Complex Integrated Supply Chain Planning with Multiple Modes Supply, Production and Distribution by ELECTRE Method

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ABSTRACT
This paper represents a model of strategic programming with limited resources in a complex supply chain. The main goal of the proposed model is to increase efficiency and effectiveness of the supply chain with respect to income increases and cost decreases. Using special objective functions, has guaranteed the lean supply, production, distribution and suppliers' selection strategies. Furthermore, it can use for production programming in the supply chain. Moreover, customer satisfaction has also been perceived, by using minimization objective functions of shortage amount and restrictions of maximum allowed shortage. In this model, objective functions have been defined in a way, which directs the supply chain to the lean. Finally, after determining strategies according to objective functions and constraints, the optimal strategies using multi-criteria decision making - ELECTRE process- have been chosen.


1. Introduction
Ensuring competitiveness in today’s globally connected marketplace is very demanding, and calls for different business strategies than what were employed by businesses in the past. Today’s businesses have to be more adaptive to change. In order to survive in competitive business, they need to be better suited to handle fluctuations in an ever-changing market than their competitors.

Production and manufacturing establishments are also faced with such challenges additionally to managing and modification their supply chains [19]. The key problem in a supply chain is management and coordinated control of programming for supply and demand, selecting the suppliers, provision of the raw materials, production and programming of the product, maintaining the goods, inventory control, distribution, delivery and service to the customers. Strategic programming of the supply chain causes that the customers can receive reliable services, with high quality, fast and with the least-cost [6].

Quantitative modeling for strategic supply chain planning continues to be a fruitful research area. The purpose of the modeling is usually to provide effective...
decision support for strategic resource allocation in the longer term, including factors such as: Selection of suppliers, configuration of manufacturers and distributors’ capacities, as well as allocation of these capacities to products and so forth [8, 9].

The presented models for optimizing the activities of the supply chain have been designed in 3 levels. The first type is the models that their objective is studying each type of the chain in a separate form. For example, we can refer to the presented models for selecting the suppliers in a supply chain, the presented models for reducing delivery time in distribution centers and so forth.

The second one is the models that only for reducing the costs create relation between producers, and distributors or the distributors and the customers. The third type of these models is the presented qualitative models for each part of the chain or combination of the two parts in the chain [8]. In studying the models related to the first and second types, we cannot declare obviously that they have the most optimum decisions and results because, these results are out of the bilateral relation between the parts and nods and sensitivity of affecting a part on another part in these types of models have not been considered.

In the study on the third type models, also due to non-quantitative decisions we cannot refer to them exactly for performance, and they only are executive for the quality of the performed affairs. In most of these models, the model has become far from the world of reality regarding various hypotheses. However, we have attempted to adopt this model with the real conditions of one supply chain to the extent that is possible [20].

Ereng et al (1999) have mentioned designing of the supply-chain network as a strategic decision making matter for the production-distribution models. In this decision making, they determine the number of the places for suppliers of raw materials, products, inventory in the process and distribution facilities for a period of time [2].

Yang et al (2002) have created a compositional model for production-distribution. They have studied a set of producers, distributors and the customers, and developed their model with a supposition for completing a product in several phases of production at each plant.

Moreover, they have mentioned that there is a possibility for making an integrated model for supply, produce, and distribution [3]. Scott et al (2003) have developed on the combination of the affiliates of transportation and storage (production-distribution) in the supply chain based on the simulator model [5].

Fu et al (2004) have studied the improvement of quality for supply chain with consideration of prioritizing in suppliers. In this model using the quantitative models and formulas in expressing the problem and statistical looking to the manner of selecting the customers, optimization of the supply chain has been carried out. Finally, the research findings were studied with performing a case study. One-dimensional observation of the problem and abundant constraints are the defects for the presented model [7].

Graeme (2005) has presented a qualitative model concentrated on the time of delivering goods to the customer for improvement of the supply chain using six-sigma. This method has tried to improve the function of the supply chain without considering mathematical programming [10].

Mohammadi et al. (2011) presented a model for the capacitated single allocation hub covering location problem. The objective of this model is introduced to minimize service times in the hubs [21].

Hongyan (2007) has presented a mathematical model for strategic programming of the limited sources in the supply chain.

This model with a comprehensive approach attempted to optimize the supply chain with the purpose of reducing the costs; this model is the closest model to our research, but in this method, other purposes for the supply-chain management such as selecting the suppliers, reducing the waste materials, increasing the customers’ satisfaction, etc. have not been considered [13].

Yang et al (2006) have an approach of coordinating the supply-chain plan by allocating response-time among the node firms and assigning production time and logistics time rationally to each node firm [3].

Leung et al (2008) have considered the problems of coordinating serial and assembly inventory systems with private information where end-item demands are known over a finite horizon. At the core of the solution procedure is a supplier–buyer link model that can be used as a building block to form other supply chain configurations [17].

