulse has a beam waist \( w_0 = 20 \text{ \mu m} \) at the frequency \( \nu = 3.7 \times 10^{14} \text{ Hz} \). This corresponds to Rayleigh range about 1.6 mm. In this case, the laser pulse length, \( c\tau_r \approx 3 \text{ \mu m} \), is much shorter than its focal spot size.

From this figure we can see the distributions of different quantities behind the laser pulse. Indices (a), (b) and (c) illustrate the plasma density variation, the corresponding wake electric field, and the corresponding 2D distributions of electron density respectively.

The steepening of the electric field and an increase in the period of the wake field are apparent for the moderately nonlinear situation, \( a_0 \approx 2 \), as compared to the linear case, \( a_0 \approx 0.2 \). Here we find that the plasma wavelength, \( \lambda_p \), are 14.7 \( \text{ \mu m} \) and 15.6 \( \text{ \mu m} \), when the maximum electrostatic wakefield amplitudes get increase up to \( 4.5 \times 10^9 \text{ V} \cdot \text{m}^{-1} \) and \( 100 \times 10^9 \text{ V} \cdot \text{m}^{-1} \) respectively for the low and high intensities. The behavior of these oscillations was qualitatively described in the references [18,19]. It is evident from this figures that the ratio of the wakefield amplitude is about 25 by increasing the laser intensity by a factor of 10.

It is worth to notice that, the laser pulse has a Gaussian shape, and it peaks on axis. Then the plasma wavelength is larger than it is off axis. This causes the wave fronts of the plasma wake to become curved and take on a "horse-shoe" shape [20,21]. This effect can be seen in Fig. 2(c). In fact, the transverse non uniformity of the pulse causes the formation of horseshoe structures that can be used to focus and accelerate electrons. The induced focusing of the EM radiation leads to fast and strong modulation of the pulse.

3.1. Effect of laser parameters

An important parameter for the wake excitation in the discussion of the laser plasma interaction is the laser pulse duration. Effect of this parameter on the Wakefield amplitude is shown in Fig. 3. The curves are plotted for the conditions \( \nu = 3.7 \times 10^{14} \text{ Hz}, w_0 = 15 \text{ \mu m} \) and \( n_0 = 5 \times 10^{24} \text{ m}^{-3} \) for the different values of the laser intensities \( a_0 = 0.5 \) (a), and \( a_0 = 2 \) (b). It is evident from this figure that the growth rates depend on the duration of the laser pulse. That is, the amplitude of the wakefields is smaller than the maximum values when the laser pulses are longer or shorter than the optimum pulse lengths, which are \( L \approx 0.5 \lambda_p \), and \( L \approx 0.8 \lambda_p \) respectively. It is worth to notice that the pattern of wakefield excitation differs significantly for laser pulses longer and shorter than the plasma period. The long laser pulse gets self-modulated with the plasma period, and the resonance between this self modulation and the plasma frequency leads to effective wakefield excitation. The corresponding regime of particle acceleration is called self-modulated laser wakefield acceleration (SM-LWFA). These results are in good agreement with those in references [22, 23].
Wakefield excitation by an intense …

Besides the intensity and duration of the laser pulse, another important parameter governing the behavior of the laser–plasma interaction is the value of laser spot size. This dependence is shown in the Fig. 4 for the different laser parameters. The background density $n_0 = 5 \times 10^{24} \text{ m}^{-3}$ and $\tau_L = 10 \text{ fs}$ are the same as previous. One can see that all profiles show the same decreasing behavior with increasing the focal spot size. That is, to reach the high intensities required for maximum wake excitation, the laser pulse should be focused to the small focal region to allowing fields of hundreds of GV/m. It seems for the wide spot size, the lateral variation of refractive index function $\eta(x) = \eta(x)/\partial x$ proportional to $\propto (-x^2/\omega_0^2)$ is less effective as a plasma positive lens and then weaker electric wake is generated. The curves (b) and (c) have been drawn for two different laser frequencies. In fact an increase in laser frequency leads to an enhancement in the wakefield amplitude. This effect is presented in Fig. 5 for the visible interval of frequencies $3 - 8 \times 10^{14} \text{ Hz}$. However, this can be the result of the approaching the group velocity to speed of light, as the frequency is increased. In this case the phase velocity of the plasma wave is typically equal to the group velocity of the laser pulse and the plasma acts as a transformer, converting the maximum transverse laser field into the plasma waves.

