BANACH JOURNAL of MATHEMATICAL ANALYSIS

Banach J. Math. Anal. 11 (2017), no. 1, 1–20
http://dx.doi.org/10.1215/17358787-3721183
ISSN: 1735-8787 (electronic)
http://projecteuclid.org/bjma

FRAMES AND REPRESENTING SYSTEMS IN FRÉCHET SPACES AND THEIR DUALS

J. BONET,[1] C. FERNÁNDEZ,[2] A. GALBIS,[2] and J. M. RIBERA[1]

Communicated by D. Bakić

Abstract. Frames and Bessel sequences in Fréchet spaces and their duals are defined and studied. Their relation with Schauder frames and representing systems is analyzed. The abstract results presented here, when applied to concrete spaces of analytic functions, give examples and consequences about sampling sets and Dirichlet series expansions.

1. Introduction and preliminaries

The purpose of this article is twofold. On the one hand, we study Λ-Bessel sequences $(g_i)_i \subset E'$, Λ-frames, and frames with respect to Λ in the dual of a Hausdorff locally convex space E, in particular for Fréchet spaces and complete (LB)-spaces E with a sequence space Λ. We investigate the relation of these concepts with representing systems in the sense of Korobeinik (see, e.g., Kadets and Korobeinik [13]) and with Schauder frames that were investigated by the authors in [8]. On the other hand, our article emphasizes the deep connection of frames for Fréchet and (LB)-spaces with sufficient and weakly sufficient sets for weighted Fréchet and (LB)-spaces of holomorphic functions. These concepts correspond to sampling sets in the case of Banach spaces of holomorphic functions. Our general results in Sections 2 and 3 permit us to obtain as a consequence examples and results in the literature in a unified way in Section 4, emphasizing their structural aspects.

Section 2 of our article is inspired by the work of Casazza, Christensen, and Stoeva [10] in the context of Banach spaces. Their characterizations of Banach

Received Jul. 13, 2015; Accepted Feb. 12, 2016.
2010 *Mathematics Subject Classification.* Primary 46A04; Secondary 42C15, 46A13, 46E10.
Keywords. frames, representing systems, Fréchet spaces, (LB)-spaces, weakly sufficient sets.