Francas et al (2008) have studied on the network design problem of a firm, which produces new products and reproduces returned products in its facilities [16].

Wang et al (2008) have a business model of capacity planning and resource allocation in which consists of two profit-centered factories [15].

Jawahar et al (2008) have considered a two-stage distribution problem of a supply chain that is associated with a fixed charge [14].

Sarker et al (2008) have an optimal policy for production and procurement in a supply-chain system with multiple non-identical buyers [18].

In the last decade, considerable attention has been paid to the supply-chain management problems. Some of the research results on supply-chain management problems were reported and reviewed in Hongyan Li et al (2007) [13].

A study on the balanced allocation of customers to multiple distribution centers was found in Zhou et al (2002). However, they dealt with an un weighted
allocation of customers to distribution centers without considering the demands of different customers; and what they were to balance is the “average” shipping cost of each distribution center, which cannot lead to a true balanced workload allocation state. In this paper, a weighted sum of the demand and the shipping time is utilized to represent the delivery workload of a DC, which serves as the proxy of general workload and is embedded in the bi-level model to study the cost minimization problem [4].

Park (2005) presented a method for integrated production and distribution planning. He investigated the effectiveness of the integration through a computational study with the objective of maximizing the total net profit. This is considered one of the best production-distribution models in the literature, because it is a relatively realistic model considering multiple capacity constraints within a multi-period planning horizon. Moreover, the model involves some fixed costs at different operation stages. Having proposed a MIP model, Park then presented alternative solutions and compared them in a computational study. In addition, sensitivity analysis was carried out on capacities and fixed costs.

However, this study assumed that the plants have an unlimited storage capacity, and the firm can change the fleet size freely without extra cost, but in real operations, these assumptions are not often realized. Additionally, the model did not take changeover cost and production batch size constraints into account at the production stage.

Moreover, the solution procedures have some limitations, for example, the decoupled models do not always give feasible solutions, since they ignore the interactions of different operation stages. Although the problem considered a supply-chain network configuration, including multi-plants, multi-retailers, multi-items, and multi-period environment, the key disadvantage is that no raw material procurement activities were considered [11]. Hongyan Li et al. (2007) [13] proposed a capacity allocation problem is embedded in the bi-level model to study the cost of each distribution center, which cannot lead to a true balanced workload allocation state. In this paper, a weighted sum of the demand and the shipping time is utilized to represent the delivery workload of a DC, which serves as the proxy of general workload and is embedded in the bi-level model to study the cost minimization problem [4].

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The presented model has studied strategic planning of limited sources for the procedures of the supply chain with regard to the costs of raw materials, production, transportation, distribution, shortage, waste materials and with consideration of the limitations for capacity of limited sources.

Furthermore, with considering the purpose functions, optimization of the incomes and minimization of the costs, we have attempted to increase efficiency in the supply chain. In following, the introduced suggested model and purpose functions and limitations have been defined. Then by using multi-standard decision making, a method has been presented for combination of purpose functions and selection of optimum production strategies.

2. Proposed Model

In this model, a supply-chain network based on raw material flow, work in process flow and product flow as depicted in Fig. 1 will be considered. This complex supply chain network includes multiple suppliers, production center, contractors and distributions centers. These contractors have some abilities to supply products in process and final products. Moreover, they have direct relation with the producers (for the sale of products in the process) and distributors (for the sale of products). At the end of the model some distributors have been considered as well, that they can be regarded as representatives for the plant sale and a mediate between the producers and the customers (Fig. 1).

![Fig. 1. Supply Chain.](image-url)
The main purpose of the model is to determine the manner of optimum allocation for the sources in general supply chain with consideration of the limited capacity of the suppliers, producers, distributors and transportation with considering the lean conditions in the chain.

In this model, “production capacity” is determined considering the available time for each producer in each period and also warehouses’ capacity and capacity of each distributor as well as the constraints in dispatch and transportation.

In addition, the capacity of each producer is independent of other producers. The capacity of the producers is equal to the maximum access space and with regard to the existing demands in each time period. Moreover, the producers’ capacity is with considering the maximum raw material that each supplier can provide and available capacity for the raw materials in the warehouse is determined in each time period. In addition to the above constraints, some other factors have been considered such as changes in the production rate, the production type and the production capacity for the products in the process as well as the minimum economic capacity.

Overall, the study is based on the following assumptions:

- The final product of each department is not held for all periods. For example: all products are transported to distribution centers in each period, and product inventory incurs in distribution parts at the end of each period.
- The demands generated at each distribution center are independent of each other.
- Each product can be produced in at least one production centers, and each production center can produce at least one products.
- The distribution centers cannot supply their shortage from other distribution centers.
- The cost of waste materials in each place is allocated to the same place.