![Fig. 4. Maximum amplitude of excited wake potential as a function of focal spot size when $n_0 = 5 \times 10^{22} \text{ m}^{-3}$ and $\tau_L = 10 \text{ fs}$. (a) $a_0 = 0.5$, $v_L = 3.7 \times 10^{14} \text{ Hz}$ (●); (b) $a_0 = 3$, $v_L = 3.7 \times 10^{14} \text{ Hz}$ (▲) and (c) $a_0 = 3$, $v_L = 5 \times 10^{14} \text{ Hz}$ (▲).](image)

3.2. Effect of the plasma density

In Fig. 6, we show the response of plasmas of different densities to a laser pulse with strength $a_0 = 2$ and $v_L = 3.7 \times 10^{14} \text{ Hz}$, $\tau_L = 10 \text{ fs}$. It indicates that the wake potential amplitude for a fixed parameters of laser goes through its maximum value as the background density is raised. However, its slope gets steeper after the maximum compared with the lower densities. The density of the plasma is in the range of $10^{-1} - 10^{-2} n_c$, where $n_c$ is the critical density. One can see that when $n_0 = 10^{25} \text{ m}^{-3}$, the maximum value of the wake potential can reach $\Phi_W \approx 0.3 \Phi_0$. Where $\Phi_0 = m_e c^2/e$ is the normalized scalar potential. At higher densities, however, the laser pulse group velocity is reduced and electron dephasing can limit the energy gain [9]. The net result is that both limits are not so efficient for laser-wake excitation.

![Fig. 5. Maximum plasma wake potential versus laser frequency with $n_0 = 5 \times 10^{24} \text{ m}^{-3}$, $\tau_L = 10 \text{ fs}$, and $w_0 = 15 \mu m$ for (a) $a_0 = 0.5$ (●) and (b) $a_0 = 1$ (▲).](image)

![Fig. 6. (a) Normalized wake potential as a function of plasma density with $v_L = 3.7 \times 10^{14} \text{ Hz}$, $\tau_L = 10 \text{ fs}$, $a_0 = 2$ and $w_0 = 20 \mu m$; (b) CS function $\tau_N$ normalized by pulse length.](image)

Another effect of the background density is that the plasma oscillation can be visualized with different shape of charge separation (CS) as it is shown in Figs. 7-9. That is, the periodic oscillations in the wake owing to compression of the plasma concentration will be significant at higher density. All physical quantities are the same as Fig. 6 except for the densities of $n_0 = 0.05 \times 10^{25} \text{ m}^{-3}$ (Fig. 7), $n_0 = 1 \times 10^{25} \text{ m}^{-3}$ (Fig. 8), and $n_0 = 5 \times 10^{25} \text{ m}^{-3}$ (Fig. 9) respectively. It is found that CS width as a function of $n_0$ gives a large wakefield amplitude at $n_0 = 1 \times 10^{25} \text{ m}^{-3}$ which corresponds to $\text{CS}/L \approx 3.7$. The wakefield in this case is called the resonant wake-
In order to excite a wakefield with highest amplitude for acceleration to a maximum bunch energy, it is important to mutually match the plasma density and the pulse duration of the laser. For the lower density, CS width or the electron bunch are much larger than the laser pulse length with small amplitude when compared with other cases. This trend is also shown in Fig. 6 (b). However at higher plasma densities, the plasma wavelength becomes very short, so that the width of CS that is excited by the laser becomes much smaller.

Fig. 7. (a) Top: the normalized scalar potential \( \Phi_W/\Phi_0 \); bottom: electron density \( n \); (b) The corresponding 2-D scalar potential and (c) 2-D electron density separation for \( n_0 = 0.05 \times 10^{25} m^{-3} \), \( \tau_L = 10 fs \), \( V_L = 3.7 \times 10^{14} Hz \), and \( w_0 = 20 \mu m \).

Fig. 8. (a) Top: the normalized scalar potential \( \Phi_W/\Phi_0 \); bottom: electron density \( n \); (b) The corresponding 2-D scalar potential and (c) 2-D electron density separation for \( n_0 = 1 \times 10^{25} m^{-3} \), \( \tau_L = 10 fs \), \( V_L = 3.7 \times 10^{14} Hz \), and \( w_0 = 20 \mu m \).

Fig. 9. (a) Top: the normalized scalar potential \( \Phi_W/\Phi_0 \); bottom: electron density \( n \); (b) The corresponding 2-D scalar potential and (c) 2-D electron density separation for \( n_0 = 5 \times 10^{25} m^{-3} \), \( \tau_L = 10 fs \), \( V_L = 3.7 \times 10^{14} Hz \), and \( w_0 = 20 \mu m \).

4. Conclusion

In this work, we have analyzed the effect of the different physical parameters on the fs pulsed laser-plasma interaction. Our conclusions can be summarized as follows:

Wakefield generation strongly depends on the laser intensity, pulse duration, beam spot size and also plasma density. For a Gaussian pulse envelope, increasing the laser intensity and frequency enhance the excited plasma wakefield. A reverse behavior occurs when the wider spot size is used. Our study also shows the dependence of wakefield amplitude on the laser pulse duration. Its maximum value is happened in certain \( L/\lambda_p \). On the other hand, the plasma density can be affected the wakefield positions behind the laser pulse. The lower densities exhibit the larger electron bunch. When the background electron density is changed, the wakefield amplitude for fixed parameters of laser goes trough a maximum. Hence, an optimized and highly amplitude could be achieved by adjusting the plasma density when the laser pulse duration as well as its focal spot size set to the small values.

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Wakefield excitation by an intense ...

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