2.1 Model Formulation

Considering the presented explanations, the suggested model is defined as follows:

2.2. Parameters

Raw material: The raw material that is acquired in the supply centers.

Products: the product that is produced in the production centers and is sent to the distribution centers.

Products in the process: the products which are produced in the production centers and are sent to other production centers as raw material.

$i$: Counter of the centers for supplying raw materials ($i=1,2,\ldots,I$).

$j, p$: Counters of the production centers for products in the process and the products ($P=J, j=1,2,\ldots, J, p=1,2,\ldots, P$).

$k$: Counter of the centers for distribution of the products ($k=1,2,\ldots,K$).

$q$: Counter of the contractors ($q=1,2,\ldots,Q$).

$n$: Counter of the raw material ($n=1,2,\ldots,N$).

$m$: Counter of the products in process ($m=1,2,\ldots,M$).

$l$: Counter of the products ($l=1,2,\ldots,L$).

$d_{lk}$: The rate of demand for product $l$, in the $k$ distribution center

$p_{rl}$: The price for each unit of the product $l$.

$ctfd_{lk}$: The transportation cost for each unit of the product $l$ from the production center $j$ to the distribution center $k$.

$ctsf_{mj}$: The transportation cost for each raw material $n$ from supply center $i$ to the production center $j$.

$ctff_{mp}$: The transportation cost for each product in the process $m$ from the production center $j$ to the production center $p$.

$ctcf_{lkq}$: The transportation cost for each unit of the product $l$ purchased from the contractor $q$ by the distribution center $k$.

$ctcf_{mpq}$: The transportation cost for each product in the process $m$ purchased from the contractor $q$ by the production center $j$.

$cp_{lj}$: The cost for production of each unit of the product $l$ by the production center $j$.

$cb_{ljq}$: The cost for purchasing of each unit of the product $l$ from the contractor $q$.

$ctw_{mqp}$: The cost for purchasing of each unit of product in the process $m$, from the contractor $q$.

$cdc_{lk}$: The cost for distribution of each unit of the product $l$ in distribution center $k$.

$ca_{lj}$: The cost for setup of the production center $j$ for producing the product the $l$.

$cta_{mq}$: The cost for setup of the production center $j$ for producing the products in process $m$.

$chwsl_{lk}$: The cost for storage of each unit of the product $l$ by the distribution center $k$.

$chwfp_{mp}$: The cost for storage of each unit of products in process $m$ in $p$ production center.

$chwrs_{pq}$: The cost for storage of each unit of raw material $n$ in production center $j$.

$csh_{ljk}$: The cost for shortage of each unit of the product $l$ by the distribution center $k$.

$csu_{mq}$: The cost for obtaining each unit of raw material $n$ in supply center $i$.

$chw_{pq}$: The cost for producing each unit of products in process $m$ in the production center $j$. 
The cost for the wastes in the production center $i$ for raw material $n$.

$csvrs_{ni}$: The cost for the wastes in production centers $p$ for products in process $m$.

$csvf_{lj}$: The cost for the wastes in the production center $j$ for producing the product $l$.

$csvdp_{kl}$: The cost for the wastes in the distribution center $k$ for the product $l$.

$csvcd_{qj}$: The cost for the wastes from the contractor $q$ for the product $l$.

$csvcf_{mq}$: The cost for the wastes from the contractor $q$ for the product in the process $m$.

$rmn$: The rate of raw material $n$ that is consumed for producing the product $l$.

$rmwn$: The rate of raw material $n$ that is consumed for producing the product $m$.

$mnw$: The rate of products in process $m$ that is consumed for producing the product $l$.

$svrs_{ni}$: The cost for the wastes in obtaining raw material $n$ in supply center $i$.

$svpf_{lj}$: The cost for the wastes in producing the goods $l$ in $j$ production center.

$svwfn$: The percent of the wastes in producing the products in process $m$ in $j$ production center.

$svdp$: The percent of the wastes from distribution of the goods $l$ in the distribution center $k$.

$svlp$: The percent of the wastes from the contractor $q$ for the product $l$.

$sympc_{mq}$: The percent of the wastes from the contractor $q$ for the products in process $m$.

$tpa$: The access production time of $j$ production center.

$tsp$: The setup time of $j$ production center for producing the product $l$.

$tp$: The required time for producing the product $l$ in $j$ production center.

$tsw$: The setup time of $j$ production center for producing the products in process $m$.

$tpw$: The required time for producing the products in process $m$ in $j$ production center.

$vwht$: The capacity of warehouse for distribution center $k$.

$vwhtj$: The capacity of warehouse for production center $j$.

$vnp$: Necessary volume for warehousing each unit of the product of $l$.

$vnmw$: Necessary volume for warehousing each unit of products in process of $m$.

$vnq$: Necessary Capacity for warehousing each unit of raw material of $n$.

$M^1$: A large number that is greater than the total amount of the productions.

$M^1_{ji}$: A large number that is greater than the total amount of products in process $m$.

$M^{4_{ij}}$: A large number that is greater than the total amount of the demands for the products.

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$M^{4_{ij}}$: A large number that is greater than the total amount of he products.

$b_{ji}$: It is equal to one in case that the $j$ plant can produce the $l$ product; otherwise it is equal to zero.

$c_{mk}$: It is equal to one in case that the $j$ plant can produce the products in process of $m$; otherwise it is equal to zero.

$e_{qijk}$: It is equal to one in case that the contractor $q$ can supply $l$ product, otherwise it is equal to zero.

$e_{qijk}$: It is equal to one in case that the contractor $q$ can supply the products in process of $m$; otherwise it is equal to zero.

Variables:

$x_{lj}$: The number of producing the $l$ type product in the $j$ production center.

$X_{mk}$: The number of producing the products in process of $m$ type in the $j$ production center.

$nlpc_{lk}$: The number of product the $l$ type from the contractor $q$ for the $k$ distribution center.

$nmpe$q$mc$: The number of producing the products in process of $m$ type from the contractor $q$ for the $j$ production center.

$nsf_{mq}$: The number of delivering raw material type $n$ from supply center $i$ to the $j$ production center.

$npdy_{jk}$: The number of delivering product type $l$ from production center $j$ to the $k$ distribution center.

$npf_{mjk}$: The number of delivering the products in process of $m$ type from the $j$ production center to $p$ production center.

$ndc_{mk}$: The number of distribution of product type $l$ in distribution center $k$.

$nwhs$: The remained amount of product type $l$ in distribution center $k$ at the end of the period.

$nwhf$: The remained amount of products in process type $m$ in distribution center $p$ at the end of the period.

$nsf$: The shortage amount of product type $l$ in distribution center $k$ at the end of the period.
The remained amount of raw material type \( n \) in production center \( j \) at the end of the period.

\[ nwhrs_{nj} \]

The warehouse of products in process type \( m \) entered into production center \( j \) at the end of the period.

\[ nwhcpf_{mj} \]

\[
X_j = \begin{cases} 
1 & \text{if} \ x_j > 0 \\
0 & \text{otherwise} 
\end{cases} 
\]

\[
\bar{X}_{nj} = \begin{cases} 
1 & \text{if} \ \bar{x}_n > 0 \\
0 & \text{otherwise} 
\end{cases} 
\]

**Objective Functions:**

\[
\max Z_1 = \left( \sum_{l=1}^{L} \sum_{j=1}^{J} pr \sum_{k=1}^{K} (d_{lk} - nsh_{lk}) \right) - \\
\left( \sum_{l=1}^{L} \sum_{j=1}^{J} cP_{lj} \times x_j \right) + \\
\left( \sum_{m=1}^{M} \sum_{j=1}^{J} cW_{mj} \times \bar{X}_{nj} \right) + \\
\left( \sum_{m=1}^{M} \sum_{l=1}^{L} cdc_{ml} \times ndc_{ml} \right)
\]

The equation (1) expresses the objective function for optimization of the income and costs, Section 1 in equation (1) expresses the objective function for optimization of the income, Section 2 in equation (1) indicates the objective function for minimization of the costs related to acquiring raw materials from supply centers, producing of work in process products and products in the production centers, distribution of the products in distribution centers, and purchase of work in process products and products from the contractors, Section 3 in equation (1) indicates the objective function for minimization related to transportation costs in the whole supply chain, Section 4 in equation (1) indicates the objective function for minimization related to start up of production centers for producing of work in process products and products, Section 5 in equation (1) indicates the objective function for minimization related to raw materials, work in process products and products, storage costs, Section 6 in equation (1) indicates the objective function for minimization of costs related to shortage of products in distribution centers, Section 7 in equation (1) indicates the objective function for minimization of the costs related to the created wastes in the supply chain. The equation (2), indicates the objective function for minimization of the shortage number from each type of the products in distribution centers. The equation (3) indicates the objective function for minimization of the stored number from each type of raw materials in the production centers. The equation (4) indicates the objective function for minimization of the stored number from each type of work in the process products in production centers.
\[
\begin{align*}
\text{min } Z_{2_l} &= \left( \sum_{k=1}^{K} nsh_{k_l} \right) \quad \forall l = 1, 2, \ldots, L \\
\text{min } Z_{3_n} &= \left( \sum_{j=1}^{J} nhwr_{s_n} \right) \quad \forall n = 1, 2, \ldots, N \\
\text{min } Z_{4_m} &= \left( \sum_{p=1}^{P} nhwr_{s_p} \right) \quad \forall m = 1, 2, \ldots, K \\
\text{min } Z_{5_l} &= \left( \sum_{k=1}^{K} nhwc_{s_l} \right) \quad \forall l = 1, 2, \ldots, L \\
\text{min } Z_{6_n} &= \left( \sum_{i=1}^{I} svs_{r_n} \times \sum_{j=1}^{J} nsf_{s_n} \times \sum_{k=1}^{K} svs_{r_n} \right) \quad \forall n = 1, 2, \ldots, N \\
\text{min } Z_{7_m} &= \left( \sum_{k=1}^{K} svs_{r_k} \times \sum_{j=1}^{J} nff_{s_n} \times \sum_{k=1}^{K} nff_{s_n} \right) \quad \forall m = 1, 2, \ldots, M \\
\text{min } Z_{8_j} &= \left( \sum_{k=1}^{K} svs_{r_k} \times \sum_{j=1}^{J} nff_{s_n} \times \sum_{k=1}^{K} nff_{s_n} \right) \quad \forall j = 1, 2, \ldots, J \\
\text{min } Z_{9_l} &= \left( \sum_{k=1}^{K} svs_{r_k} \times \sum_{l=1}^{L} nfd_{r_l} \times \sum_{k=1}^{K} nfd_{r_l} \right) \quad \forall l = 1, 2, \ldots, L \\
\text{min } Z_{10_m} &= \left( \sum_{q=1}^{Q} \left( svlp_{c_q} \times \sum_{j=1}^{J} nlp_{s_q} \right) \times \sum_{p=1}^{P} mmp_{p} \right) \quad \forall m = 1, 2, \ldots, M \\
\text{min } Z_{11_l} &= \left( \sum_{q=1}^{Q} \left( svlp_{c_q} \times \sum_{k=1}^{K} nlp_{s_q} \right) \times \sum_{p=1}^{P} mmp_{p} \right) \quad \forall l = 1, 2, \ldots, L \\
\end{align*}
\]

The equation (5) indicates the objective function for minimization of the stored number from each type of products in distribution centers. The equation (6) indicates the objective function for minimization of the sent waste rate from each type of raw materials from the supply centers to producing centers. The equation (7) indicates the objective function for minimization of the sent waste rate from each type of work in process products from production centers to other producing centers. The equation (8) indicates the objective function for minimization of the sent waste rate from each type of products from production centers to distribution centers.

The equation (9) indicates the objective function for minimization of the sent waste rate from each type of products from distribution centers to all the customers. The equation (10) indicates the objective function for minimization of the purchased and sent waste rate from each type of work in process products from the contractors to all the production centers. The equation (11) indicates the objective function for minimization of the purchased and sent waste rate from each type of purchased goods from the contractors to all the distribution centers.

Constraints:

\[
\begin{align*}
\sum_{j=1}^{J} (tpq_{ij} \times x_{ij} + tsp_{ij} \times X_{ij}) + \\
\sum_{m=1}^{M} (twp_{ij} \times x_{ij} + tsp_{ij} \times X_{ij}) \leq tpa_{ij} \quad \forall j = 1, 2, \ldots, J \\
\sum_{i=1}^{I} \sum_{j=1}^{J} nfd_{r_k} \times vnp_{i} \times (1 - svp_{r_j}) + \\
\sum_{i=1}^{I} \sum_{j=1}^{J} nlp_{s_q} \times svp_{i} \times (1 - svlp_{s_q}) \leq vwhd \quad \forall k = 1, 2, \ldots, K \\
\sum_{i=1}^{I} \sum_{j=1}^{J} nsf_{s_n} \times vnr_{i} \times (1 - svs_{r_n}) + \\
\sum_{i=1}^{I} \sum_{j=1}^{J} nff_{s_n} \times vnr_{i} \times (1 - svr_{s_n}) + \\
\sum_{i=1}^{I} \sum_{j=1}^{J} nmmp_{p} \times vnr_{i} \times (1 - svmp_{s_q}) \leq vwhf_{ij} \quad \forall j = 1, 2, \ldots, J
\end{align*}
\]
\[ x_{ij} \leq M \cdot x_{ij} \cdot b_{ij} \quad \forall l = 1,2,\ldots,L \& \forall j = 1,2,\ldots,J \] (15)

\[ \bar{x}_{mj} \leq M \cdot \sum_{l=1}^{L} c_{mj} \cdot x_{lj} \cdot b_{lj} \quad \forall m = 1,2,\ldots,M \& \forall j = 1,2,\ldots,J \] (16)

\[ \left( \sum_{j=1}^{J} nf_{d_{jk}} \cdot (1 - svf_{gj}) + \sum_{q=1}^{Q} nlp_{c_{qk}} \cdot (1 - svlpc_{nj}) \right) - \frac{d_{jk}}{(1 - spd_{jk})} \right) = nwhs_{jk} - nsh_{jk} \quad \forall l = 1,\ldots,L \& \forall k = 1,\ldots,K \] (17)

\[ ndc_{jk} \cdot (1 - svpd_{jk}) = d_{jk} - nsh_{jk} \quad \forall l = 1,\ldots,L \& \forall k = 1,\ldots,K \] (18)

\[ \sum_{k=1}^{K} nf_{d_{jk}} = x_{ij} \quad \forall l = 1,2,\ldots,L \& \forall j = 1,2,\ldots,J \] (20)

\[ nwhs_{nj} = \left( -1 \right) \cdot \left( \sum_{l=1}^{L} \sum_{m=1}^{M} r_{m,ij} \cdot x_{lj} \cdot b_{lj} + \sum_{m=1}^{M} \sum_{n=1}^{N} rw_{n,ij} \cdot \bar{x}_{nj} \right) \]

\[ + \sum_{l=1}^{L} nsl_{j} \cdot (1 - svs_{nj}) \right) \quad \forall n = 1,2,\ldots,N \& \forall j = 1,2,\ldots,J \] (21)

\[ nwhc_{mj} = \left( \sum_{q=1}^{Q} nmpc_{q,j} \cdot (1 - smpc_{nj}) + \sum_{p=1}^{P} nmp_{m,j} \cdot (1 - svmpc_{nj}) \right) \]

\[ \forall m = 1,2,\ldots,M \& \forall j = 1,2,\ldots,J \] (22)

\[ \bar{x}_{mj} = nwhf_{mj} + \sum_{l=1}^{L} nwf_{ml} \cdot x_{lj} + \sum_{p=1}^{P} nff_{mp} \]

\[ \forall m = 1,2,\ldots,M \& \forall j = 1,2,\ldots,J \] (23)

\[ \sum_{j=1}^{J} nlp_{c_{qk}} \leq M \cdot c_{q} \quad \forall l = 1,2,\ldots,L \& \forall q = 1,2,\ldots,Q \] (24)

\[ \sum_{j=1}^{J} nmp_{m,j} \leq M \cdot f_{m} \quad \forall m = 1,2,\ldots,M \& \forall q = 1,2,\ldots,Q \] (25)

The relation (12) indicates the limitation related to access time in each plant. The relation (13) indicates the constraints related to the capacity of warehouse volume in each distribution center. The relation (14) indicates the constraints related to the capacity of warehouse volume in each production center. The relation (15) indicates one logical limitation related to the production or non-production of one product in a production center. The relation (16) indicates one logical limitation related to the production or non-production of one work in process products in a production center. The equation (17) indicates logical constraints for calculating the shortage and remained amounts for each product in each distribution center. The equation (18) indicates one logical limitation for calculating the sent amounts from each product to the customer in each distribution center. The relation (19) indicates controlling constraints for optimal performance of the constraints in the equation (17). The equation (20) indicates one logical limitation for determining the production rate and number of sent amounts from each product and from each production center to the distribution centers. The equation (21) indicates logical constraints for calculating the remained amounts from each raw material in each production center. The equation (22) indicates one virtual warehouse for each type of work in process products purchased from the contractors and received from other production centers and their maintenance for the next period during all the time periods. The equation (23) indicates logical constraints for calculating the remained amounts from each type of work in process products at each production center. The relation (24) indicates the limitation related to the possibility or non possibility for supply of each product by each contractor for the distribution centers. The relation (25) indicates the limitation related to the possibility or non possibility for supply of each work in process product by each contractor for the production centers. The main purpose of the presented model can be the following cases:

- Calculation of the optimum allocation of the limited sources in the integrated supply chain.
- Directing the supply chain toward the lean considering the equation (2 to 11).
- Obtaining the rate of optimum production of each product in every production center.
- Obtaining the rate of optimum production of each work in process product in every production center.
- Obtaining the rate of optimum purchase of each work in process product in every contractor center for every production center.
- Obtaining the rate of optimum purchase of each product in every contractor center for every distribution center.
• Obtaining the rate of optimum delivery of each raw material from every supply center to every production center.
• Obtaining the rate of optimum delivery of each product, from every production center to every distribution center.
• Obtaining the rate of optimum delivery of each semi-made material from every production center to every other production center.
• Obtaining the rate of optimum distribution and delivery of each product from every distribution center to every customer.
• Obtaining the rate of optimum storage of each product in every distribution center.
• Obtaining the rate of optimum storage of each work in process product in every production center.
• Obtaining the rate of optimum storage of each raw material at every production center.
• Optimum production programming
• Controlling the inventory for reducing the costs
• Selecting the suppliers, producers and contractors as well as their evaluation
• Considering the factor of inflation and increasing the price in the programming
• And finally very high efficiency for making the optimum decision related to one multi-stage supply chain

3. The Proper Strategies for SUPPLY Produce and Distribute

In this paper, the optimal solution is carried out by the multi-criteria decision-making procedure (ELECTRE). The ELECTRE method has recognized in both qualitative and quantitative criteria. When a set of alternatives must be ranked according to a set of criteria reflecting the decision maker’s preferences, ELECTRE as a multi-criteria decision-making can be applied. There are two main parts to an ELECTRE application: first, the construction of one or several outranking relations, which aims at comparing in a comprehensive way each pair of actions; second, an exploitation procedure that elaborates on the recommendations obtained in the first phase.

Relationships between alternatives and criteria are described using attributes concerned to the characteristics of alternatives that are relevant according to the established criteria. In multi-criteria decision problems, although logical and mathematical conditions required to determine an optimum do not exist, a solution representing a good compromise according to the conflicting criteria established can be individuated. ELECTRE method is based upon pseudo-criteria. A pseudo-criterion allows, by using proper thresholds, to take into account the uncertainty and ambiguity that can affect the evaluation of the performance, so that, if the difference in the performance of two alternatives is minimal, according to a certain criterion, such as alternatives can be considered indifferent according to that criterion. Another peculiarity which differentiates ELECTRE from other methodologies is that it is not compensative, which means that a very bad score in one objective function is not compensated by good scores in other objectives. In other words, the decision maker will not choose an alternative if it is very bad compared to another one, even on a single criterion. This occurs if the difference between the values of an attribute of two alternatives is greater than a fixed veto threshold [1].

ELECTRE is based upon outranking relations: an alternative \( a \) outranks another alternative \( b \) if sufficient reasons exist to assert that \( a \) is as good as \( b \) and good reasons to reject such assertion do not exist. Outranking is therefore based upon a concordance/discordance principle, which consists in the verification of the existence of a concordance of criteria in favor of the assertion that an alternative is as good as another one, and upon the verification that strong discordance among the score values that may reject the previous assertion does not exist [12].

For each criterion, the following thresholds are introduced:
- \( q_j \) : Indifference threshold,
- \( p_j \) : Preference threshold,
- \( v_j \) : veto threshold.

Where: \( q_j \leq p_j \leq v_j \).

By these thresholds, the following six preference relations between alternatives \( a \) and \( b \) may be established, referring to the values \( g_j(a) \) and \( g_j(b) \) of the attribute \( j \):

1. \( (a I b) \) : \( a \) is indifferent to \( b \) with respect to the criterion \( j \) if \( |g_j(a) - g_j(b)| \leq q_j \).
2. \( (a WP b) \) : \( a \) is weakly preferred to \( b \) with respect to the criterion \( j \) if \( q_j \leq g_j(a) - g_j(b) \leq p_j \).
3. \( (a SP b) \) : \( a \) is strongly preferred to \( b \) with respect to the criterion \( j \) if \( g_j(a) - g_j(b) \geq p_j \).
4. \( (a NR b) \) : the assertion that \( a \) outranks \( b \) cannot be refused with respect to the criterion \( j \) if \( g_j(b) - g_j(a) \leq v_j \).
5. \(a \,	extit{WR} \, b\): the assertion that \(a\) outranks \(b\) is weakly refused with respect to the criterion \(j\) if \(p_j \leq g_j(b) - g_j(a) \leq v_j\).

6. \(a \,	extit{SR} \, b\): the assertion that \(a\) outranks \(b\) is strongly refused with respect to the criterion \(j\) if \(g_j(b) - g_j(a) \geq v_j\).

For each criterion, thresholds \((q_j, p_j, v_j)\) can either be fixed values or functions of the performance, according to the expression (26).

\[ s_j(a) = a_j g_j(a) + \beta_j \]  

The previous equations (1)–(3) are named “concordance” equations and are used to evaluate the reasons favorable to the assertion that alternative \(a\) outranks alternative \(b\), according to criterion \(j\). Expressions (4)–(6) are named “discordance” expressions and are used to measure the strong reasons that lead to reject the assertion that \(a\) outranks \(b\) with respect to criterion \(j\).

Suppose that:

- \(Str_r\): The \(r\) production strategy, that satisfy the constrains from 12 to 25. (\(r = 1,2,\ldots,R\)).
- \(ZStr_r\): The amount of objective function for production strategy \(r\) (. \(r = 1,2,\ldots,R \& s = 1,2,\ldots,11\)).
- \(c_s(Str_a, Str_b)\): With considering the \(s\) index, the \(Str_a\) production strategy is preferred to the \(Str_b\).
- \(d_s(Str_a, Str_b)\): With considering the \(s\) index, the \(Str_a\) production strategy isn’t any preference to the \(Str_b\).
- \(C(Str_a, Str_b)\): The matrix for preference the production strategy \(Str_a\) to the \(Str_b\).
- \(S(Str_a, Str_b)\): The preference credibility of the \(Str_a\) production strategy than \(Str_b\) for all indexes.

Concordance, indicated by \(c_s(Str_a, Str_b)\), is equal to 1 if \(ZStr_a\) is greater than \(ZStr_b\) or, in any case, expression (1) is verified, is equal to 0 if \(ZStr_{ab} - ZStr_{sa} \geq p_s\), while it is evaluated by the equation (27) when \(q_s \leq ZStr_{ab} - ZStr_{sa} \leq p_s\):

\[ c_s(Str_a, Str_b) = \frac{p_j + ZStr_{sa} - ZStr_{ab}}{p_s - q_s} \]  

Discordance, indicated with \(d_s(Str_a, Str_b)\), is 0 when expression (4) is verified, 1 when expression (6) is verified, while it is expressed by the equation (28) when expression (5) is verified:

\[ d_s(Str_a, Str_b) = \frac{ZStr_{ab} - ZStr_{sa} - p_s}{v_s - p_s} \]  

For each pair of strategies \(Str_a\) and \(Str_b\), the values of concordance \(c_s(Str_a, Str_b)\) with respect to each criterion \(s\), are aggregated in the global concordance matrix, by means of a weight \(k_s\) assigned to each criterion. The generic element of such a matrix is expressed by equation (29):

\[ C(a,b) = \sum s k_s \times c_s(Str_a, Str_b) \]  

A further step consists in the definition of the credibility of “\(Str_a\) outranks \(Str_b\)”, that summarizes the information expressed by concordance and discordance:

\[ d_s(Str_a, Stg) \leq C(Str_a, Stg) \land \forall s \]  

\[ S(Str_a, Stg) = \left\{ \begin{array}{ll} C(Str_a, Stg) & \text{if } d_s(Str_a, Stg) \leq C(Str_a, Stg) \\ \prod_{(Str_a,Stg) \leq C(Str_a, Stg)} 1 - d_s(Str_a, Stg) & \text{otherwise} \end{array} \right. \]  

The next step of the method is the so-called descending distillation: based on the credibility parameter, the strategies are ranked in descending order.

A further threshold is considered (equation (31)):

\[ \hat{\lambda} = \max_{\forall s \neq Str_a, Str_b} S(Str_a, Str_b) \]  

A credibility level \(\hat{\lambda}'\), less but close to \(\hat{\lambda}\), is established so that the interval \((\hat{\lambda} - \hat{\lambda}')\) can be considered as an indifference interval of credibility. A Boolean matrix is then calculated as equation (32):

\[ B(Str_a, Str_b) = \begin{cases} 1 & \forall Str_a, Str_b | S(Str_a, Str_b) > \lambda' \\ 0 & \text{otherwise} \end{cases} \]  

Finally, for each \(Str_a\), the difference \(Q(Str_a)\) between the number of \(Str_{j}\) that are outranked by \(Str_a\) at level \(\lambda'\) or higher (i.e. the \(Str_{j}\) having \(B(Str_j, Str_a) = 1\)) and the number of \(Str_{k}\) that outrank the \(Str_a\) at level \(\lambda'\) or higher (i.e. the \(Str_{k}\) having \(B(Str_a, Str_k) = 1\)), is calculated. The first distillates are the strategy \(Str_a\) having:

\[ Q(Str_a) = \max_{\forall b} Q(Str_b) \]
If the set containing all the strategies, for which the previous equation is verified, has a cardinality higher than 1, the described procedure is applied recursively until the set contains only one strategy or a group of strategies that cannot be differentiated further. In this last case, an ascending distillation can be applied, ranking the strategies in ascending order. This new ranking, coupled with that obtained by descending distillation, leads to a unique final ranking. Among the different versions of the ELECTRE method, ELECTRE III [1] has been employed.

4. Conclusion

In this paper, a model was presented for strategic planning of a multiple supply chain. The main purpose of the model is obtaining optimum strategies in the supply chain with considering purpose functions for minimization of raw materials costs, production, transportation, distribution, shortage and losses, optimization of the incomes, and minimization of the shortage and losses. In this model, the limitations of capacity for provision, production, distribution, transportation and stores, minimum economic volume of production and lean have been considered. The important point is this that in addition to considering the functions that directs the strategies of the supply chain toward lean; simultaneously lean of the model has been guaranteed using the 6-sigma statistical tool. The integrated approach of the model to the whole supply chain is a major privilege of the presented model in comparison with other methods. Finally, optimum strategies of provision, production, distribution, transportation, stores and selection of the suppliers were selected by using the procedure for multi-standard decision making.

References


